A tournament model: the manager's behavior in the presence of reputation concerns

Mark Treurniet

Erasmus School of Economics

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Abstract

This paper studies the acquisition and use of information by a manager who has to award a bonus in order to incentivize his employees. We analyze two ways of information acquisition and show how reputation concerns may improve the manager's effort to collect information. Moreover, we show how they may lead to distorted decisions.

Keywords: tournament, endogenous manager, reputation concerns, information acquisition

1 Introduction

Generally, interests of an organization and its employee are not perfectly aligned. The employee exerts costly effort, but the organization takes the benefits. In order to properly incentivize the employee to exert an optimal amount of effort, the employee's reward is often based on his output. The more the employee produces, the higher is the compensation that he receives for exerting effort. However, if compensation is based on output, measuring output is essential.

Since measuring output of individual employees may be hard and costly, Lazear and Rosen (1981) suggest that compensation can be based on relative performance rather than actual output. The key idea of their tournament theory is that the most productive employee gets a bonus. Since output is increasing in effort, workers are motivated to exert effort in order to increase their probability of getting the bonus.

However, Lazear and Rosen simply assume that the most productive employee gets the bonus. In this paper we endogenize and analyze the behavior of the manager who (i) has to collect information on the relative performance of his employees and (ii) has to decide on the award of the bonus.

The manager may have personal preferences that distract him from choosing the optimal alternative from the perspective of the organization. Prendergast and Topel (1996) argue that firms are social institutions where personal relations may play an important role: the manager may award the bonus to the employee he favors. In this paper, we argue that besides this conventional favoritism reputation concerns can be another reason why a manager may decide to distort the award of the bonus.

Our research intends to investigate under what conditions reputation concerns encourage the manager to collect valuable information. Furthermore, we examine whether reputation concerns may incentivize the manager to distort his decision on the award of the bonus.

To this end, we extend the model of Lazear and Rosen with a manager who has to award the bonus. In order to motivate the employees, the most productive employee should get the bonus. If a capable manager exerts effort, he increases the probability that he learns which employee is most productive. The way in which effort affects the manager's beliefs about the productivity of his employees depends on the information acquisition technology. We distinguish between two different information acquisition technologies, which we distract from Swank and Visser (2011).

In case of Perception Technology, the manager always gets a perception about which employee is most productive. If the manager is more able and increases his effort, the probability that his perception is correct increases. Since the manager always gets a perception, informative and uninformative signals are indistinguishable. Therefore, the manager generally does not know whether the signal is informative or not. He only knows the likelihood that a certain employee is most productive. As an example, we can think of a manager who has to judge the performance of a teacher: the more classes a high-able manager visits and the more he discusses with the teacher and his students, the better his perception about the teacher's performance will be.

In case of Conclusive Evidence Technology, the manager either finds conclusive evidence indicating that a certain employee is most productive or he finds no evidence at all. If the manager is more able and increases his effort, the probability that the manager finds conclusive evidence increases. We can, for example, think of a manager in a hospital that has to evaluate performance of cleaners who have to remove some bacteria. Once he chemically tests the presence of a certain bacterium, he knows with certainty whether this bacterium has persisted or not.

After the manager has received a signal about which employee is most productive, he awards a bonus to one of the employees.

Our results show that the manager's competence plays a crucial role in motivating employees. If the employees know in advance that the manager is unable to assess which employee is most productive, they will not be motivated to exert effort.

Furthermore, this paper provides new insights in the implications of reputation concerns under both information acquisition technologies. Swank and Visser suggest that reputational concerns are of less interest under the Perception Technology than under the Conclusive Evidence Technology. However, we have reasons to change some critical assumptions and find totally different results.

Swank and Visser assume that under the Perception Technology both alternatives are equally likely to be signalled to the manager. This assumption ensures that the likelihood of getting a certain perception is independent from the ability and effort level of the manager. Therefore, reputation concerns do not induce the manager to exert more effort or distort his decision. However, if a certain employee is more experienced and has more knowledge about the manager and his productivity tests, he may be more capable to influence the manager's perception about the relative productivity, especially if the manager is low-able or does not exert effort in order to become well informed. Therefore, we relax this crucial assumption, by stating that the manager's perception may be biased in favor of one of the employees. We show that if the uninformative signal is biased against the less experienced employee, a high-able manager is more likely to find this employee most productive than a low-able manager. Therefore, awarding the bonus to this employee yields a higher reputation. Consequently, the manager may increase his effort in order to increase the probability of getting an informative signal indicating that the less experienced employee is most productive. Eventually, he may even distort the award of the bonus in favor of the less experienced employee.

In contrast with Swank and Visser, we do not have a reason to expect that the organization is in advance biased against one of the alternatives. The firm does not prefer one of the employees to get the bonus if no evidence is found. Therefore, we show that if the manager finds no evidence, he can mix in such a way that both decisions yield equal reputations. Consequently, under the Conclusive Evidence Technology reputation concerns do not motivate the manager to exert more effort or to distort the decision.

Another main contribution of this paper is that it adds an explanation for distortion in the award of bonuses by managers besides conventional favoritism. Since more experienced employees may be more capable to influence the perception of the manager in their favor and since low-able managers are more sensitive for this kind of interference, the manager may distort the award of the bonus in order to improve his reputation.

2 Model

A manager has to award a bonus $B > 0$ to one of his employees $i \in \{1, 2\}$. We denote the decision as $X \in \{1, 2\}$. The employees work for the firm and exert effort in order to produce output.

The employees value getting the bonus and assign costs to exerting effort. More precisely, the utility of the employee i equals:

$$U_i = I_{\{X=i\}} B - c_i(e_i)$$

where

$$I_{\{X=i\}} = \begin{cases} 1, & \text{if } X = i \\ 0, & \text{if } X \neq i \end{cases}$$

indicates whether employee i received a bonus and

$$c_i(e_i) = \frac{1}{2}\theta e_i^2$$

denotes the employee's cost of effort function.

Throughout this paper, we assume that the participation constraint is met, i.e. the employees want to work for the firm, and consider the bonus B as given.

The employee's effort e_i is unobservable for the firm and its manager. However, effort influences output and the manager can obtain information about which employee is most productive. Based on the model of Lazear and Rosen (1981), we assume that, for $e_i \approx e_j$, the probability that employee i is more productive than employee j equals[1]:

$$\Pr(q_i > q_j) = \frac{1}{2} + \varphi(e_i - e_j) \tag{1}$$

The parameter φ can be interpreted as a measure of uncertainty. If φ is high, then effort has a big impact on which employee is most productive. However, if φ is low, luck plays a bigger role.

Note that since the employees have symmetric preferences, they will exert equal effort in equilibrium and thus, their probability to be the most productive employee equals $\frac{1}{2}$ in equilibrium.

The ability of the manager a is either high H or low L. The prior probability that managers are high-able equals: $\Pr(a = H) = \pi \in (0, 1)$. At the beginning of the game, none of the players has any more information about the ability of the manager.

Before deciding which employee to award the bonus, the manager can exert effort e_M to assess which employee is most productive. The manager receives a signal s about which employee is most productive. The signal and its meaning depend on the information acquisition technology, which we distract from Swank and Visser (2012).

1. In case of the Perception Technology, the signal $s \in \{1, 2\}$ constitutes a perception of which employee is most productive. If managers are high-able, more effort increases the

[1] We can derive this probability function more formally: suppose that the output of the employees depends on both effort and random factors. More precisely, assume that the output of employee i equals: $q_i = e_i + \varepsilon_i$, where ε_i is a random term with zero expectation. Then, the probability that employee i is more productive than employee j equals:

$$\Pr(q_i > q_j) = \Pr(e_i + \varepsilon_i > e_j + \varepsilon_j) = \Pr(\varepsilon_i - \varepsilon_j > e_j - e_i)$$

Let $G(\cdot)$ be the distribution function and $g(\cdot)$ be the density function of $\varepsilon_i - \varepsilon_j$. Then, the probability that employee i is more productive than employee j equals $G(0)$ if both employees exert equal effort. Since employees are similar, we assume this probability to be $\frac{1}{2}$. Furthermore, the derivative of this probability with respect to the effort of employee i equals:

$$\frac{\partial \Pr(q_i > q_j)}{\partial e_i} = g(0) \equiv \varphi$$

From the properties of the density function follows that $\varphi > 0$. Now, for $e_i \approx e_j$, the probability that employee i is more productive than employee j reduces to (1).

probability that the perception is correct.

Depending on their experience in the firm, the employees may differ in their knowledge about the tests that the manager may perform in order to assess which employee is most productive. If one of the employees knows more about these tests, then he may be more capable to influence the tests. In that case, the probability that the manager gets an uninformative signal indicating that this employee is most productive exceeds the prior probability that he is the most productive employee. Therefore, we relax the assumption of Swank and Visser by allowing that the probability distribution of an uninformative signal differs from the actual probability distribution, which means that the manager's effort affects the likelihood of getting a certain perception. Without loss of generality, we assume that the probability that the manager gets an uninformative signal $s = 1$ is non-strictly smaller than the probability that employee 1 is more productive than employee 2.

In particular, the conditional probability distribution of getting a certain signal can be represented as follows:

$$\begin{aligned}
\Pr\left(s = 1 | q_1, q_2, a = L, e_M\right) &= \alpha \\
\Pr\left(s = 1 | q_1 > q_2, a = H, e_M\right) &= \alpha + (1 - \alpha) e_M \\
\Pr\left(s = 2 | q_2 > q_1, a = H, e_M\right) &= (1 - \alpha) + \alpha e_M
\end{aligned}$$

Hence, for $e_i \approx e_j$, the unconditional probability of getting signal $s = 1$ is:

$$\Pr\left(s = 1 | e_M\right) = \alpha + \pi\left[\Pr\left(q_1 > q_2\right) - \alpha\right] e_M = \alpha + \pi\left[\frac{1}{2} + \varphi\left(e_1 - e_2\right) - \alpha\right] e_M$$

Recall that since employees have symmetric preferences, they will exert equal effort in equilibrium. Hence, in equilibrium the latter probability equals:

$$\Pr\left(s = 1 | e_M\right) = \alpha + \pi\left(\frac{1}{2} - \alpha\right) e_M$$

Clearly, the unconditional likelihood of getting signal $s = 1$ depends on the manager's ability and increases in his effort if $\alpha < \frac{1}{2}$. However, if $\alpha = \frac{1}{2}$, the unconditional likelihood of receiving signal $s = 1$ is constant and independent from the manager's effort and ability.

The manager's cost of effort function $c_M\left(e_M\right)$ is strictly convex and satisfies $c_M\left(0\right) = 0$,

$c_M'(0) = 0$ and $c_M'(1) = \infty$.

2. In case of the Conclusive Evidence Technology, the signal $s \in \{1, 2, \varnothing\}$ either reveals which employee is most productive with certainty ($s \in \{1, 2\}$) or signals that no conclusive evidence is found ($s = \varnothing$). Incorrect information is never obtained. A low-able manager will never find evidence, whereas a high-able manager's probability of finding evidence is increasing in his effort.

In particular, the conditional probability distribution of getting a certain signal can be represented as follows:

$$\begin{aligned}
\Pr(s = \varnothing | q_1, q_2, a = L, e_M) &= 1 \\
\Pr(s = i | q_i > q_j, a = H, e_M) &= e_M \\
\Pr(s = \varnothing | q_i > q_j, a = H, e_M) &= 1 - e_M
\end{aligned}$$

The manager's cost of effort function $c_M(e_M)$ is strictly convex and satisfies $c_M(0) = 0$, $c_M'(0) = 0$ and $c_M'(1) = \infty$.

After the manager exerted effort and received a signal about which employee is most productive, the manager has to decide which employee to award the bonus. A decision strategy σ is a function that translates the managers effort and his signal into the probability that employee 1 receives the bonus: $\sigma(e_M, s) = \Pr(X = 1 | e_M, s)$.

The manager wants his employees to be productive. Therefore, he values the output of his employees. Secondly, the manager may have internal motivation to take the right decision. Moreover, the current decision is likely to influence future output. This will especially be the case if (i) productive employees are more likely to be productive in the future as well and (ii) employees are more likely to stay within the firm if they receive the bonus. Thus, the manager likes to take the right decision, i.e. to assign the bonus to the most productive employee. Thirdly, the manager cares about his reputation. He wants to be seen as capable by his boss, since this may influence his career at his current employer or, via references, at other firms. Finally, the manager assigns costs to his own effort. More precisely, the utility of the manager equals:

$$U_M = \kappa(q_1 + q_2) + \rho\widehat{\omega}(X = i | s, e_M) + \lambda\widehat{\pi}(X = i | e_M^*, \sigma) - c_M(e_M)$$

where

$$\widehat{\omega}\left(X=i|s=k,e_M\right)\equiv \Pr\left(q_i>q_j|s=k,e_M\right)=\frac{\Pr\left(s=k|q_i>q_j,e_M\right)}{\Pr\left(s=k|e_M\right)}\frac{1}{2}$$

denotes the probability that the most productive employee receives the bonus conditional on the signal,

$$\widehat{\pi}\left(X=i|e_M^*,\sigma\right)\equiv \Pr\left(a=H|X=i,e_M^*,\sigma\right)=\frac{\Pr\left(X=i|a=H,e_M^*,\sigma\right)}{\Pr\left(X=i|e_M^*,\sigma\right)}\pi$$

represents the manager's reputation, which is the posterior probability that he is high-able conditional on the manager's decision, and $c_M\left(e_M\right)$ is the cost of effort function of the manager.

We define the reputational gap as the difference in reputation of awarding the bonus to employee 1 and awarding the bonus to employee 2: $\widehat{\pi}\left(X=1|e_M^*,\sigma\right)-\widehat{\pi}\left(X=2|e_M^*,\sigma\right)$.

The timing of the model is as follows:

1. Nature determines the manager's ability a.
2. The manager chooses his effort e_M.
3. The employees observe the manager's effort e_M and choose their effort level e_i.
4. The manager receives a signal s about which employee is most productive and chooses which employee receives the bonus X.
5. The boss observes the manager's decision X and updates his beliefs about the manager's ability.
6. Payoffs are realized.

Our model is a dynamic game with incomplete information. In equilibrium, (i) the boss optimally uses information, i.e. his beliefs about the manager's ability must be obtained by Bayes' rule whenever possible, and his beliefs must coincide with the manager's effort and decision strategy, (ii) given the boss' beliefs and what is decided before (the manager's effort and the employees' effort), the decision strategy maximizes the manager's payoff, (iii) given what is decided before (the manager's effort) and what will be decided later (the decision strategy), the employees' effort maximize their payoff and (iv) given the boss' beliefs and what will be decided later (the employees' effort and the decision strategy), the manager's effort maximizes his payoff.

3 Results

We define an undistorted decision strategy as a decision strategy in which the manager maximizes the probability that the most productive employee receives the bonus given the manager's signal. This means that the decision strategy maximizes the incentives to exert effort for the employees given the manager's effort.

3.1 Perception Technology

3.1.1 Undistorted decision strategy

Under the Perception Technology, the undistorted decision strategy σ_U means that the manager always follows his signal: $\sigma_U(e_M, s=1) = 1$ and $\sigma_U(e_M, s=2) = 0$.

Given the undistorted strategy σ_U, Bayes' rule implies:

$$\begin{aligned}
\widehat{\pi}\left(X=1|e_M^*,\sigma_U\right) &= \frac{\alpha+\left(\frac{1}{2}-\alpha\right)e_M^*}{\alpha+\pi\left(\frac{1}{2}-\alpha\right)e_M^*}\pi \\
\widehat{\pi}\left(X=2|e_M^*,\sigma_U\right) &= \frac{(1-\alpha)-\left(\frac{1}{2}-\alpha\right)e_M^*}{(1-\alpha)-\pi\left(\frac{1}{2}-\alpha\right)e_M^*}\pi
\end{aligned} \tag{2}$$

Note that if the likelihood of receiving a certain signal is constant and independent from the manager's effort and ability ($\alpha=\frac{1}{2}$), both types of managers award the bonus to employee 1 with equal probability. Therefore, the posterior probability that the manager is high-able is independent from which employee receives the bonus, so both decisions will hold the same reputation: $\widehat{\pi}\left(X=1|e_M^*,\sigma_U\right)=\widehat{\pi}\left(X=2|e_M^*,\sigma_U\right)$. However, if $\alpha<\frac{1}{2}$, a high-able manager is more likely to get signal 1 and hence, awarding the bonus to employee 1 yields a higher reputation than awarding the bonus to employee 2: $\widehat{\pi}\left(X=1|e_M^*,\sigma_U\right)>\widehat{\pi}\left(X=2|e_M^*,\sigma_U\right)$.

Given the undistorted decision strategy σ_U, the expected payoff of each employee i when choosing effort equals:

$$EU_i = \Pr\left(s = i|e_M\right) B - \frac{1}{2}\theta e_i^2$$

Differentiating the above expression with respect to e_i yields:

$$e_i^* = \frac{\pi\varphi e_M B}{\theta} \tag{3}$$

This equation explicitly defines the employees' equilibrium effort e_i^* as a function of π, φ, e_M, B and θ. Clearly, both π, φ, e_M and B increase the employee's marginal benefit of effort and therefore, the employee's effort. θ increases the marginal cost of effort and hence, decreases the employee's effort.

Furthermore, the manager's expected payoff when choosing effort equals:

$$\begin{aligned} EU_M \quad = \quad & \kappa\left(e_1^* + e_2^*\right) + \Pr\left(s = 1|e_M\right)\rho\widehat{\omega}\left(X = 1|s = 1, e_M\right) \\ & + \Pr\left(s = 2|e_M\right)\rho\widehat{\omega}\left(X = 2|s = 2, e_M\right) \\ & + \Pr\left(s = 1|e_M\right)\lambda\widehat{\pi}\left(X = 1|e_M^*, \sigma_U\right) \\ & + \Pr\left(s = 2|e_M\right)\lambda\widehat{\pi}\left(X = 2|e_M^*, \sigma_U\right) - c_M\left(e_M\right) \end{aligned}$$

Differentiating the above expression with respect to e_M yields the first-order condition[2]:

$$\frac{2\kappa\pi\varphi B}{\theta} + \rho\pi\frac{1}{2} + \pi\left(\frac{1}{2} - \alpha\right)\lambda\left[\widehat{\pi}\left(X = 1|e_M^*, \sigma_U\right) - \widehat{\pi}\left(X = 2|e_M^*, \sigma_U\right)\right] = c_M'\left(e_M^*\right) \tag{4}$$

This equation implicitly defines the manager's equilibrium effort e_M^* as a function of κ, π, φ, B, θ, ρ, α and λ. Clearly, the wish to have productive employees κ and the wish to make the right decision ρ increase the manager's effort. If $\alpha = \frac{1}{2}$, effort does not increase the probability of taking a certain decision and both decisions yield equal reputations, so the wish to come across as able λ does not affect the manager's effort. However, if $\alpha < \frac{1}{2}$, the manager's effort increases the probability of getting the higher reputation and hence, the wish to come across as able λ does increase the manager's effort. The marginal effect of the manager's effort on the probability of getting the higher reputation increases in the distance between α and $\frac{1}{2}$. Therefore, the manager's effort increases in this distance.

[2]For the details of the derivation, we refer to the Appendix.

The effect of the prior probability of being a high-able manager π on the manager's effort can either be positive or negative, depending on the value of π. The prior probability of being a high-able manager π increases the manager's effort, since (i) the employees exert more effort, since their effort is more likely to affect getting the bonus, (ii) the manager's effort faster increases the probability of making the right decision, (iii) if $\alpha < \frac{1}{2}$, the manager's effort faster increases the probability of getting the higher reputation and (iv) if $\alpha < \frac{1}{2}$ and π is low, the reputational gap is bigger. However, if $\alpha < \frac{1}{2}$ and π is high, then the reputational gap decreases in π. Therefore, the total effect of π on the manager's effort is positive for low π and may be negative for high π if $\alpha < \frac{1}{2}$.

The parameter φ and the bonus B increase the manager's effort, since it increases the marginal benefit on the employee's effort. Likewise, the employee's cost of effort parameter θ decreases the manager's effort.

Finally, note that if the manager's effort increases, the reputational gap will increase. As a result, the manager's effort will increase even further.

So far we have assumed that the manager chooses $X = 1$ if and only if $s = 1$. However, we derived that if $\alpha < \frac{1}{2}$, awarding the bonus to employee 1 yields a higher reputation than awarding the bonus to employee 2. Therefore, if the wish to come across as able λ is big, the manager becomes tempted to choose $X = 1$ if $s = 2$. Consequently, the undistorted strategy is part of an equilibrium if and only if $\rho\widehat{\omega}\left(X = 1|s = 2, e_M\right) + \lambda\widehat{\pi}\left(X = 1|e_M^*, \sigma_U\right) \leq \rho\widehat{\omega}\left(X = 2|s = 2, e_M\right) + \lambda\widehat{\pi}\left(X = 2|e_M^*, \sigma_U\right)$ or:

$$\lambda \leq \frac{\rho\left[\widehat{\omega}\left(X = 2|s = 2, e_M\right) - \widehat{\omega}\left(X = 1|s = 2, e_M\right)\right]}{\widehat{\pi}\left(X = 1|e_M^*, \sigma_U\right) - \widehat{\pi}\left(X = 2|e_M^*, \sigma_U\right)} \equiv \overline{\lambda} \tag{5}$$

Proposition 1 summarizes the above discussion.

Proposition 1 *Consider an agent with a Perception Technology. Suppose that $\lambda \leq \overline{\lambda}$, where $\overline{\lambda}$ is given by (5). Then, a unique equilibrium exists in which*
(i) the boss' beliefs about the manager's ability are given by (2);
(ii) the manager always follows his signal: $\sigma_U\left(e_M, s = 1\right) = 1$ and $\sigma_U\left(e_M, s = 2\right) = 0$;
(iii) the employees' effort levels e_i^* are given by (3) and
(iv) the manager's effort level e_M^* satisfies (4).
If $\alpha = \frac{1}{2}$, then the manager's effort level and thus, also the employees' effort levels are

independent *from the reputation concerns* λ*. However, if* $\alpha < \frac{1}{2}$*, then the manager's effort level and thus, also the employees' effort levels are* ***increasing*** *in the reputation concerns* λ*.*

3.1.2 Distorted decision strategy

Now suppose that $\alpha < \frac{1}{2}$ and $\lambda > \overline{\lambda}$. As shown above, the manager prefers awarding the bonus to employee 1 over awarding the bonus to employee 2 and an equilibrium in mixed strategies exists. The distorted decision strategy σ_D is given by: $\sigma_D(e_M, s=1) = 1$ and $\sigma_D(e_M, s=2) = \gamma$, where γ is implicitly defined by $\rho\widehat{\omega}(X=1|s=2,e_M) + \lambda\widehat{\pi}(X=1|e_M^*,\sigma_D) = \rho\widehat{\omega}(X=2|s=2,e_M) + \lambda\widehat{\pi}(X=2|e_M^*,\sigma_D)$ or:

$$\begin{aligned} \lambda\left[\widehat{\pi}(X=1|e_M^*,\sigma_D) - \widehat{\pi}(X=2|e_M^*,\sigma_D)\right] &= \rho\left[\widehat{\omega}(X=2|s=2,e_M) - \widehat{\omega}(X=1|s=2,e_M)\right] \\ &= \rho\frac{e_M}{(1-\alpha) - \pi\left(\frac{1}{2}-\alpha\right)e_M}\frac{1}{2} \end{aligned} \tag{6}$$

with

$$\begin{aligned} \widehat{\pi}(X=1|e_M^*,\sigma_D) &= \frac{\left[\alpha + \left(\frac{1}{2}-\alpha\right)e_M^*\right] + \gamma\left[(1-\alpha) - \left(\frac{1}{2}-\alpha\right)e_M^*\right]}{\left[\alpha + \pi\left(\frac{1}{2}-\alpha\right)e_M^*\right] + \gamma\left[(1-\alpha) - \pi\left(\frac{1}{2}-\alpha\right)e_M^*\right]}\pi \\ \widehat{\pi}(X=2|e_M^*,\sigma_D) &= \frac{(1-\alpha) - \left(\frac{1}{2}-\alpha\right)e_M^*}{(1-\alpha) - \pi\left(\frac{1}{2}-\alpha\right)e_M^*}\pi \end{aligned} \tag{7}$$

One can verify that the higher is λ, the higher is γ. Furthermore, the higher is e_M, the lower is γ.

Anticipating the distorted decision strategy σ_D, the employees' expected payoff when choosing effort equals respectively:

$$\begin{aligned} EU_1 &= \left[\Pr(s=1|e_M) + \gamma\Pr(s=2|e_M)\right]B - \frac{1}{2}\theta e_1^2 \\ EU_2 &= (1-\gamma)\Pr(s=2|e_M)B - \frac{1}{2}\theta e_2^2 \end{aligned}$$

Differentiating the above expressions with respect to e_i yields:

$$e_i^* = (1-\gamma)\frac{\pi\varphi e_M B}{\theta} \tag{8}$$

This equation explicitly defines the employees' equilibrium effort e_i^* as a function of γ,

π, φ, e_M, B and θ. If the distortion γ increases, then the marginal effect of the employee's effort on the probability of getting the bonus decreases and thus, the employee exerts less effort.

Furthermore, the manager's expected payoff when choosing effort equals:

$$\begin{aligned} EU_M &= \kappa\left(e_1^* + e_2^*\right) + \Pr\left(s=1|e_M\right)\rho\widehat{\omega}\left(X=1|s=1,e_M\right) \\ &+\gamma\Pr\left(s=2|e_M\right)\rho\widehat{\omega}\left(X=1|s=2,e_M\right) \\ &+\left(1-\gamma\right)\Pr\left(s=2|e_M\right)\rho\widehat{\omega}\left(X=2|s=2,e_M\right) \\ &+\left[\Pr\left(s=1|e_M\right)+\gamma\Pr\left(s=2|e_M\right)\right]\lambda\widehat{\pi}\left(X=1|e_M^*,\sigma_D\right) \\ &+\left(1-\gamma\right)\Pr\left(s=2|e_M\right)\lambda\widehat{\pi}\left(X=2|e_M^*,\sigma_D\right) - c_M\left(e_M\right) \end{aligned}$$

Differentiating the above expression with respect to e_M yields the first-order condition[3]:

$$(1-\gamma)\left[\frac{2\kappa\pi\varphi B}{\theta} + \rho\pi\frac{1}{2} + \pi\left(\frac{1}{2}-\alpha\right)\lambda\left[\widehat{\pi}\left(X=1|e_M^*,\sigma_D\right) - \widehat{\pi}\left(X=2|e_M^*,\sigma_D\right)\right]\right] = c_M'\left(e_M^*\right) \tag{9}$$

This equation implicitly defines the manager's equilibrium effort e_M^* as a function of γ, κ, π, φ, B, θ, ρ, α and λ. Clearly, the wish to have productive employees κ and the wish to make the right decision ρ increase the manager's effort. If the wish to come across as able λ increases, then the distortion increases. Since information becomes less valuable to the manager if he increases distortion γ, this causes effort to decrease. Therefore, by (6), the value of the reputational gap decreases. Thus, the higher are the reputation concerns λ, the higher is the distortion and the lower is the manager's effort. As the reputation concerns tend to infinity, then the distortion converges to 1 and the manager's effort converges to 0, which means that the agent almost fully distorts and does not exert effort in order to improve the signal.

The prior probability of being a high-able manager π increases the manager's effort, since (i) the employees exert more effort, since their effort is more likely to affect getting the bonus, (ii) the manager's effort faster increases the probability of making the right decision (iii) the manager's effort faster increases the probability of getting the higher reputation and (iv) by (6), the reputational gap is bigger. The increasing effort causes the distortion to decrease,

[3] For the details of the derivation, we refer to the Appendix.

which again increases effort.

The parameter φ and the bonus B increase the manager's effort, since it increases the marginal effect on the employee's effort. Hence, by (6), distortion γ will decrease in order to increase the reputational gap. Likewise, the employee's cost of effort parameter θ decreases the manager's effort and increase distortion.

Finally, both the marginal effect of the manager's effort on the probability of getting the higher reputation and, by (6), the reputational gap increase in the distance between α and $\frac{1}{2}$. Therefore, the manager's effort increases in this distance. Again, distortion decreases and effort increases even further.

Proposition 2 summarizes the above discussion.

Proposition 2 *Consider an agent with a Perception Technology. Assume that* $\alpha < \frac{1}{2}$. *Let* $\overline{\lambda}$ *satisfy (5) and assume that* $\lambda > \overline{\lambda}$. *Then, a unique equilibrium exists in which*
(i) the boss' beliefs about the manager's ability are given by (7);
(ii) the manager always follows signal $s = 1$, *but with probability* γ *does not follow signal* $s = 2$: $\sigma_D (e_M, s = 1) = 1$ *and* $\sigma_D (e_M, s = 2) = \gamma$, *where* γ *is defined by (6);*
(iii) the employees' effort levels e_i^* *are given by (8) and*
(iv) the manager's effort level e_M^* *satisfies (9).*
The manager's effort level and thus, also the employees' effort levels are ***decreasing*** *in the reputation concerns* λ.

Remark 3 *If* $\alpha = \frac{1}{2}$, *both decisions yield equal reputations. Thus, the reputational gap equals zero and reputation concerns will not induce the manager to distort the award of the bonus. This particular case corresponds to the Perception Technology of Swank and Visser (2012).*

3.2 Conclusive Evidence Technology

Under the Conclusive Evidence Technology, the undistorted strategy σ_U means that the manager follows his signal if he finds conclusive evidence: $\sigma_U (e_M, s = 1) = 1$ and $\sigma_U (e_M, s = 2) = 0$.

If the manager finds no evidence, the probability that the most productive employee receives the bonus given the manager's effort level is independent from which employee gets the bonus: $\widehat{\omega}(X=i|s=\varnothing,e_M)=\frac{1}{2}$. Hence, the manager can mix between awarding the bonus to employee 1 and awarding the bonus to employee 2: $\sigma_U(e_M,s=\varnothing)=\gamma$.

Given the undistorted decision strategy σ_U, Bayes' rule implies:

$$\begin{aligned}
\widehat{\pi}(X=1|e_M^*,\sigma_U) &= \frac{e_M^*\frac{1}{2}+\gamma(1-e_M^*)}{\pi e_M^*\frac{1}{2}+\gamma(1-\pi e_M^*)}\pi \\
\widehat{\pi}(X=2|e_M^*,\sigma_U) &= \frac{e_M^*\frac{1}{2}+(1-\gamma)(1-e_M^*)}{\pi e_M^*\frac{1}{2}+(1-\gamma)(1-\pi e_M^*)}\pi
\end{aligned} \tag{10}$$

Given that no evidence is found, the manager maximizes his utility by maximizing his reputation. Therefore, the manager awards the bonus to employee 1 with probability $\frac{1}{2}$ if no conclusive evidence is found, ensuring that $\widehat{\pi}(X=1|e_M^*,\sigma_U)=\widehat{\pi}(X=2|e_M^*,\sigma_U)=\pi$. This means that the unconditional probability that the manager awards the bonus to employee i equals: $\Pr(X=i|e_M)=\Pr(s=i|e_M)+\frac{1}{2}\Pr(s=\varnothing|e_M)=\frac{1}{2}$.

Given the undistorted decision strategy σ_U, the expected payoff of each employee i when choosing effort equals:

$$EU_i=\left[\Pr(s=i|e_M)+\frac{1}{2}\Pr(s=\varnothing|e_M)\right]B-\frac{1}{2}\theta e_i^2$$

Differentiating the above expression with respect to e_i yields:

$$e_i^*=\frac{\pi\varphi e_M B}{\theta} \tag{11}$$

This equation explicitly defines e_i^* as a function of π, φ, e_M, B and θ. Clearly, both π, φ, e_M and B increase the employee's marginal benefit of effort and therefore, the employee's effort. θ increases the marginal cost of effort and hence, decreases the employee's effort.

Furthermore, the manager's expected payoff when choosing effort equals:

$$
\begin{aligned}
EU_M &= \kappa\left(e_1^* + e_2^*\right) + \Pr\left(s = 1|e_M\right)\rho\widehat{\omega}\left(X = 1|s = 1, e_M\right) \\
&+ \frac{1}{2}\Pr\left(s = \varnothing|e_M\right)\rho\widehat{\omega}\left(X = 1|s = \varnothing, e_M\right) \\
&+ \frac{1}{2}\Pr\left(s = \varnothing|e_M\right)\rho\widehat{\omega}\left(X = 2|s = \varnothing, e_M\right) \\
&+ \Pr\left(s = 2|e_M\right)\rho\widehat{\omega}\left(X = 2|s = 2, e_M\right) \\
&+ \Pr\left(X = 1|e_M\right)\lambda\widehat{\pi}\left(X = 1|e_M^*, \sigma_U\right) \\
&+ \Pr\left(X = 2|e_M\right)\lambda\widehat{\pi}\left(X = 2|e_M^*, \sigma_U\right) - c_M\left(e_M\right)
\end{aligned}
$$

Differentiating the above expression with respect to e_M yields the first-order condition[4]:

$$
\frac{2\kappa\pi\varphi B}{\theta} + \rho\pi\frac{1}{2} = c_M'\left(e_M^*\right) \tag{12}
$$

This equation implicitly defines the manager's equilibrium effort e_M^* as a function of κ, π, φ, B, θ and ρ. Clearly, the wish to have productive employees κ and the wish to make the right decision ρ increase the manager's effort. The wish to come across as able λ does not affect the manager's effort, since both decisions yield equal reputations.

The prior probability of being a high-able manager π increases the manager's effort, since (i) the employees exert more effort, since their effort is more likely to affect getting the bonus, and (ii) the manager's effort faster increases the probability of making the right decision.

The parameter φ and the bonus B increase the manager's effort, since it increases its marginal benefit on the employees' effort. Likewise, the employee's cost of effort parameter θ decreases the manager's effort.

Since in equilibrium awarding the bonus to employee 1 yields the same reputation as awarding the bonus to employee 2, the manager has no incentive to play a distorted decision strategy.

Proposition 4 summarizes the above discussion.

Proposition 4 *Consider an agent with a Perception Technology. Then, a unique equilibrium exists in which*

[4]For the details of the derivation, we refer to the Appendix.

(i) the boss' beliefs about the manager's ability equal the prior probability that the manager is high-able π;

(ii) the manager always follows conclusive evidence, and equally mixes between the employees if he finds no evidence: $\sigma_U(e_M, s = 1) = 1$, $\sigma_U(e_M, s = 2) = 0$ and $\sigma_U(e_M, s = \varnothing) = \frac{1}{2}$;

(iii) the employees' effort levels e_i^* are given by (11) and

(iv) the manager's effort level e_M^* satisfies (12).

The manager's effort level and thus, also the employees' effort levels are ***independent*** *from the reputation concerns λ.*

4 Discussion

The timing of our model implies that the employees base their effort e_i on the manager's effort e_M, which he has chosen in an earlier stage. In practice, a commitment problem may arise: once the employees have chosen their effort level, the manager may want to decrease his effort. However, the manager can commit to certain effort by taking measures in advance, like, for example, setting up information systems, installing video cameras or hiring consultants that will assess the performance of the employees. Since the manager values taking the right decision, he will in the end choose to benefit from the earlier effort by exerting additional effort (analyzing administrative records, watching the videos, reading the reports) as long as this is not too costly. Furthermore, if the manager and the employees are bound in a continuing professional relationship or if the manager's promised and final effort choices are visible for potential new employees, the manager may have incentives to exert the full effort he promised to exert.

In practice, the information acquisition technology of the manager is often a combination of the Perception Technology and the Conclusive Evidence Technology. A manager can either find conclusive evidence, get a perception or find no clue at all. Mathematically, we can combine both information acquisition technologies by assuming that with some given probability the manager gets a perception and otherwise the manager gets a signal like in the Conclusive Evidence Technology.

We find no qualitative different results when we analyze this more general information acquisition technology. The manager still always follows conclusive evidence. If the likelihood of receiving a certain perception is constant and independent from the manager's effort and ability ($\alpha = \frac{1}{2}$), the manager will also follow perceptions and equally mix between the employees if he finds no clue at all. However, if the manager is less likely to get a perception suggesting that employee 1 is more productive than employee 2 ($\alpha < \frac{1}{2}$) and reputation concerns λ increase, he will initially increase the probability that he awards the bonus to employee 1 and hence, choose for the better reputation if he finds no clue at all. If α is sufficiently lower than $\frac{1}{2}$ and the manager's value for reputation λ increases, he even starts to distort by awarding the bonus to employee 1 if his perception suggests that employee 2 is more productive than employee 1.

5 Conclusion

In this paper, we endogenized the information acquisition and award of the bonus by the manager in a tournament model. We used two common ways of modelling information acquisition.

First of all, the competence of the manager is crucial in motivating the employees. If the manager is more able and exerts more effort to assess the productivity of the employees, the employees are more strongly motivated to exert effort, since their effort is more likely to affect the probability of getting the bonus. Depending on the information acquisition technology, the effect of the manager's ability on the employees' effort may eventually become negative, since uncertainty about the manager's ability may enforce the effect of reputation concerns on the manager's effort.

If a manager forms a perception and if both employees are equally likely to be identified as most productive (Swank and Visser, 2012), then the ex-ante probability of getting a certain perception is independent of the manager's ability. Therefore, reputation concerns do not induce the manager to exert extra effort or distort the award of the bonus. However, if the more experienced employee is more capable to influence the manager's perception, awarding the bonus to the less experienced employee will yield a better reputation. Therefore, the manager may increase effort in order to increase the probability that he will get the better reputation. Eventually, he may even distort the decision. Consequently, we can conclude that reputation concerns may be another explanation for distorted awards of bonuses besides conventional favoritism.

In our tournament setting, we find that in case of the Conclusive Evidence Technology, both the manager's and the employees' effort is independent from reputation concerns and there is no incentive to distort the decision. Swank and Visser assume in their model that the principal (in our setting the manager's boss) is biased against one of the choices. Consequently, under the Conclusive Evidence Theory, the undistorted decision strategy implies that the agent (in our setting the manager) chooses against this option if he finds no evidence. This, however, means that a low-able agent is more likely to choose for the ex-ante preferred alternative. Therefore, Swank and Visser find that this choice yields a lower reputation, the wish to come across as able initially increases effort and agents will distort the decision if reputation concerns are sufficiently high. Since we do not have a reason to expect that the organization is in advance biased against one of the employees, we do not find a reputational gap and hence, we find no impact of reputation concerns on effort and no incentive to distort the decision.

In this paper, we studied the effects of the manager's wish to come across as able on his effort and decision strategy in presence of different information acquisition technologies. However, a manager may have several kinds of reputation concerns. Managers may, for example, differ in their motivation to make the right decisions and hence, in their willingness to follow their perception. Further research is needed to examine the effects of the joint occurrence of both kinds of reputation concerns.

Furthermore, we assumed in this paper that the information acquisition technology is given. In practice, however, a manager may be able to choose whether he wants to improve his perception or search for conclusive evidence. Moreover, the employer may be able to enforce the use of a certain information acquisition technology. Further research may address the question which information acquisition technology the manager and the firm will choose.

6 Appendix

6.1 Perception Technology

Anticipating the distorted decision strategy σ_D, the manager's expected payoff when choosing effort equals[5]:

$$\begin{aligned} EU_M &= \kappa\left(e_1^* + e_2^*\right) + \Pr\left(s=1|e_M\right)\rho\widehat{\omega}\left(X=1|s=1,e_M\right) \\ &\quad +\gamma\Pr\left(s=2|e_M\right)\rho\widehat{\omega}\left(X=1|s=2,e_M\right) \\ &\quad +\left(1-\gamma\right)\Pr\left(s=2|e_M\right)\rho\widehat{\omega}\left(X=2|s=2,e_M\right) \\ &\quad +\left[\Pr\left(s=1|e_M\right)+\gamma\Pr\left(s=2|e_M\right)\right]\lambda\widehat{\pi}\left(X=1|e_M^*,\sigma_D\right) \\ &\quad +\left(1-\gamma\right)\Pr\left(s=2|e_M\right)\lambda\widehat{\pi}\left(X=2|e_M^*,\sigma_D\right) - c_M\left(e_M\right) \end{aligned}$$

In order to differentiate the above expression with respect to e_M, we use that the derivative of $\Pr\left(s=k|e_M\right)\rho\widehat{\omega}\left(X=i|s=k,e_M\right)$ with respect to e_M equals:

$$\begin{aligned} & \frac{\partial}{\partial e_M}\left[\Pr\left(s=k|e_M\right)\rho\widehat{\omega}\left(X=i|s=k,e_M\right)\right] \\ = & \frac{\partial}{\partial e_M}\left[\Pr\left(s=k|e_M\right)\rho\frac{\Pr\left(s=k|q_i>q_j,e_M\right)}{\Pr\left(s=k|e_M\right)}\frac{1}{2}\right] \\ = & \frac{\partial\Pr\left(s=k|e_M\right)}{\partial e_M}\rho\frac{\Pr\left(s=k|q_i>q_j,e_M\right)}{\Pr\left(s=k|e_M\right)}\frac{1}{2} \\ & +\Pr\left(s=k|e_M\right)\rho\frac{\frac{\partial\Pr\left(s=k|q_i>q_j,e_M\right)}{\partial e_M}\Pr\left(s=k|e_M\right)-\Pr\left(s=k|q_i>q_j,e_M\right)\frac{\partial\Pr\left(s=k|e_M\right)}{\partial e_M}}{\left[\Pr\left(s=k|e_M\right)\right]^2}\frac{1}{2} \\ = & \frac{\partial\Pr\left(s=k|e_M\right)}{\partial e_M}\rho\frac{\Pr\left(s=k|q_i>q_j,e_M\right)}{\Pr\left(s=k|e_M\right)}\frac{1}{2} \\ & +\rho\frac{\frac{\partial\Pr\left(s=k|q_i>q_j,e_M\right)}{\partial e_M}\Pr\left(s=k|e_M\right)-\Pr\left(s=k|q_i>q_j,e_M\right)\frac{\partial\Pr\left(s=k|e_M\right)}{\partial e_M}}{\Pr\left(s=k|e_M\right)}\frac{1}{2} \\ = & \frac{\partial\Pr\left(s=k|e_M\right)}{\partial e_M}\rho\frac{\Pr\left(s=k|q_i>q_j,e_M\right)}{\Pr\left(s=k|e_M\right)}\frac{1}{2} \\ & +\rho\frac{\partial\Pr\left(s=k|q_i>q_j,e_M\right)}{\partial e_M}\frac{1}{2}-\frac{\partial\Pr\left(s=k|e_M\right)}{\partial e_M}\rho\frac{\Pr\left(s=k|q_i>q_j,e_M\right)}{\Pr\left(s=k|e_M\right)}\frac{1}{2} \\ = & \rho\frac{\partial\Pr\left(s=k|q_i>q_j,e_M\right)}{\partial e_M}\frac{1}{2} \end{aligned}$$

[5]Note that the manager's expected payoff under the undistorted decision strategy is a restricted version ($\gamma=0$) of the manager's expected payoff under the undistorted decision strategy.

After substituting $e_i^* = \frac{\pi\varphi e_M B}{\theta}$, $\Pr(s=1|e_M) = \alpha + \pi\left(\frac{1}{2}-\alpha\right)e_M$, $\Pr(s=2|e_M) = (1-\alpha) - \pi\left(\frac{1}{2}-\alpha\right)e_M$, $\Pr(s=1|q_1>q_2,e_M) = \alpha + \pi(1-\alpha)e_M$, $\Pr(s=2|q_1>q_2,e_M) = (1-\alpha) - \pi(1-\alpha)e_M$ and $\Pr(s=2|q_2>q_1,e_M) = (1-\alpha)+\pi\alpha e_M$, differentiating the above expression with respect to e_M yields:

$$(1-\gamma)\left[\frac{2\kappa\pi\varphi B}{\theta} + \rho\pi\frac{1}{2} + \pi\left(\frac{1}{2}-\alpha\right)\lambda\left[\widehat{\pi}\left(X=1|e_M^*,\sigma_D\right) - \widehat{\pi}\left(X=2|e_M^*,\sigma_D\right)\right]\right] = c_M'\left(e_M^*\right)$$

6.2 Conclusive Evidence Technology

Anticipating the undistorted decision strategy σ_U, the manager's expected payoff when choosing effort equals:

$$\begin{aligned}
EU_M \quad = \quad & \kappa\left(e_1^*+e_2^*\right) + \Pr(s=1|e_M)\,\rho\widehat{\omega}\left(X=1|s=1,e_M\right) \\
& +\frac{1}{2}\Pr(s=\varnothing|e_M)\,\rho\widehat{\omega}\left(X=1|s=\varnothing,e_M\right) \\
& +\frac{1}{2}\Pr(s=\varnothing|e_M)\,\rho\widehat{\omega}\left(X=2|s=\varnothing,e_M\right) \\
& +\Pr(s=2|e_M)\,\rho\widehat{\omega}\left(X=2|s=2,e_M\right) \\
& +\Pr(X=1|e_M)\,\lambda\widehat{\pi}\left(X=1|e_M^*,\sigma_U\right) \\
& +\Pr(X=2|e_M)\,\lambda\widehat{\pi}\left(X=2|e_M^*,\sigma_U\right) - c_M\left(e_M\right)
\end{aligned}$$

In order to differentiate the above expression with respect to e_M, we use the expression of the derivative of $\Pr(s=k|e_M)\,\rho\widehat{\omega}\left(X=i|s=k,e_M\right)$ with respect to e_M from the last subsection:

$$\frac{\partial}{\partial e_M}\left[\Pr(s=k|e_M)\,\rho\widehat{\omega}\left(X=i|s=k,e_M\right)\right] = \rho\frac{\partial\Pr\left(s=k|q_i>q_j,e_M\right)}{\partial e_M}\frac{1}{2}$$

After substituting $e_i^* = \frac{\pi\varphi e_M B}{\theta}$, $\Pr(s=i|q_i>q_j,e_M) = \pi e_M$, $\Pr(s=\varnothing|q_i,q_j,e_M) = \pi e_M$ and $\Pr(s=i|e_M) = \frac{1}{2}$, differentiating the above expression with respect to e_M yields:

$$\frac{2\kappa\pi\varphi B}{\theta} + \rho\pi\frac{1}{2} = c_M'\left(e_M^*\right)$$

7 References

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