Welfare benefits from dynamic premia, asset allocation and age of retirement in a defined contribution pension scheme

Author: Joep Steffens (348157)

Date: 03-03-2017

Supervisors: AAG D. Boeijen MSc. (PGGM) and prof. dr. D. van Dijk
Co-reader: C. Frey PHD

Abstract

Defined contribution (DC) schemes are on the rise in the Netherlands due to poor results of defined benefits (DB) schemes. DC is relatively new implying that there is room for improvement of the commonly used static lifecycle investment schemes. The digital era has made it possible for pension funds to invest in tailor-made pensions, based on the participants' preferences. This research investigates whether relaxing premia, asset allocation and the accumulation phase length results in welfare gains for the pension grantee. Economic scenarios, based on historical data, are simulated and are used to calculate the performance of different strategies each based on a different utility function. Results are evaluated on the whole distribution of the pension replacement ratio and the means used to achieve the pension. This model is extended by including risks on labour income. An extensive sensitivity analysis is performed, providing intuition on how parameters affect the results and the magnitude of these effects. The research concludes that differentiating in pension strategies based on personal preferences results in additional welfare and that the investment climate should be taken into account.

Master Thesis Quantitative Finance, Erasmus School of Economics

1

1 Introduction

Pension systems in Europe can be categorized in two important categories. The first category is the Bismark-system, which is mostly used in continental Europe. In the Bismark-system the largest part of ones pension comes from the so called first pillar[1]. This first pillar consists of government benefits which are often related to the past income and years served in the labour force. The second pillar, the additional pension, is of little importance in countries with a Bismark system. The second category is known as the Beveridge-system and is used in Scandinavian countries, on the British isles and in the Netherlands. In these countries the first pillar is equal for everybody and should guarantee subsistence. The second pillar in the Beveridge-system is related to the past earnings and years spent in the labour force and is much larger compared to the second pillar in the Bismark-system. Most countries with a Beveridge-system use a defined benefits (DB) scheme based on principles of solidarity between generations and colleagues.

A DB scheme is a retirement plan where benefits are computed using the length of employment and salary history as inputs. These benefits are guaranteed to some extent by the pension fund or the employer and therefore do not directly depend on the returns on the invested funds, unlike the DC schemes researched in this paper. In the Netherlands many sectors or companies have their own, often mandatory, DB scheme. Due to the collective nature of the scheme risks are shared between generations.

These DB schemes have been under quite some pressure lately due to the misleading name of the system, which provides a participant with a false sense of security. When the overall coverage rate in a DB scheme is too low, pension funds will have to stop correcting pensions for inflation or even start performing cuts. Ergo, the benefits are not always 'defined'. In recent times pension funds have been forced to take the previously described measures, due to a persisting low interest rate, equity returns unlike the historical average and an increase in the life expectancy. In the 'Nationale Pensioendialoog' the future of the Dutch pension system is discussed and pensions have become a major electoral battle for the Dutch elections in March 2017. Currently the majority of Dutch pension schemes are DB schemes. The disappointing results in DB schemes and the resulting reduced confidence in the system further strengthened the trend in public opinion and politics towards defined contributions (DC) schemes, with liberal parties calling for a system in which every participant has a personal pension pot. In such a DC system every individual participant will build up a pension pot from their own contributions together with the acquired investment return. When the participant retires the pension will be financed from this

---

[1]This first pension pillar is defined by Investopedia as: A standardized, state-run pension system, which offers basic coverage and is primarily focused on reducing poverty.

The second pillar is defined as: A funded system that recipients and employers pay into; this includes pension funds and defined-contribution accounts/plans.

The third and final pillar is defined as: Voluntary private funded accounts, including individual savings plans, insurance, etc.

personal pot, which also makes the system a lot clearer for participants. The high complexity of the current DB system is a source of distrust in pension funds. Participants already receiving their pension are afraid of unjustified cuts and lack of indexation, while younger participants are afraid that the current pot will be squandered, leaving little for when they retire.

The first DC schemes used a static mix of stocks and bonds throughout the entire period, e.g. holding 50 percent of both asset categories.Merton et al. (1992) i.a. showed that it is optimal to have a riskier portfolio for a young investor with a long horizon and high human wealth compared to an older investor with a shorter horizon and less human wealth[2]. The second generation of DC schemes improved on the first by allowing for lifecycle investing. Lifecycle investing allows for risks to be high when young and lowers the risk when the retirement comes closer, thus reducing the risk of losing a substantial part of a pension near the retirement whilst benefiting from high expected returns in the early accumulation years.

A DC scheme with lifecycle investing is still a pretty static strategy. It holds a predetermined mix of different asset types and only takes the distance to retirement into account, gradually reducing risk previous to it. Nowadays a lot of computing power is available allowing for an increase in the flexibility and individuality of investment strategies. It is interesting to research the magnitude of the welfare gains for individual participants that go with involving participants more in the choices regarding their pension, thereby obtaining a tailor made pension plan fitted to the particpants' personal needs[3]. In a very individualistic way of implementing the DC system all risks regarding the accumulation of wealth are to be dealt with by the individual participant. This implies that the participant should decide the height of the premia, the moment of retiring and with how much risk the pension pot should be invested. Needless to say, this system would not suit most participants as they lack the knowledge or skills to make an informed and prudent decision regarding these topics, since a pension remains a very complex financial product[4].

This leads us to the question how pension funds can optimally support participants in a DC system. Based on preferences declared by the participant and the risk attitude associated with said preferences an econometric model could provide a personal strategy regarding the premia, risk appetite and accumulation phase length in which the personal preferences are optimally honored. Such a scheme is better known as a 'managed DC' scheme to insiders.

---

[2]Human capital or wealth is defined as the expected net present value of an individuals future labor supply. Risks concerning human capital include the risk of losing ones job, the insecurity about the evolution of labour income and the risk of a change in the ability to provide labour.

[3]Introducing extra degrees of freedom can theoretically only increase performance.

[4]Merton, R. C. (2014). The crisis in retirement planning. Harvard Business Review, 92(7), 42-50. Merton compares the choices regarding ones' pension to a surgeon asking complex medical questions to a patient being wheeled into an operating room.

1.1 Problem definition and approach

The aim of this research is to investigate whether allowing for dynamics and individual preferences in premia, accumulation phase length and asset allocation can significantly improve the performance of a DC scheme and what the magnitude of these welfare gains for a participant would be.

The model used in this research will allow a yearly reevaluation of the premium, accumulation phase length and asset allocation. This allows a participant to update his pension scheme based on realized returns, income changes and changes in the investment climate. Allowing for individual choices ensures a tailored solution focussed on creating a pension fitted to the relevant personal evaluation criteria.

Behavioural finance has shown that people weigh perceived losses more heavily compared to gains of the same size[5]. For pension recipients the results in bad states of the world, losses, should thus be weighed more heavily when determining an optimal strategy than results in good states of the world, gains. Therefore it is important to consider low percentile results and means for different strategies and to compare these results against several benchmarks to evaluate performance relative to a static strategy. Since there is a multidimensional problem at hand the differences in premia, accumulation phase length, risk and acquired pension are translated into a common unit via the means of utility functions. This is the most tricky part of the research since it is difficult and subjective to compare a one percent increase in the pension replacement ratio to a one percent decrease in premia. An economic scenario generator is used to simulate the performance of these different utility functions or strategies.

1.2 Practical relevance

The research shows that managed DC schemes outperform static strategies. They are able to do so by taking the investment climate into account, i.e. a different asset allocation strategy when interest rates are at 1 or 6 percent ceterus paribus. Furthermore the research concludes that via different routes different distributions of pensions can be obtained by choosing between utility functions. Thus involving the participant and the personal risk attitude in choosing the pension scheme is helpful. These insights are of high interest to the pension sector and via their fiduciary duty[6] in the interests of all participants in a pension scheme. Knowing which relaxations improve the performance of a pension scheme and with what magnitude, in a model with transparent assumptions, adds to the current pension system discourse in the Netherlands.

As stated previously DB schemes are under a lot of pressure due to the fast increase in life expectancy, the financial crisis and the prolonged low interest rates. The Dutch government, hereinafter referred to as 'government'

---

[5]Kahneman and Tversky developed their prospect theory in 1979. It presents a critique of expected utility theory. They conclude that the value function is generally steeper for losses than for gains, among other interesting insights.

[6]The legal obligation to act in the best interest of another party.

for simplicity, also recognizes these problems currently facing the DB schemes. Already, changes have been implemented making some of the rules for pension funds more flexible, such as spreading out financial shocks over a time period and linking the first pillar of an individuals' pension to the life expectancy[7]. The government is also working with the pension sector to research whether a new type of pension contract might result in welfare benefits for all parties. In the so called Nationale Pensioendialoog and a report written by the Dutch Social and Economic Council (SER) DC schemes with some risk sharing features of the current DB system are compared. These movements underline the practical relevance of this research, because the currently researched DC schemes could be improved making them more attractive.

The main contribution of the research is combining pension evaluation methods found in the literature with a state of the art economic scenario generator, enabling this research to form an overall opinion on the optimal form of a DC framework. Furthermore it relaxes asset allocation, premia and the age of retirement whereas most of the literature on the subject of DC only investigates the effect of a relaxed investment scheme. This research therefore provides a unique insight into welfare gains from relaxing premia and the age of retirement compared to a static DC strategy. Finally a transparent and extensive sensitivity analysis provides intuition on which assumptions or parameters are the most important. Naturally this is only possible in this research's unique managed DC framework containing all relevant parameters.

2   Literature

The basis for lifetime portfolio selection problems, closely related to forging an optimale pension scheme, was laid in 1969 by the renowned nobel prize laureate Robert Merton in Merton (1969) and by P.A. Samuelson in his companion paper Samuelson (1969). In his paper Merton derives optimal portfolios under stochastic returns on assets in a continuous-time model, whereas Samuelson develops a similar model in discrete time. Both the constant absolute risk-aversion case and the constant relative risk-aversion case are discussed and are shown to have explicit solutions by Merton. Though Merton's solution for the multi period portfolio selection problem was not specifically targeted at pension schemes there is a clear overlap, reflected by the many papers in the literature list who still use it as an important reference. Merton's approach will serve as the basis for the lifetime portfolio selection problem, important for building a model ranging over 50 years and determining the asset mix every year.

Building on Merton's model Chang (1999) takes an approach combining stochastic simulations and dynamic optimization in order to determine the op-

---

[7]The 'Algemene Ouderdomswet' (AOW) is a law that provides a state backed pension when an eligible person reaches a certain age. Reacting to an increase in life expectancy, driving up the costs of these pensions, the government has gradually increased the age for a person to be entitled to this pension for cohorts close to the eligible age while linking it to the life expectancy for younger cohorts.

timal funding policy for a DB pension scheme. The study's results show a significant advantage for the dynamic approach over the traditional deterministic pension valuation. Indicating that gains can be made by allowing for a dynamic funding strategy in a pension scheme, which is of high interest for the optimization of premia in the model.

A useful extension for when transforming Merton's multi period portfolio selection solution to the selection of an optimal pension scheme is incorporating the annuitization risk[8]. Vigna et al. (2001) include both investment risk and annuitization risk faced by the scheme member, which they analyze in a DC pension scheme using dynamic programming. Their results show a large variability of pension levels achieved at retirement when a variable annuity conversion rate, i.e. a variable annuity obtained for a fixed lump sum, is considered suggesting further research is needed to tackle this problem, while the investment risk is appropriately reduced by the dynamic lifestyle strategy. Their results signify the importance of incorporating annuitization risk since the annuity conversion rate could provide vastly different pensions for similar lump sums.

Another important and relevant extension is the incorporation of the risk the human capital faces[9]. The approach from Cairns et al. (2006) is considered who develop an optimal asset allocation model for the accumulation phase of a DC pension scheme where the riskiness of human capital is taken into account. An important assumption they make is that this risk on salary is non-hedgeable. They score their model on the expected terminal utility which is thus optimized. Their research shows that under their framework it is optimal to invest in three different portfolios. The first is the minimum risk portfolio relative to the salary, the second the minimum risk portfolio relative to the salary times the annuity rate and the third is an efficient risky portfolio relative to the salary and salary divided by the annuity rate.

The model also needs to allow for flexibility in the choice of premia and the accumulation phase length. Merton et al. (1992) examines the effect of the labor-leisure choice in portfolio and consumption decisions over a life cycle. Note that the flexibility in work effort, including picking the moment of retirement, is similar to choosing the optimal percentage of premia which should be paid whilst determining the ideal moment of retirement. Merton et al. (1992) discovers an intimate relation between labour and investment choices. This also helps explain why young people may take greater investment risks than the old. Young people

---

[8]At the point of retiring the pension grantee can choose to convert his accumulated pension assets into an periodic income for the rest of his or her life or for a variable periodic income with a fraction of the capital still invested. The default in this research is a participant that converts the total lump sum at retirement into an annuity. This annuity takes away the risk of outliving ones savings among other risks, since a periodic income is guaranteed for the remainder of the participants' life. However the height of this periodic income one could get for a lumpsum, the annuitization, heavily depends on the inflation rate and the yield curve, with fictional interest rates such as the Ultimate Forward Rate for non liquid maturities. Another important risk regarding the annuitization is the period over which the income is expected to be payed and hence life expectancy or macro longevity risk.

[9]In this research it will denote the risk of losing ones job and the uncertainty about future wage increases.

have a full life ahead of them in which they still have a lot of labour flexibility. Their work also explains why consumption is relatively smooth despite volatility in asset prices, which is less relevant for this research.

In Vigna (2014) a target-based approach is used for the optimization problem in the accumulation phase for a DC pension scheme in which a quadratic loss function is used to penalize accumulating less wealth than the set target. Choosing the target-based approach results in not having to estimate the notoriously hard to estimate risk-aversion coefficient for an individual. Their target-based approach is flexible and adaptable with regards to changes in the member's needs and preferences or the realized returns. They show that this approach is efficient in a mean-variance setting. They compare their approach against several tactics that optimize expected utility and show that for longer durations, which are typical for DC frameworks, there is a significant mean-variance inefficiency in the utility optimization techniques. Which implies that combining a mean-variance approach and a utility framework may not be a good idea. Blake et al. (2013) choose a different evaluation method to find the optimal investment strategy namely a loss aversion framework. They use stochastic investment and labour income processes and a path dependent fund target. Based on realized returns the equity allocation is either increased or decreased. They obtain the trivial but nonetheless interesting result that target-driven strategies have a considerable higher chance of attaining the target replacement ratio in comparison to the benchmark strategies which maximize expected utility. These strategies will be tested and compared against other strategies and benchmarks in this research.

Blake et al. (2001) estimate values-at-risk in the accumulation phase of DC schemes for a range of asset return models and strategies. They find that DC schemes can be extremely risky compared to DB schemes. Their values-at-risk are very sensitive to the asset allocation strategy that is used. The research also finds that a static high equity strategy outperforms all dynamic strategies, whereas conservative bond based strategies require substantially higher contributions for the same pension to be achieved. It is important to understand the sensitivity of the results to input parameters which might explain why a high equity strategy outperforms others. The result that DC schemes can be extremely risky compared to DB schemes is reason to take scores for several percentiles into account in this research when comparing strategies and selecting strategies based on percentile expectations might also be a good strategy.

Maccheroni (2009) proposes a portfolio selection method based on monotone preferences which coincide with mean-variance preferences when mean-variance preferences are economically meaningful. If ceterus paribus the expected return for the risky asset in the most positive state increases, resulting in an increased variance and higher expected return, an investor may decrease his weight in the risky asset due to the increase in variance. This is not an economically meaningful decision since any rational investor would strictly prefer the world with the higher expected return and would not decrease his weight for the risky asset. Maccheroni (2009) use truncated mean-variance decisions to counter this problem, which the mean variance strategy in this research will also incorporate.

7

3   Methodology

The methodology can be split into three large parts. The first part has to do with the simulation of economic scenarios, the second part describes how a pension is accumulated and transferred into an annuity and finally the third part contains the utility functions that choose an optimal pension strategy for each of the economic scenarios. An individual's risk aversion, an individual's premium-consumption balance, an individual's willingness to work/retire and an individual's utility function associated with different levels of the pension can all deviate from assumptions used in this research. This means that an optimal strategy is subjective in all those four dimensions associated with picking an optimal strategy. Differences may occur between the rich and the poor, between workers in different industries or between countries when the first pillar does not guarantee subsistence. Even individuals with the exact same pension pot, wage and perspective may have different preferences. Determining a realistic way of simulating economic scenarios is also subjective, due to the subjective nature of economics.

Econometrics in finance tries to mathematically model real world financial processes as accurately as possible. The main goal of this econometric approach in this research is to discover where the current static pension strategy can be improved with regard to DC pensions. Since there is ex ante no way of knowing the true value of e.g. the parameter for the mean of equity returns, every parameter will be estimated based on historical data and economic expectations. This research tries to be as transparent as possible with regard to all of the used assumptions and parameters and performs an extensive sensitivity analysis to illustrate what happens when a particular parameter is changed. The point this paragraph tries to stress is perhaps best illustrated by the following quote from Albert Einstein: "As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality."

3.1   Economic scenarios

To analyze whether relaxations in premia, retirement age and asset allocation strategy make sense we compare utility values for different simulated economic scenarios over the complete parameter space. In the first analysis this space will only allow for one parameter to change. This will be extended to relaxing two parameters and finally all three parameters will be relaxed. The investor may invest in two different assets, namely risky equity and a risk-free bond which has a level equal to the interest rate. 21 different lifecycles are considered with weights in equity ranging from 0 percent up to 100 percent, 15 years before retirement the weight in the risky equity will be gradually reduced towards zero. A graphical illustration of these lifecycles can be found in Figure 1.

In terms of the accumulation phase length (APL) the default option considered is an accumulation period of 45 years after which the retiree is expected to live another 20 years. Both longer and shorter accumulation periods are one on one translated to shorter and longer life expectancies when retired, meaning
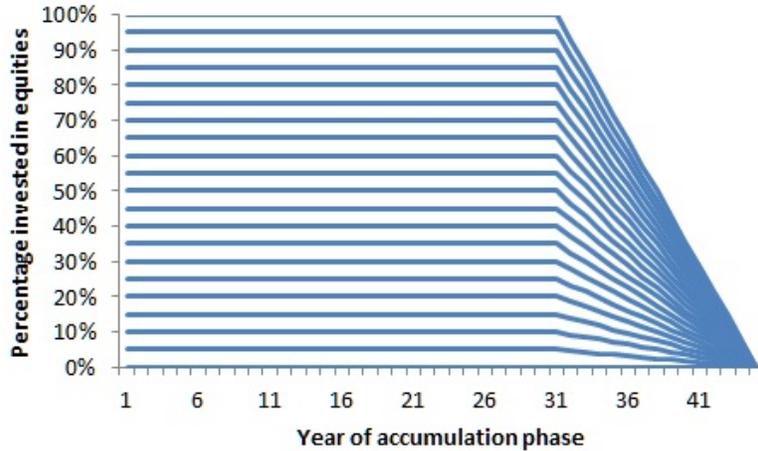
Figure 1: Overview of the 21 lifecycles considered for a fixed accumulation phase length of 45 years.

a accumulation phase of 46 years leaves a decumulation phase of 19 years etc. In order to have some certainty about an individual's accumulation- and decumulation phase length, the accumulation phase length is always between 43 and 47 years. With regard to premia a similar policy is adopted with a median percentage of 11 percent, allowing deviations up to 2 percent. The research will also look into a premium ranging over a much wider domain from 0 to 25 percent. To prevent large changes in premia the a stability restriction is put in place allowing which allows the premium to differ at most 2 percent from the value chosen in the previous year. In order to avoid large transaction costs a similar stability restriction can be put in place for the chosen lifecycle.

The economic scenarios that are used consist of 1,000 scenarios for the interest rate, the inflation and equity returns. These economic scenarios have an annual frequency and are constructed as follows.

For simplicity the interest rate term structure which describes the level of risk-free bonds is assumed to be flat. The investments in bonds will be nominally matched. This means that the part of the portfolio that is invested in bonds pays the same nominal amount in each year of the expected retirement period. This is fictional for very long illiquid maturities, nevertheless long maturities are available with similar characteristics and the interest rate term structure is relatively flat for long maturities. Shorter maturities do become relevant when close to retirement, i.e. close to retirement there might be significant differences in yields. Nonetheless modeling a complex term structure 45 years in advance would result in mere speculation. Only the interest rate level for the decumulation years is relevant. Thus the part of around 20 years of the term structure that is relevant shifts every year. All in all a flat term structure is

9

considered enough for the purposes of this research. The term structure follows an AR(1) path with mean reversion and normal shocks and is simulated as described below:

$$r_t = r_{t-1} + m_r * (\bar{r} - r_{t-1}) + \sigma_r * \varepsilon_r, \qquad (1)$$

in which r_t is the level of the risk-free rate or the bond level, m_r is the degree of mean reversion and $\bar{r}$ is the mean to which the process reverts. In this research ε_x always denotes a standard normal distributed shock.

In most of the literature a constant risk-free rate is assumed. However, since the model deals with pensions, the risk-free rate is important for determining the yearly allowance one could buy for a lumpsum. This is called annuitization risk and is one of the most important problems a pension scheme faces. Working with a constant risk-free rate would greatly reduce this risk, in that case only the inflation rate would provide annuitization risk. For the setup used in this research an AR(1) model seems the best fit, since it describes a relatively stable process with a certain degree of mean reversion which is able to replicate empirical evidence. In extreme scenarios the interest rate on the government bonds can take on negative values, given the recent developments with many negative interest rates for different maturities this is not considered a problem. When one keeps in mind that there is mean reversion, dependent on the distance to the (positive) mean, extreme values are not expected to last long. It is important to know that this mean reversion allows an investor to time the market, since there is a long term mean to which the process reverts. Market timing is discussed in Appendix A and the benefits of this effect for the managed strategies will be investigated in the sensitivity analysis.

As stated previously investments in fixed income securities are nominally matched, which implies the following formula for the return on bonds defined as R_t:

$$R_t = \sum_{i=APL+1}^{APL+LE} \left(1 - \frac{1}{(1+r_t)^i} \right) / \sum_{i=APL+1}^{APL+LE} \left(1 - \frac{1}{(1+r_{t-1})^{i-1}} \right) - 1, \qquad (2)$$

where APL is the accumulation phase length, LE is the conditional life expectancy after retirement, with the return calculated over all nominal values in the payout phase of the pension.

To model the inflation scenarios another AR(1) model is considered based on the ability of this model to replicate the inflation path from recent history. Again the model allows for periods of deflation which are not expected to last long due to mean reversion, but nevertheless have been observed empirically. The inflation is important for determining the annuitization in the year of retirement and to determine the accumulated wealth X_t in real terms. The inflation thus follows an AR(1) path with mean reversion and normal shocks as described below:

$$Infl_t = Infl_{t-1} + m_{Infl} * (I\bar{n}fl - Infl_{t-1}) + \sigma_{Infl} * \varepsilon_{Infl}, \qquad (3)$$

in which m_{Infl} is the degree of mean reversion and $I\bar{n}fl$ is the mean to which the process reverts. The shocks ε_{Infl} and ε_r are assumed to have a fixed correlation coefficient $\rho_{Infl,r} = 0.5$, based on historical correlations. All other shocks are assumed to be uncorrelated, unless explicitly stated otherwise in e.g. a sensitivity analysis.

Earlier this research observed that it is important to be able to model returns in bad states of the world. Equity is notorious for it's negative skewness. Participants might not understand the whole pension scheme they are involved in but they do know what a stock market crash is. In order to be able to model a negatively skewed distribution, thus account for an occasional crash, two regimes of equity returns are considered. With a high probability, p, the equity returns are drawn from a bullish regime which has a mediocre positive return on average. With a small probability, $1 - p$, the equity returns are drawn from a bearish regime with a fairly large negative mean. Thus the model can account for an occasional market crash, underlining the riskyness, especially short term i.e. close to retirement, of investing in equity compared to bonds.

Mathematically the equity return is a mixture of two normal distributions to allow for skewness and leptokurtosis and is simulated with the help of the formulas below:

$$S_t = I_{u_t < p}(\mu_L + 0.5 * \sigma_L^2 + \sigma_L * \varepsilon_S) + I_{u_t \geqslant p}(\mu_H + 0.5 * \sigma_H^2 + \sigma_H * \varepsilon_S) \quad (4)$$

$$u_t \sim U(0, 1), \qquad (5)$$

where S_t is the return on equity, $I_{a<b}$ is an indicator function equal to one if the condition holds and zero otherwise, p is the chance of being in the low mean regime and where $0.5 * \sigma_X^2$ is added to the drift in order to be able to interpret μ_x as the geometric mean.

For more advanced models human wealth will also be risky. Growth factors for the salary vary over time, since the expected increase in salary for a 25 year old is different to the expected salary growth for a 65 year old. However salary growth is assumed to be non-negative. The risky human wealth growth factor is log-normally distributed to guarantee positive shocks and is simulated as follows:

$$g_t \sim LogN(ln(l_t/\sqrt{1 + (v/l_t^2)}), \sqrt{ln(1 + (v/l_t^2))}), \qquad (6)$$

in which l_t is the average wage growth for a person at time t and v determines the variance of the growth rates. This specification makes sure the geometric mean of the wage growth shocks is equal to l_t.

The salary for period t, L_t, is the previous salary multiplied by the growth factor if the indicator functions are equal to one, which corresponds to the chance of an individual keeping his job:

$$L_t = I_{p1_t > jobloss_t} * L_{t-1} * g_t, \tag{7}$$

in which $p1_t$ is uniformly distributed on a zero to one interval and $jobloss_t$ is the chance of losing ones job.

3.2 Pension accumulation

The premium is then calculated by subtracting a franchise from the wage, which is a standard amount over which no pension is build up, and then multiplying by the premium percentage ω:

$$premium_t = (L_t - franchise) * \omega. \tag{8}$$

The accumulated wealth W_t is updated via the following equation in which ϕ denotes the percentage invested in government bonds :

$$W_t = (1-\phi)*(W_{t-1}+premium_t)*(1+S_t)+\phi*(W_{t-1}+premium_t)*(1+R_t). \tag{9}$$

$annuity_t$ is the amount of wealth that is needed to pay a real amount of 1 each year to a grantee after retirement, with the year in which the pension starts building up as base year. This is calculated as follows:

$$annuity_{t=APL} = \sum_{i=APL+1}^{APL+LE} \left(\left(\frac{1+Infl_i}{1+r_i} \right)^i * \prod_{j=AgeS}^{APL} (1+Infl_j) \right), \tag{10}$$

where $AgeS$ is the age when the participant starts building up the pension. The summation term sums over every year in which an amount is payed. This is more expensive for high inflation and less expensive for a high interest rate. The product term corrects for the inflation during the accumulation period, since W_t is not corrected for inflation.

The final resulting pension or replacement ratio, for which a value of 1 would mean receiving 100 percent of the final wage as pension, is calculated as follows:

$$PRR = \frac{W_{APL}}{annuity_{APL}}/L_{APL}. \tag{11}$$

PR is equal to PRR without dividing by the final wage to obtain the yearly pension which is paid:

$$PR = \frac{W_{APL}}{annuity_{APL}}, \tag{12}$$

where annuity is the cost of buying a yearly allowance for the rest of ones life.

3.3   Utility

To be able to fairly compare differences in premia, working years, risk and expected values all is translated in a common unit, utility. This allows strategies to pick the combination of premium, accumulation phase length and lifecycle that results in the highest utility. First the pension pay-offs, premium and accumulation phase length need to be translated into utility. These numbers can be plugged in into different utility functions each with a unique way of determining what is best. Since participants have different preferences, several utility functions are considered to understand how goals can be achieved and which utility functions perform well. The utility functions in equations 16 and17 are mean-variance based. Equations 18 and 19 are based on loss aversion. Equation 20 considers a power utility function and equation 21 considers results in different percentiles. Finally equation 22 contains a utility function focussed on obtaining a minimum pension replacement ratio. An extensive motivation of each utility function and which function could be used for which participant is discussed in Appendix A.1. In the subsection below the utility functions are briefly motivated.

The total utility of all yearly pension payoffs can be calculated as follows:

$$U(Pen) = \sum_{i=1}^{LE} PR/(1+\delta)^i, \qquad (13)$$

where δ is the rate of substitution between a period and the next period. The cash flows are discounted to account for an individual's impatience to consume and the chance of living long enough to obtain said cash flows. This rate is assumed to be higher than the long term risk-free rate, otherwise an extremely rational individual, under the assumptions of the model, would consume nothing.

The utility of differences in premia is calculated as follows:

$$U(Pr) = -\sum_{i=AgeS}^{APL} (premium * (1+\delta)^{APL-i+1})/annuity_{APL}/E(L_{APL}), \quad (14)$$

thus the future value of every unit of wealth invested as premium is calculated to find the utility of all these premia at retirement.

The utility of differences in the accumulation phase length is calculated as follows:

$$U(Ra) = (minimumAPL - APL) * \xi, \qquad (15)$$

with ξ the factor which corrects for taxes and the pension premium paid over the final wage as well as the utility of having more leisure time available and $minimumAPL$ is the minimum age for which a person can choose to retire.

The following utility functions are applicable for optimization in a multidimensional parameter space. Each function has a unique way of determining the optimal balance of the expected value of the pension, the accumulation phase length, premia and risk. To be able to compare e.g. a situation where an extra premium is payed resulting in a higher pension to a case in which no extra premium is payed resulting in a lower pension, everything will need to be translated into a common unit. Utility will serve as this common unit.

The first utility function is maximized according to mean-variance preferences to find the optimal combination:

$$U_1(T, Ra, Pr) = U(Pr) + U(Ra) + E(U(Pen)) - 1/2 * \gamma * var(U(pen)). \quad (16)$$

An investor with mean-variance preferences likes a high expected value, but dislikes a large variation in his expected pension. Premia and the accumulation phase length are subtracted from the expected pension value with certainty. A risk neutral investor is assumed to only care about the expected value of the pension scheme, ignoring the variance associated with a scheme.

The risk neutral variant of this utility function simplifies to:

$$U_{1b}(T, Ra, Pr) = U(Pr) + U(Ra) + E(U(Pen)). \quad (17)$$

Another method of picking an optimal strategy is using an absolute loss function. An investor with an absolute loss function has a pension replacement ratio target at an arbitrary level of e.g. 1. This investor gets the same utility from a pension replacement ratio of 1.5 as he would for a level of 2. When the target is not achieved the 'pain' or difference in utility is the same for 0.1 compared to 0.3 as for 0.7 compared to 0.9. This utility function is modeled as follows:

$$U_2(T, Ra, Pr) = -I_{target>E(PRR)} * (target - E(PRR)) + (U(Ra) + U(Pr)) * \varphi, \quad (18)$$

where I is an indicator function equal to 1 if the condition is met and φ is a scalar translating differences in premia and the accumulation phase length to the pain experienced from not reaching the pension replacement ratio target.

A variant on this absolute loss function is the quadratic loss function. In this framework an investor assigns greater pain to a loss when the target is further away. In the example from the absolute loss function the difference in utility between 0.1 compared to 0.3 is 0.32 whereas the difference between 0.7 compared to 0.9 is 0.08 and thus smaller. In this example an investor the marginal utility of an extra premium or working longer would be 0.32 / 0.08= 4 times as much in the case where he is further away from his target. Mathematically this utility function looks as follows:

$$U_3(T, Ra, Pr) = I_{target>E(PRR)} * (target - E(PRR))^2 + (U(Ra) + U(Pr)) * \varphi, \quad (19)$$

where I again is an indicator function equal to 1 if the condition is met and φ again is a scalar translating differences in premia and the accumulation phase length to the pain experienced from not reaching the pension replacement ratio target.

The assumption that an investor derives no extra utility from higher pension replacement ratios might be too restrictive. His loss aversion might also not be quadratic or absolute. Realistic assumptions are that his utility function is concave and monotone increasing. Mathematically this means the first derivative of the function should be positive on the whole interval whereas the second derivative should be negative, resulting in an asymptote. This asymptote can be interpreted as 'enough is enough', at some point a person derives only very little marginal utility from a higher pension. A graphical depiction of such a function can be found in Figure 22 in Appendix B which is of the form -1/x for the default setting, the risk aversion parameter $\eta = 2$.

The mathematical formula associated with this utility function can be found below:

$$U_4(T, Ra, Pr) = (E(PRR - \lambda)^{(1-\eta)} - 1)/(1 - \eta) + U(Ra) + U(Pr), \quad (20)$$

where λ is a tuning parameter.

Another way of picking an optimal strategy is to look at the performance in certain percentiles. A strategy which performs well in the first percentile can be a good choice for an investor who cares about good performance in a horror scenario or 'bad state' of the world. Since risk aversion is assumed for a participant in a pension scheme, the strategy optimizing performance based on percentile results will look at scenarios worse or equal to the median. The expected pension replacement ratios in five percentiles are taken into account and extra premia and extra years of work are penalized:

$$U_5(T, Ra, Pr) = E(min(PRR, prctile(1, 5, 10, 25, 50), 1.5)) + (U(Pr) + U(Ra)) * \theta, \quad (21)$$

where θ corrects for the fact that the mean of the whole distribution is lower than the mean of the five lower half percentile results that are considered. Since the summed utility value should be corrected for the amount of percentiles considered and the fact that for e.g. an extra premium the expected value in the lower half percentiles is smaller than the overall expected value.

Another possible strategy is setting a minimum pension replacement ratio which should be achieved with a certain chance. Given that a number of strategies manage to achieve the minimum pension replacement ratio with enough certainty, the strategy with the highest median utility is chosen. The mathematical representation of this strategy is depicted below:

$$U_6(T, Ra, Pr) = U(prctile(50)) + U(Pr) +\quad s.t.\ \ E(I_{PRR>minimum}) > 1 - \alpha, \tag{22}$$

if $E(I_{PRR>minimum}) < 1 - \alpha \quad \forall\, \mathbb{R}$ then

$$U_6(T, Ra, Pr) = U(prctile(5)). \tag{23}$$

4 Data

To obtain economic scenarios from which performances of different pension strategies can be simulated, first realistic economic parameters are needed. This includes parameters for the asset return, inflation and interest rate both in terms of expected value and volatility and higher moments when relevant in order to model processes. The data set on which the parameters will be based has an annual frequency and ranges over a time period starting in 1891 up until 2015. This data is obtained from the internal PGGM database which in turn collected the data from Global Financial Data (GFD). The old data will probably not be representative for the current economy, however if a very stable pattern for a time series is observed over more than a century this could provide additional confidence in the chosen economic parameters. Additionally, knowing which extreme shocks happened in the past century and what triggered them can provide extra insights. The same holds for correlations e.g. between the interest rate and the inflation rate. The investment model only allows for one equity return and one fixed-income return. However bonds from three different countries are considered in order to form a broad picture which can lead to robust parameter choices. For the equity return the developed markets index is considered which contains about 90 percent of the total market capitalization, the other 10 percent being emerging markets. Table 1 provides summary statistics for the full sample period.

Since the economy has changed a lot over the past decades, the economic parameters will mainly be based on a sub sample of the full data set ranging from 1971 up until 2015 provided in Table 2.

Table 1: Summary statistics over the period of 1891-2015 for asset returns, inflation rates and the 10 year government bond level.

	Starting year	Mean	Median	Standard deviation	Skewness	Kurtosis	First order auto-correlation
Equity return Developed Markets	1891	11.5%	14.3%	16%	-0.57	3.3	0.02
InflationNL	1891	2.9%	2.5%	5%	0.35	5.4	0.56
10y German bond	1891	5.3%	5.3%	2.0%	-0.06	2.8	0.83
10y Japanese bond	1891	5.3%	5.4%	2.5%	0.32	3.9	0.89
10y US bond	1891	4.5%	3.6%	2.4%	1.57	5.3	0.94

Table 2: Summary statistics over the period of 1971-2015 for asset returns, inflation rates and the 10 year government bond level.

	Starting year	Mean	Median	Standard deviation	Skewness	Kurtosis	First order auto-correlation
Equity return Developed Markets	1971	12.3%	16.6%	17%	-0.85	3.4	-0.08
InflationNL	1971	3.3%	2.4%	3%	1.2	3.5	0.88
10y German bond	1971	5.8%	6.0%	2.5%	-0.33	2.3	0.92
10y Japanese bond	1971	4.1%	4.6%	2.8%	0.14	1.4	0.97
10y US bond	1971	6.5%	6.4%	2.9%	0.48	2.8	0.90

4.1 Equity returns

The mean for returns of Developed Markets, which is the simple average or arithmetic mean return thus not the geometric mean return, is considerably higher with respect to the level of the bond returns. This difference may be interpreted as a risk premium or equity premium, since equity returns have a much larger standard deviation. The Developed Markets' returns reflect the stylized fact that more large negative returns are observed, resulting in a negatively skewed distribution, and have a kurtosis quite close to the theoretical value of the normal distribution, implying limited tail probability mass. As expected the first order autocorrelation is low for equity returns. For the sub sample a negative first order autocorrelation is observed suggesting good years follow bad years and vice versa. Since there is no clear interpretation why this

would be the case and that it is only a modest autocorrelation this empirical property is ignored.
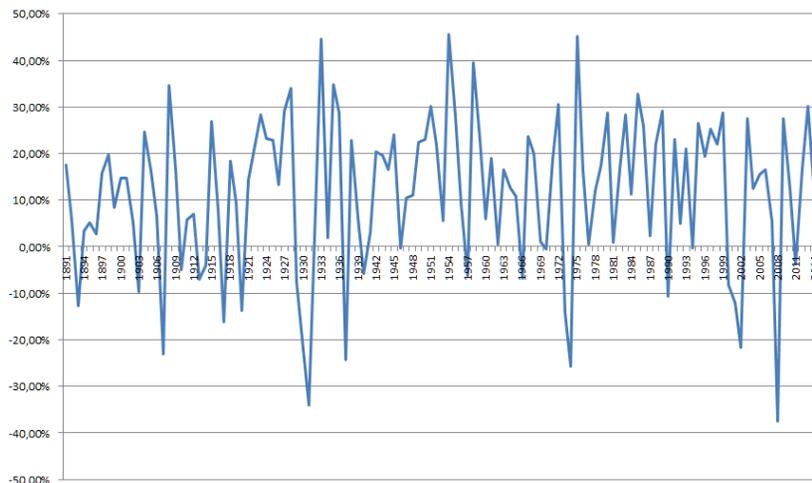

Figure 2: Returns for Developed Markets over time

Figure 2 shows the returns for the Developed Markets over time. The figure shows little evidence of heteroskedasticity in year on year asset returns, as do the standard deviations in Tables 1 and 2. The aggregated nature of the time series might be the reason not much leptokurtosis or volatility clustering is observed, which are both often apparent in financial time series. Furthermore the absence of autocorrelation in the equity returns also indicates that incorporating different regimes for equity returns would be of limited relevance, since there is no empirical evidence found for the existence of latent regimes.

Since the total market capitalization of the Developed Markets outweighs the total market capitalization of the Emerging Markets by a factor of nine to one and pension funds invest in both categories in roughly the same ratio the behaviour of the returns for Developed Markets is most important in the simulation of equity returns. The above statistics and figure provide evidence that there is skewness present in the returns of the developed markets and some leptokurtosis connected to this skewness. Because of the large weight of developed markets in a pension fund's equity portfolio, equity returns are simulated from a mixture of normal distributions accounting for these properties[10].

The parameters for this mixture of normals are estimated using the historical data for Developed Markets and the Expectation Maximization algorithm. However, as Table 3 shows, the estimated parameters are extremely sensitive to the chosen period.

The values for the arithmetic mean of equity returns in Tables 1, 2 and

---

[10]For ABP, the largest Dutch pension fund, the allocation towards Developed markets equity outweighed the allocation towards Emerging markets by 3 to 1 in 2015.

Table 3: Estimated means, standard deviations and probabilities for three different periods for a Gaussian mixture model with 2 regimes using the EM algorithm.

	1891-2015	1931-2015	1971-2015
Mean Low	0,02	-0,29	0,04
Mean High	0,19	0,15	0,23
Sigma Low	0,17	0,07	0,18
Sigma High	0,11	0,14	0,06
P low	0,44	0,05	0,57
P high	0,56	0,95	0,43

thereby the resulting estimates in Table 3 converted into geometric means are much higher compared to the geometric mean regularly used by pension funds. Dutch law prescribes a conservative geometric mean of maximally 6.75 percent when calculating recovery plans for pensions funds and pension funds tend to be conservative investors. These large differences induce us to consider a conservative geometric mean for asset returns (6.5 percent) in the default scenario, as well as a higher geometric mean based on the historical data (8.5 percent) in an optimistic scenario. A change in the geometric mean is dealt with by shifting the distribution as a whole i.e. shifting the entire distribution e.g. 2 percent to the right.

Since the parameter estimates are very sensitive to the chosen sample and the historical means are considered irrealistic by conservative investors, the choice is made to uphold the expert guesses for the recession probability and the standard deviations used as input for the Expectation Maximization algorithm, these values can be found in subsection 5.1.1. This enables the scenario generator to closely mimic the historical distribution in terms of skewness and extreme values, whilst using a lower mean. Figure 3 compares the two distributions. It shows that the desired lower mean for the simulated returns is realized by simulating less large positive returns than were observed historically.

4.2   Inflation

Figure 4 shows the path of the inflation rate over the past 125 years. The picture confirms the findings from Table 1 that there is a slightly positive skewness indicating that years of high inflation have been observed more than periods of high deflation. Furthermore the figure shows that the inflation rate has been relatively stable since about 1980. This phenomenon is often called the 'Great Moderation' in economics. The Great Moderation was a period in which the volatility of a lot of macroeconomic time series was greatly reduced. Greater independence of central banks, which allowed them to follow macroeconomic stabilization measures, is often cited as the main reason for this reduction in
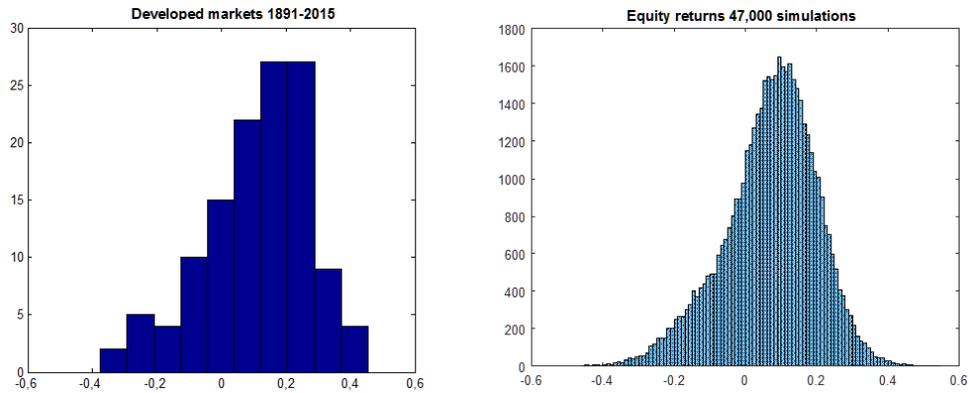
Figure 3: Histograms of historical and simulated equity returns.

volatility[11]. In more recent years the European Central Bank (ECB) aimed at an annual inflation rate of just below two percent, whilst preventing deflation and high volatility. The ECB has a lot of means for achieving its goal, such as quantitative easing[12], which aim to further stabilize the inflation rate. This increased stability becomes very clear when comparing the first order autocorrelation for the sub sample against the full sample autocorrelation. Nowadays the inflation rate is expected to be very closely related to the rate observed in the previous year, where this relation was much less strong in the past.
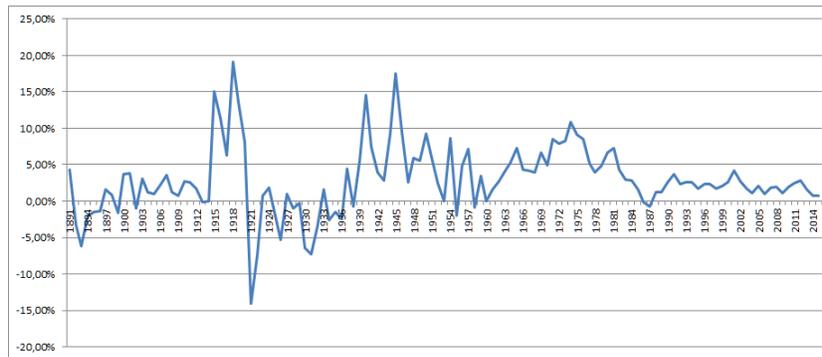
Figure 4: Inflation in the Netherlands since 1891

The inflation rate goal of the ECB is clearly reflected in the EU 30 year Break Even Inflation which is obtained from the Bloomberg database. This

---

[11]This was most notably brought forward by former Federal Reserve president Ben Bernanke in his speech "The Great Moderation" on February 20, 2004.

[12]Buying financial assets from commercial banks and other financial institutions, thus raising the prices of those financial assets and lowering their yield, while simultaneously increasing the money supply.

rate, shown in Figure 5, can be interpreted as the expected average inflation rate for the next 30 years. Since 2012 the ECB has been struggling to keep up the inflation rate, with the Break Even Inflation for shorter horizons close to zero or even negative. The gap between the rates shows the disbelief in the capacity of the ECB to achieve their goals in the next couple of years. In the long run however an expectation of 1.75 percent is observed which is close to the current policy goals of the ECB.
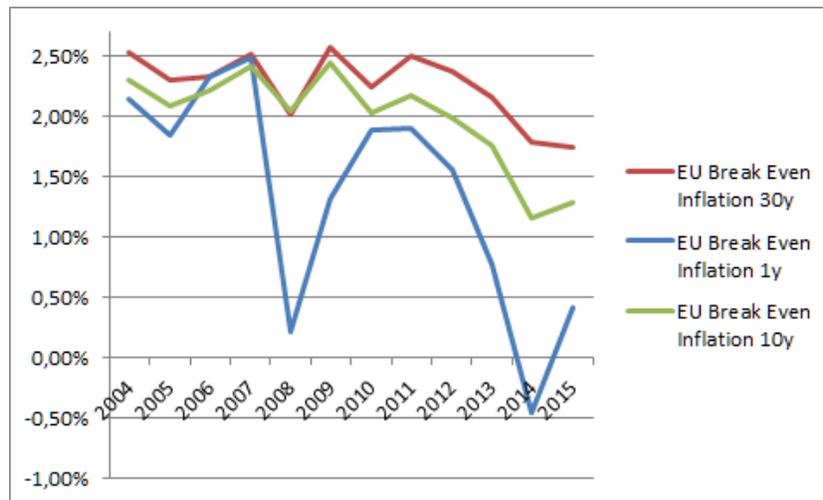

Figure 5: Break even inflation for different periods

The analyses above show a change in the volatility regime of the inflation rate following the period of the Great Moderation. Since then the fluctuations have been a lot smaller due to several causes. Based on these insights the simulations will contain a mean (1,75 percent) reverting inflation process with relatively small normal shocks and will allow for periods of deflation.

4.3   Interest rates

From Table 1 one can distill that the returns on 10 year government bonds have been roughly similar for the US, Japan and Germany in terms of mean. For the shorter period of Table 2 these means are considerably higher for the US and Germany. In Figure 6 the evolution of the interest rates over time is shown. As the figure shows, as well as the first order autocorrelations for both periods, 10 year government bond rates can be approximated rather good by the value in the previous year due to their relatively stable process. Though the average interest rate is not much higher in the shorter period, the returns on bonds have been very high in the past decades. This is due to the two parts which add up to the total return on a bond. The first simply being the level of the interest rate. The second part has to do with the development of the interest rate level.

When the interest rate goes down the bond with the old higher rate increases in value and vice versa. This second effect has had a considerable effect on the returns in the shorter period with interest rate plummeting throughout the period, thus driving up returns. With the theoretical bottom of nil percent in sight, and the practical bottom estimated at around minus one percent, these high returns are no longer durable. Nominal interest rates are closely related to the inflation rates. The real interest rate, which is the nominal rate corrected for inflation, is closely related to long term macroeconomic expectations. Ever since the financial crisis of 2008 interest rates have not been able to recover and suffer from a lack of trust in the economy in the long run. The demographic challenge of an aging population, which is one of the drivers that has kept Japanese interest low for a long time, is unprecedented in the EU and the US. The 50 year swap rate of just 1.57 percent in 2015, down from 5.45 percent in 2001, indicates the uncertainty and possible perseverance of the low interest rate regime.
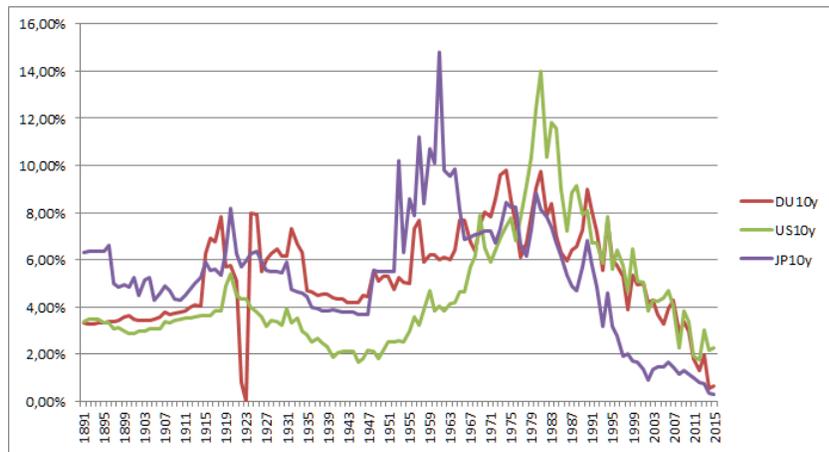
Figure 6: Development of 10 year government bond rates since 1891

The uncertainty regarding the low interest rates induces one to compare performance of the DC system for two different parameter sets of the interest rate. The first and default option has a high mean based on historical data (4 percent), whereas the second has a low mean based on recent developments and poor macroeconomic expectations (2 percent). Both will have a degree of mean reversion (10 percent) and will allow the interest rates to be negative. Due to the mean reversion values below the practical bottom will be rare, therefore unrealistically negative rates will not form a problem. Another option is to model the mean to which the process reverts to follow a random walk in a region or in a unbounded region, which is researched in a sensitivity analysis.

The variance of interest rates can be quite different between periods as Figure 7 shows. During the early years of the sample interest rates were relatively stable for all three countries considered. The first real increase in variation is

observed during and after the first world war. Especially for Germany a large increase in volatility is visible in the same period which was notorious for its hyperinflation in Germany. After the volatility seems to have cooled of during the 1930's it spikes up again after the second world war, this time mostly in Japan. Interesting to see is that volatility has never again been as low as during the second world war. For the US year on year differences have been much larger during the second part of the sample compared to the first. Volatility for Japan has been lowest the past 10 years, which could be due to the interest rate being close to its theoretical border and hence providing not much room for movement. These observations suggest a positive relation between the level of the interest rate and the volatility of interest rates.
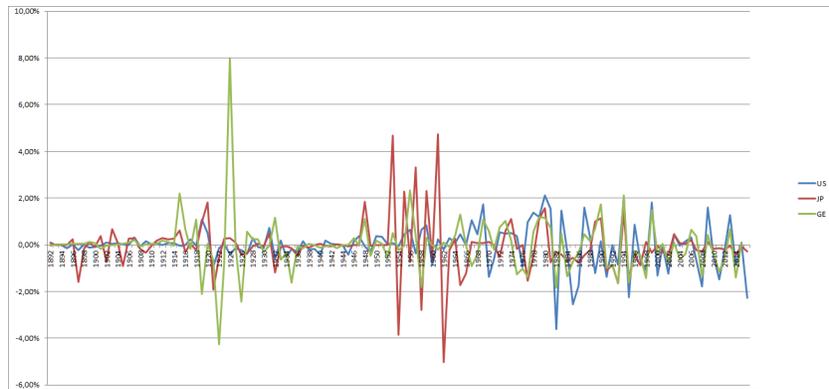
Figure 7: First differences of 10 year government bond rates since 1891

4.4 Human Wealth

To model the income of a participant in a pension scheme a growth factor for the salary is needed dependent on the age. PGGM provided data on the average realized increase in salary for all active participants in their database. This data is represented by the red line in Figure 8. The green line is the smoothed version which will be used in the model. A negative salary growth close to retirement is deemed undesirable for the purposes of this research. Negative salary growth would reduce the denominator of the pension replacement ratio providing a third positive effect on this ratio for continuing working, besides a shorter decumulation phase and a longer accumulation phase. Furthermore a yearly pay cut for someone near retirement might not be realistic. Therefore salary growth shocks are simulated from a lognormal distribution and the salary growth mean is always nil percent or larger, thus always avoiding a pay cut.

It is important to know that almost all participants of PGGM work in the health sector. Nevertheless these salary growth estimates can grosso modo serve as a proxy for employees in every sector. The salary growth is without the

mandatory increase in salary following the collective employment agreement (CAO), which is assumed to be in step with inflation.
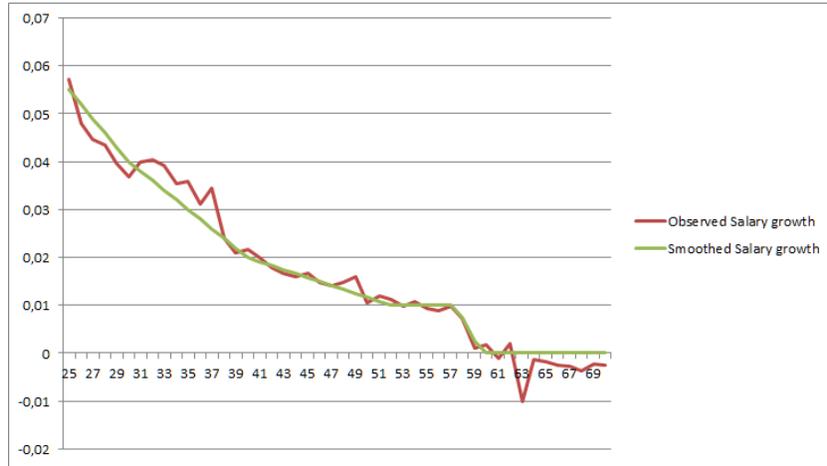
Figure 8: Salary growth curve exluding collectively agreed raises

4.5 Correlations

Whilst modeling these economic parameters, it is important to consider correlations. The most well known correlation is the correlation between the interest rate and the inflation rate. Together they determine the real interest rate. When inflation is high or expected to be high, lenders require a higher nominal rate to counter the devaluing effect of the inflation in order to obtain a decent real interest rate. Inflation, interest rate and equity returns, even the salary growth curve and shocks, are all related to macroeconomic factors such as the economic growth inter alias. Therefore the historical correlations between said parameters are analyzed.

Table 4: Correlations between 1971-2015 for inlation, interest rates and equity returns.

	Inflation NL	Germany 10y yield	US 10y yield	Japan 10y yield	ERDM
Inflation NL	1,00	0,64	0,35	0,68	-0,10
GE 10y rate	0,64	1,00	0,82	0,91	0,05
US 10y rate	0,35	0,82	1,00	0,84	0,19
JP 10y rate	0,68	0,91	0,84	1,00	0,12
Equity returns Developed markets	-0,10	0,05	0,19	0,12	1,00

In Table 4, as expected, a strong positive correlation between the Dutch

inflation rate and the interest rate on foreign 10 year state bonds is apparent. The correlation is much higher for the German rate compared to the US rate. This is not an unexpected result since Germany is the most important trading partner for the Dutch economy and both are subject to many similar monetary and political shocks, especially since the Euro/ECB and the Lisbon treaty. The observed correlation between the Dutch inflation rate and the Japanese rate is even higher for this time period suggesting similar factors influencing both rates.

The correlation between the Dutch inflation rate and both asset classes is slightly negative in the sample period. The correlation computed over the whole period between the Dutch inflation rate and the Developed markets returns is a small positive value of 0.038. This suggest no strong lasting correlation between the (Dutch) inflation rate and asset returns.

The correlation between the interest rates and asset classes is positive. Especially the US rate shows a moderate correlation with asset returns in recent years. A possible interpretation is that the US 10 year treasury rate is globally seen as a proxy for the risk-free rate. Thus in times where stocks are becoming more attractive the risk-free rate is also expected to increase since there is less demand for risk-free assets. Another way of putting it is that the equity risk premium is expected to stay roughly the same therefore the risk-free rate should rise when expected equity returns rise. Despite this evidence the correlation will not be standard in this research. The Dutch inflation and interest rates are more closely related to the German rate which shows less of these characteristics. However the correlation between equity returns and interest rates will be covered in the sensitivity analyses.

Correlations between variables in the same categories are positive and quite high as expected. Interest rates are more closely related when compared to equity returns due to the more systematic character of their risk, compared to more idiosyncratic risk in different equity classes.

In the economic scenarios only the correlation between shocks to the interest rate and inflation rate will be incorporated by default and will have a value of 0.5. A correlation between equity returns and both inflation and interest rates is investigated as a sensitivity analysis with values based on the observed correlations in Table 4.

5 Simulation

The next section will describe the input parameters used in the simulation as well as the algorithm that is used by the simulation to pick the optimal choices for an individual in each period. An extensive explanation of the parameters used in the different strategies with numerical examples can be found in Appendix A.1.

5.1 Parameters

Throughout the results section one basic economic scenario generator is used. For the final model the research will look into several variants of this default scenario. For example the persistence of the current low interest rate low inflation environment. The changes that these other scenarios make on the basic economic scenario generator are specified in the relevant section, all parameters that are not mentioned remain the same as specified in this section.

5.1.1 Equity returns

The mean of stock returns can refer to either the arithmetic mean or the geometric mean. The arithmetic mean is not equal to the geometric mean[13]. The arithmetic mean minus half of the variance is equal to the geometric mean, which is defined as the average return in this research. Equation 4 accounts for these characteristics with μ_X equal to the geometric mean and σ_X equal to the variance of stock returns.

In order to take the empirically found skewness, and to a lesser extent leptokurtosis, into account; a mixture of normal distributions is used. Since the EMGM algorithm for estimating the parameters was extremely sensitive to the chosen sample, the expert guesses were upheld since they could replicate the empirically found distribution as Figure 3 showed. This means that for the default scenario generator the following parameters are used: The 'recession' probability p is equal to 0.16. The geometric mean conditional on being in a recession, μ_L, is -0.1185, with a standard deviation, σ_L, of 0.1. The probability of being in a bullish regime, $1-p$, is logically 0.84. The geometric mean for this regime, μ_H, is 0.1, with a standard deviation, σ_H, of 0.1. Therefore the unconditional geometric mean is equal to 0.065 in the basic scenario. With a standard deviation, σ, of 0.13, a skewness equal to -0.45 and an excess kurtosis of 0.22. Since equity returns are considered unpredictable, no autocorrelation between regimes or returns is assumed. Furthermore no correlation with movements in the interest rate, inflation rate or wage related developments is assumed.

5.1.2 Interest rate and inflation rate

For the interest rate and inflation a burn in period will be used. This means that the interest rate and inflation rate are simulated an extra period previous to the start of the actual simulation. Doing so ensures that the different scenarios have different starting values, replicating the real world in which participants start building up their pension in periods with a different interest and inflation climate. Using only one starting value, e.g. the mean, entails that extreme rates are never observed in the early phase of the accumulation period. This is unrealistic and it can harm the research if every scenario starts similarly. I.e. if a very low starting interest rate would severely harm performance, this would not

---

[13]A return of 50 percent in period 1 and -50 percent in period 2 would result in an arithmetic mean of 0 percent whereas the average return or geometric mean is $\approx$ -13 percent.

be uncovered. To dismiss any bias a burn in period of 25 years is used after which the distribution of the interest and inflation levels has reached an equilibrium due to the mean reversion present in both time series. The expected value of e.g. the 95 percentile interest rate one year after all 1000 scenarios started at the same value is vastly different compared to the same expected value after 25 years of interest rate movements are simulated. A period of 25 years is enough for the distributions to reach an equilibrium, i.e. difference in the expected value of the 95 percentile interest rate after 25 or 26 years is negligible.

As stated earlier in the data section, the mean to which the interest rate will revert in the basic scenario generator, $\bar{r}$, is 4 percent. σ_r, the standard deviation of the process, is equal to 0.007 and the mean reversion parameter, m_r, is set at 0.1. The long term mean of 4 percent is slightly lower than the historical average for the 3 bonds considered. However with the interest rate level at a historically low point 4 percent seems a prudent choice. The mean reversion and standard deviation are closely linked. In the modern economy, after the 'great moderation', several institutions obtained independent powers to stabilize the interest rate. Therefore the standard deviation chosen is also lower than the historic average. The mean reversion is set low enough for the interest rate process to remain random, but high enough to keep in within reasonable bounds.

For the inflation rate the long term mean in the basic scenario generator, $\bar{I}$, is 1.75 percent, which is based on the EU 30 year break even inflation combined with the ECB goal of an inflation just under 2 percent. σ_I, the standard deviation of the inflation rate, is equal to 0.007 and the mean reversion parameter, m_I, is set at 0.1. Again to include some randomness in the process whilst keeping it within reasonable bounds.

The inflation rate and the interest rate combined make up the real interest rate, which is considered to be relatively stable. In order to realize this stability a positive shock to the inflation rate should often be positively correlated with the shock to the interest rate. Empirically this positive correlation was found in the data section with values ranging from 0.35 up until 0.68. The default scenario generator will use a correlation of 0.5.

5.1.3 Wage growth and job risk

The wage growth in the basic scenario generator is dependent on the phase of the accumulation period t. Each mean can be found in Figure 8 in the section 4, the data section. To allow for uncertainty of this wage shocks the log normal distribution from equation 6 is considered. The main advantage of this log normal distribution is that it does not allow for negative shocks to the salary. Such negative shocks are undesirable since negative shocks in a stable employment are uncommon. Additionally a negative shock to the salary means lowering the target pension and the minimum, which cannot be united with the assumption of an individual trying to maintain or improve the highest obtained consumption level. The variance determinant v is 0.1 percent, which is independent of the expected wage growth l_t.

The risk of losing ones job is assumed to be constant throughout the entire accumulation period and to be equal to five percent. Ergo no cyclical effects are taken into account in the default scenario generator. Including a positive correlation with the investment climate would mean that poor pensions would often be found for low final wages. Such a correlation therefore would have a soothing effect on more extreme outcomes, making it less relevant for the purposes of this research. Another problem is how one should relate changes in the investment climate to wage developments in e.g. the Dutch health care sector.

Each period a number is drawn from a standard uniform distribution. When this number is smaller than the probability of losing ones job, the participant obtains no income that year. A participant without a job is assumed to have a constant chance of 0.66 to get a new job in the following year, thus after five years without a job this chance of getting a job is still 0.66 under these specifications. The scenarios again make use of a standard uniform distribution to simulate this re-entry chance. This is a fairly crude way of modeling job risk, but there are no clear recession or expansion interpretations associated with the interest rate level or equity returns which makes it tough to include correlations.

During the period in which the participant can retire the probability of having a job is equal to one. This choice is made to prevent situations in which the participant chooses not to retire but receives no income in one or more of the final working years, which would mean the participant is de facto retired but not according to the model.

5.2   Algorithm

The algorithm used in this research simulates a 'tree' inside another 'tree' of possible paths. This means computation time can become a problem when a high number of scenarios is used. For the purposes of this research 1000 scenarios is considered enough, since smooth distribution functions are obtained with this number of scenarios.

The algorithm first generates 1000 different economic scenarios, which is the first tree with 1000 different 'branches' or participants. Each of them containing unique paths for equity returns, the interest rate etc. Each of these branches has a length equal to the maximum accumulation phase length. This tree and its 1000 branches or participants will form the basis of the simulation. This way 1000 pensions are simulated each starting in different investment climates and with unique returns throughout the accumulation phase. This ensures a broad picture of the possibilities and limitations of the considered pension schemes.

Next, using a particular branch or participant at the first point in time as starting value, another tree is simulated again containing 1000 branches with length equal to the maximum accumulation phase length minus the elapsed time. The performance of all combinations, of premia, accumulation phase lengths and lifecycles, is calculated using this newly simulated tree. Each combinations has 1000 different pension replacement ratios, which are translated to utility via the different strategies. The combination which resulted in the highest utility

is chosen for the participant and the newly simulated tree is deleted. Based on the paths in the 'basis tree' and the newly chosen strategy, the wealth is updated and the algorithm moves on to the next point in time for which again a tree will be simulated. This will be repeated until the participant in the basis tree is retired. Now the algorithm moves on to the second participant, until all 1000 branches or participants in the basis tree are retired with a certain pension replacement ratio.

For computational feasibility the research uses a dynamic strategy but does not allow the strategy to take future actions into account. Using backwards recursion or stochastic programming this could be done. However due to the large number of parameters it is very time consuming to determine a strategy for every possible combination of wealth, interest rate level, inflation rate level, and salary at a point in the future and then move backwards through the tree[14].

The following fragment of pseudo code illustrates the same algorithm:

PSEUDO CODE:

1. Create 1000 scenarios for stock returns, inflation, interest rate and human capital.

2. For scenarionumber=1 and t =1 simulate 1000 paths for stock returns, inflation and the interest rate and human capital, with respect to the current values for one of the thousand paths simulated in step 1 at time t.

3. Choose the combination (premia, accumulation phase length and lifecycle) that maximizes expected utility according to one of the strategies.

4. Update the accumulated pension wealth based on the strategy chosen in step 3 and the path simulated in step 1.

5. Set t = t+1 and move to step 2 until retirement.

6. Set scenarionumber=scenarionumber+1 and t=1 and move to step 2.

6   Results

Figure 5 provides an overview of the intermediate models and the steps towards the final model.

All simulations in this section will make use of the standard values for the default scenario generator as stated in chapter 5.1 unless explicitly stated otherwise. The default accumulation phase lasts 45 years, where after the participant is expected to receive a pension for 20 years. The default lifecycle starts with a mix containing 70 percent equity which is reduced prior to the retirement date, a graphical illustration of this investment mix is provided in Figure 21 in Appendix B. The wage of the participant is normalized to 1 each period and grows along with the realized inflation.

---

[14]The current model takes around 17 hours to simulate 1000 scenarios on a professional PGGM computer. Using backwards recursion is therefore not an option since e.g. only picking an optimal lifecycle via backwards recursion would increase the computation time from $\sum_{x=1}^{47} x$ to $\sum_{x=1}^{47} \sum_{i=1}^{x} i$.

Table 5: An overview of the intermediate models used in different subsections of the results section.

	Lifecycles	Premium	APL	Human Capital
Section 6.1	Fixed	Fixed	Fixed	Riskless
Section 6.2	Relaxed	Fixed	Fixed	Riskless
Section 6.3.1	Relaxed	Relaxed	Fixed	Riskless
Section 6.3.2	Relaxed	Fixed	Relaxed	Riskless
Section 6.4	Relaxed	Relaxed	Relaxed	Riskless
Section 6.5	Relaxed	Relaxed	Relaxed	Risky
Section 6.5.1	Stability based domain	Stability based domain	Relaxed	Risky

6.1 Static premia, accumulation phase length and lifecycles

The most simple DC strategy is setting an accumulation phase length, a fixed yearly premium and a lifecycle beforehand. The results for sticking to this strategy regardless of economic or personal circumstances are evaluated in this paragraph. Extensive results on how each of these three choices influences the expected pension replacement ratio are provided in Appendix C in paragraph 9.1.1. Choosing the fixed lifecycle with the highest equity weight unsurprisingly results in the highest expected pension value. This is by construction due to the positive equity premium included in the economic scenario generator, meaning a higher expected value for equity returns compared to the long term average of the risk-free rate or bonds, because an investor requires a higher return to be compensated for the risk faced when investing in equity compared to bonds. A less risky lifecycle starting with 60 or 70 percent equity generates more balanced returns. Though the expected value is lower, less extremely poor outcomes are observed resulting in the highest fifth percentile results for these lifecycles. Adjusting the premium, ceterus paribus, is simply a matter of multiplication. Putting in twice as much premium will result in a twice as high pension. The probability density function of pension results can therefore be shifted towards the right and slightly flattened by increasing the premium. The chance of obtaining a realistic target pension increases with the probability mass in the left tail shifted past the target due to the higher premium. This chance will increase asymptotically towards one as the left tail mass becomes smaller. For the length of the accumulation phase the positive effect of an increase is twofold. Firstly the phase over which a pension is accumulated and returns are made becomes larger and secondly the phase in which the pension is decumulated decreases. Therefore the accumulation phase length is a powerful instrument for increasing the chance of reaching a certain target pension.

The three dimensional plot, Figure 9, nicely illustrates the most important points made in this section.

The upper plane is highest for lifecycles containing more equities and high retirement ages. The figure also shows that the difference in pension replacement ratios is higher for long accumulation phases due to the twofold effect of an increase in the accumulation phase length. The lower plane shows the same
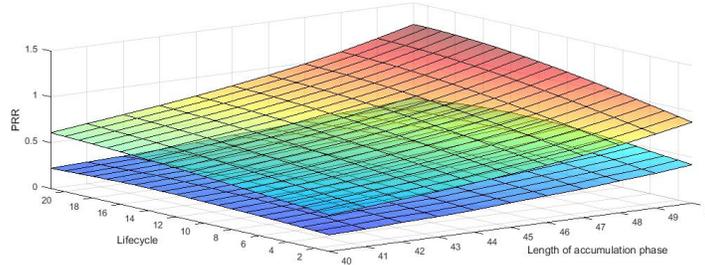
Figure 9: The upper plane describes the expected value, whereas the lower plane describes the fifth percentile. A fixed premium of 10 percent was used to construct these planes.

curve in fifth percentile results for more accumulation period lengths as Figure 23 in the individual results for fixed lifecycles in paragraph 9.1.1 in Appendix C.

6.2 Managed DC lifecycles

Managed DC chooses an optimal premium, accumulation phase length and lifecycle. These optima are reselected after every certain period of time, in this research yearly. The unmanaged DC benchmarks simply choose a premium, accumulation phase length and lifecycle and stick to it the entire accumulation phase. The first step towards managed DC is allowing for a yearly choice for a lifecycle. Though premia and the accumulation phase length are fixed, the seven utility functions or strategies are used to choose between outcome distributions of the different lifecycles. Results can be found in Table 6.

All managed strategies perform roughly the same. The only notable difference is the higher mean and median but lower percentile results for strategy 6 compared to the other strategies. This is not unexpected since strategy 6 is the risk neutral strategy and clearly holds a higher percentage of assets as can be seen in the final column. The main difference between the unmanaged benchmarks and the managed strategies is the ability to time the market. When interest rates are above their assumed long term mean, managed strategies invest more heavily in bonds and vice versa. Due to the mean reversion present in the economic scenario generator this is a profitable strategy. The managed strategies can also reduce their risk close to the retirement date. The average number of the lifecycle is higher closer to retirement however. This does not mean that the percentage of equities is higher since the lifecycles linearly decline towards zero percent equities. Nevertheless, the managed strategies are allowed to take a little extra risk close to retirement adding to their expected result, whereas the unmanaged strategies simply have to follow the decrease corresponding with their lifecycle.

31

Table 6: Table containing results for managed DC lifecycles. Mean and median denote the average and median pension replacement ratio obtained by the strategy. prctile x denotes the xth percentile pension replacement ratio obtained by the strategy. Lifecycle* denotes the mean for the first 30 years in which the lifecycles are constant and Lifecycle is the mean over the entire period. Unmanaged 1 and 2 hold an asset mix corresponding to their static lifecycles 15 and 13 respectively.

	Mean	Median	prctile 5	prctile 1	Lifecycle	Lifecycle*
Strategy 1	1,32	1,08	0,46	0,34	13,8	12,6
Strategy 2	1,30	1,10	0,48	0,34	15,0	13,6
Strategy 3	1,32	1,10	0,47	0,34	15,4	14,3
Strategy 4	1,32	1,10	0,47	0,34	15,3	14,2
Strategy 5	1,33	1,11	0,47	0,34	15,2	14,1
Strategy 6	1,38	1,14	0,46	0,32	15,3	16,8
Strategy 7	1,29	1,09	0,47	0,34	14,2	12,6
Unmanaged 1	0,95	0,82	0,38	0,29	15,0	15,0
Unmanaged 2	0,91	0,81	0,38	0,30	13,0	13,0

Table 7: Frequency of different levels of the interest rate r and the percentage invested in equities associated with these levels. Based on the first 30 years of the accumulation phase to allow full investment in equities or bonds.

	Frequency	% Equities
r <0	0,5%	98,8
0 <r <2	10,0%	91,4
2 <r <4	39,4%	72,3
4 <r <6	39,1%	46,6
6 <r <8	10,4%	17,9
8 <r	0,6%	1,5

Table 7 shows the extent to which mean reversion is used to profit by strategy 1, which is exemplary for all strategies. For the large middle categories containing almost 80 percent of the probability mass, on average 60 percent is invested in equities. For more extreme intervals, containing about 10 percent of the probability mass each, the table shows that almost all is invested in one asset class. The equities class is a little more attractive since still 17 percent is invested in equities for very high interest rate levels, whereas only 9 percent is invested in bonds for very low interest rate levels. Even though the expected return for equities (6,5 percent) might be lower than the expected return for the interest rate which is the level and the expected mean reversion bonus for levels above the mean, there is still a diversification bonus from a percentage of equities. For very low interest rate levels the difference in expected return is so high that this diversification bonus is relatively small. For very extreme levels almost the entire portfolio is made up of one asset class.

6.3 Managed DC lifecycles and premia/retirement age

The next step towards a managed DC scheme with flexible lifecycle, premium and retirement age is to allow for a flexible lifecycle as well as a flexible premium or retirement age. To be able to choose a strategy the seven utility functions, discussed in part two of the methodology, are needed.

6.3.1 Relaxed lifecycles and premia

The parameter space consists of the 21 lifecycles and five different premium percentages ranging from 9 percent until 13 percent[15]. The length of the accumulation phase remains fixed at 45 years. For each point in time all seven utility functions therefore will consider 1000 simulated paths for 21 x 5 parameter combinations and will choose the combination for which the utility function is the highest. Results are provided in Table 8.

Again all strategies show a much higher mean and median compared to unmanaged benchmarks. The same holds for the percentile results. Two strategies almost always choose the highest possible premium. The risk neutral (6) strategy decides to put in the maximum premium if the expected return on this extra premium is higher than their discount rate, which is the case for an investment in equities[16]. During the phase prior to retirement a portfolio always holds an increasing percentage of bonds, ergo the risk neutral strategy does not strictly choose the maximum premium. The mean variance (4) strategy has the lowest average premium combined with the lowest pension replacement ratio results.

---

[15]These settings are chosen since the unmanaged pension replacement ratio for the maximum premium is close to 100 percent for these settings.

[16]The discount rate is expected to be higher than the average risk free rate, but lower than the expected return on equities.

If the discount rate is lower than the risk free rate a person would always postpone consumption. If the discount rate is higher than the highest expected return for all asset classes, in this case the expected return on equities, a risk neutral person would always immediately consume everything.

Table 8: Table containing results for managed DC lifecycles and premium. Premium denotes the average premium percentage used throughout the accumulation phase. Lifecycle* denotes the mean for the first 30 years in which the lifecycles are constant.

	Mean	Median	prctile 5	prctile 1	Lifecycle	Lifecycle*	Premium
Strategy 1	1,44	1,20	0,53	0,40	13,8	12,8	11,6
Strategy 2	1,43	1,22	0,55	0,41	14,8	13,2	11,8
Strategy 3	1,35	1,15	0,53	0,39	15,3	14,1	11,0
Strategy 4	1,22	1,04	0,50	0,36	15,4	14,4	10,3
Strategy 5	1,51	1,29	0,56	0,40	15,3	14,4	12,4
Strategy 6	1,62	1,35	0,54	0,38	15,3	16,8	13,0
Strategy 7	1,45	1,24	0,54	0,40	13,7	11,8	12,6
Unmanaged 1	1,12	0,97	0,45	0,34	15,0	15,0	13,0
Unmanaged 2	0,77	0,67	0,31	0,24	15,0	15,0	9,0
Unmanaged 3	1,07	0,96	0,45	0,36	13,0	13,0	13,0
Unmanaged 4	0,74	0,66	0,31	0,25	13,0	13,0	9,0

All in all the results indicate that strategies 1 to 4 reduce premia, whereas strategies 5 to 7 keep depositing high premia. Nonetheless strategies 5 and 7 can reduce the premium a little when 'enough' pension has been accumulated. Strategies 1 to 4 are based on loss aversion or reaching a minimum, when this target is comfortably in sight they reduce premia. Strategies 5 to 7 are based on accumulating wealth and derive utility from every extra unit of money, therefore they reduce their premium later or not at all. These phenomena can be seen in Figure 10 for strategies 1 and 7. The figure containing the other strategies can be found in Appendix C in Figure 27. The worst performing strategy (4) still outperforms the best benchmarks for each pension replacement ratio indicator with over 20 percent less premium paid. The differences in mean and median for the managed strategies seem to be related to the average premium and little differences are observed for percentile results.

6.3.2   Relaxed lifecycles and accumulation phase length

The parameter space consists of the 21 lifecycles and five different lengths of the accumulation phase ranging from 43 until 47 years[17]. The premium percentage is fixed at 11 percent. The parameter space is therefore as large as it was for the previously considered premium lifecycle variant.

Unlike the results for the premia, the strategies all have a retirement age lower than the mean of the parameter space, which is 45 for these settings. Thus, no strategy tends to keep working regardless of the pension situation. This corresponds with the intuition about human behaviour the strategies should reflect. Some people may choose to put in a high premium since they expect to

---

[17]These settings are chosen since the unmanaged pension replacement ratio for the maximum premium is close to 100 percent for these settings.
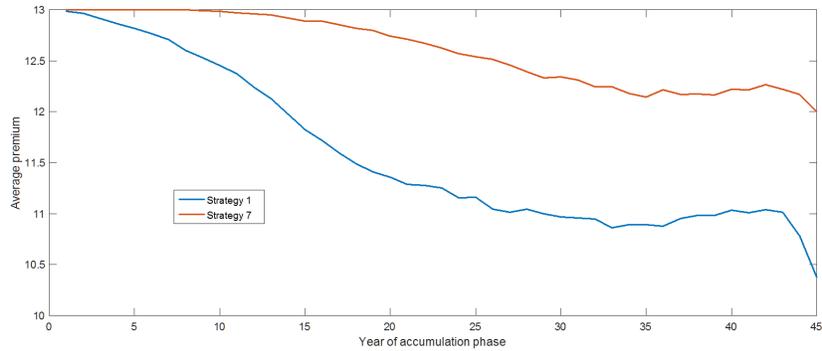
Figure 10: Average premium in percentages payed during each year of the accumulation period for strategies 1 and 7.

get a good return, whereas others might choose to reduce their premium when near their target pension. This should not hold for the accumulation phase length. Some strategies reckon with a high accumulation phase length in the early phases of the accumulation period, where there is a lot of uncertainty. When time progresses and the results can be estimated with higher certainty they reduce the accumulation phase length if the results are sufficient. In the final year(s) one can either take a pension replacement ratio with certainty or choose to work at least one year extra. The minimum strategy (1) cares about the pension being higher than a certain ratio, whereas the others require an increase in expectation often depending on the current status.

Strategies 1, 2, 5 and 7 outperform the unmanaged benchmarks in every facet. The risk neutral strategy (6) shows very poor low percentile results directly related to the risk attitude of the strategy. The absolute loss (3) and mean variance (4) strategies very often choose to retire as early as possible which has repercussions on the pension result. Strategies 1, 2 and 7 all continue working for very low results. The quadratic loss (2) and utility curve (7) strategies can get high benefits from increasing their results from a poor position, whereas the minimum (1) strategy simply works until a minimum ratio of 0.6 is achieved or the maximum age or retirement is reached. This explains why these strategies perform so well in the lower percentiles. These strategies also have less equities in their mix compared to strategies 3 to 6, which also helps explaining why they outperform said strategies. Strategy 3 still provides roughly the same pension results as an unmanaged benchmark but requires 3.7 years of working less to do so. A histogram is provided in Figure 11 for the retirement ages of strategies 1 and 7 to obtain some intuition on their behaviour and distribution. For the other strategies this histogram is provided in Appendix C in Figure 28.

The histogram clearly shows a bi-modal distribution with very little probability mass is located in the middle, especially for strategy 1. Strategy 1 either retires when the minimum pension replacement ratio is reached after 43 years, or keeps working until the maximum retirement age or the minimum pension

Table 9: Table containing results for managed DC lifecycles and the accumulation phase length. APL denotes the average accumulation phase length for a strategy. Lifecycle* denotes the mean for the first 30 years in which the lifecycles are constant.

	Mean	Median	prctile 5	prctile 1	Lifecycle	Lifecycle*	APL
Strategy 1	1,16	0,99	0,56	0,42	13,4	13,2	44,7
Strategy 2	1,10	0,93	0,56	0,42	13,5	12,8	44,1
Strategy 3	1,09	0,93	0,46	0,33	14,5	14,8	43,3
Strategy 4	1,08	0,91	0,46	0,34	14,4	14,5	43,3
Strategy 5	1,16	1,09	0,48	0,36	14,0	13,9	44,1
Strategy 6	1,32	1,07	0,40	0,29	14,2	16,7	44,4
Strategy 7	1,19	1,08	0,50	0,40	13,4	12,8	44,6
Unmanaged 1	1,06	0,93	0,47	0,35	15,0	15,0	47,0
Unmanaged 2	0,78	0,69	0,33	0,25	15,0	15,0	43,0
Unmanaged 3	1,02	0,91	0,47	0,34	13,0	13,0	47,0
Unmanaged 4	0,75	0,68	0,34	0,24	13,0	13,0	43,0
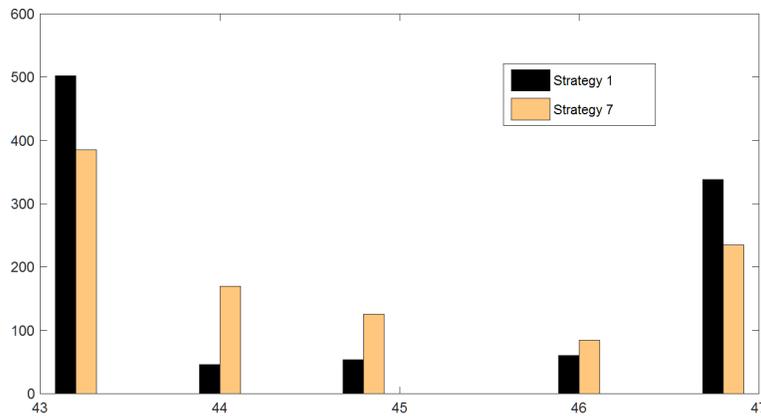
Figure 11: Histogram of realized retirement ages for selected strategies.

replacement ratio is reached. The histogram clearly shows that a considerable but small fraction reaches the minimum every year in between the two modes, but that most of the probability mass is located at the maximum duration of 47 for strategy 1. For strategy 7 a slightly different pattern is apparent. A large portion of about 40 percent has accumulated enough wealth to retire after 43 accumulation years. Thus, in the majority of cases the participant keeps working until the utility gained from working an extra year is smaller than the expected gains. In almost a quarter of scenarios strategy 7 chooses the maximum accumulation period. When premia are also relaxed the second mode of these distributions can be expected to become smaller. In tough scenarios most

strategies are inclined to put in extra premia therefore reducing the probability of the second mode.

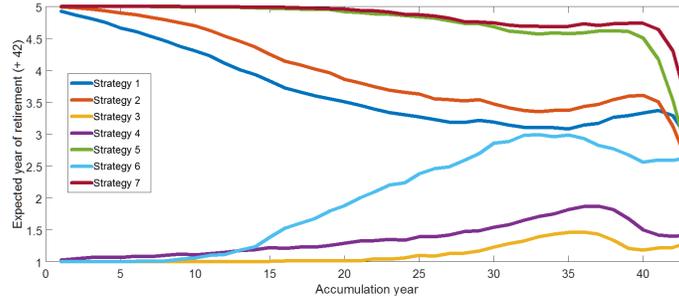Figure 12: Development of expected retirement year over time for all managed strategies.

Figure 12 shows the development of the expected year in which the participant will retire over time. Some strategies start with the latest retirement year possible and try to advance this date when returns and the economic climate make this possible, while others postpone if it is impossible. Strategies 1, 2, 5 and 7, which performed best, all take the first approach which can be characterized as more defensive. Albeit that strategies 5 and 7 decide to advance the retirement closer to the relevant period. Strategy 6 is the stranger in the midst that continues working when a high return can be made, which is more often true if there is a lot of capital already at hand.

6.3.3 Conclusion

All strategies start defensive when premia and lifecycles are relaxed. After some years a dichotomy becomes visible. Strategies 1 to 4 can be characterized as loss aversion strategies, with the other strategies based on accumulating the highest possible results. Therefore the first strategies group shows a decline in premia when loss, the chance of not reaching the target, is averted which is much less apparent or absent for the second group. For the accumulation phase length a similar division is visible. Strategies 1, 2 and 7 have a non linear loss aversion enabling them to avoid very poor results by postponing retirement.

All in all the managed strategies perform much better than the unmanaged strategies. With the risk neutral (6) strategy unsurprisingly achieving the highest mean and strategies 1, 2 and 7 most promising when the whole distribution and the use of premia and extra working years is taken into account.

6.4 Fully relaxed parameter space

The parameter space now consists of the 21 lifecycles, five different lengths of the accumulation phase ranging from 43 until 47 years and 5 different premium

Table 10: Table containing results for the fully relaxed parameter space. Lifecycle* denotes the mean for the first 30 years in which the lifecycles are constant.

	Mean	Median	prctile 5	prctile 1	Premium	Lifecycle*	APL
Strategy 1	1,23	1,04	0,65	0,47	11,19	13,48	44,42
Strategy 2	1,14	0,98	0,63	0,49	11,22	12,63	43,89
Strategy 3	1,15	1,01	0,53	0,39	11,12	14,44	43,24
Strategy 4	1,05	0,92	0,48	0,35	10,38	14,58	43,32
Strategy 5	1,24	1,17	0,55	0,42	11,51	13,52	43,99
Strategy 6	1,58	1,30	0,47	0,35	12,70	16,72	44,69
Strategy 7	1,35	1,25	0,58	0,46	11,90	13,18	44,56
Unmanaged 1	1,25	1,10	0,55	0,41	13	15	47
Unmanaged 2	0,64	0,56	0,27	0,21	9	15	43
Unmanaged 3	1,21	1,08	0,55	0,40	13	13	47
Unmanaged 4	0,62	0,56	0,28	0,20	9	13	43

levels ranging from 9 to 13 percent[18]. The parameter space is therefore three dimensional and 5 times larger as it was in the previous paragraph.

The utility curve (7) strategy is the only one outperforming the unmanaged benchmarks in all facets. Nevertheless the strategies that do not outperform unmanaged benchmark 1 in terms of the mean need a premia which is about 15 percent lower and retire about 3 years earlier. The percentile results are best for the quadratic loss (2) and minimum (1) strategy. The mean variance (4) strategy performs poorly due to the unwillingness to put in extra premia or retire later both increasing the variance of the pension replacement ratio, which is consistent with earlier results. The percentiles (5) strategy performs reasonably well on every facet without scoring the highest in any individual facet and the absolute loss (3) function seems to be outstripped by the quadratic loss (2) function among others. A participant who only cares about the expected value again favours the risk neutral strategy, whereas the utility curve (7) and percentiles (5) strategies seem to be balanced choices and strategies 1 and 2 perform best in low percentiles.

To obtain a better view of the choices made by the strategies in different circumstances the research will now provide results sorted in three different groups. The first group contains only the results in which the strategies chose the minimal retirement age and had a pension replacement ratio of over 1. The results for this first group can be found in Table 11.

One may interpret the frequency of the first group of results the chance of obtaining a care free pension. For the loss aversion strategies and the percentiles (5) strategy this chance is above forty percent and little premium is used to achieve this result. The other two strategies have a much lower chance of retiring as early as possible with a comfortable pension replacement ratio. For the risk neutral (6) strategy this is caused by choosing to continue working despite

---

[18]These settings are chosen since the unmanaged pension replacement ratio for the maximum premium is close to 100 percent for these settings.

Table 11: Results conditional on the minimum accumulation phase length being chosen with a pension replacement ratio higher than one. Freuency denotes the number of times a pension scheme met the described criteria divided by the total number of simulations, i.e. the chance of a strategy ending up in this 'group'.

	Frequency	APL	Premium	Lifecycle*	Mean
Strategy 1	0,48	43,00	10,51	12,89	1,64
Strategy 2	0,47	43,00	10,51	13,02	1,51
Strategy 3	0,51	43,00	10,46	14,04	1,55
Strategy 4	0,42	43,00	9,74	14,15	1,47
Strategy 5	0,44	43,00	10,82	13,26	1,64
Strategy 6	0,22	43,00	12,50	15,17	1,60
Strategy 7	0,32	43,00	11,15	13,39	1,83

Table 12: Results conditional on the pension replacement ratio lower than the minimum of sixty percent. Freuency denotes the number of times a pension scheme met the described criterium divided by the total number of simulations.

	Frequency	APL	Premium	Lifecycle*	Mean
Strategy 1	0,03	47,00	12,73	15,70	0,50
Strategy 2	0,04	46,36	12,36	15,32	0,51
Strategy 3	0,09	43,80	12,23	15,26	0,50
Strategy 4	0,14	43,64	11,19	15,50	0,50
Strategy 5	0,07	43,69	12,31	14,69	0,49
Strategy 6	0,12	43,03	12,55	15,95	0,48
Strategy 7	0,06	44,83	12,66	14,33	0,53

already having a comfortable pension. When the pension is already high namely, the expected return on this pension is also higher in absolute amounts. The utility curve (7) strategy only retires this early when the pension replacement ratio is very high as is visible in the conditional mean. For median pension replacement ratios enough utility can be earned to keep working just a little longer, since this strategy also derives utility from values well above the targets used in the loss aversion strategies.

The second group of results provides the chance of ending up with a pension lower than the minimum used in strategy 1, these results are displayed in Table 12.

The chance of obtaining a pension smaller than the minimum is smallest for the minimum (1) strategy which has an obvious advantage since the results are separated using this same minimum. For the minimum strategy the conditional accumulation phase length is the maximum by construction. An important result is the average premium which is close to the maximum as well for the minimum strategy. The only other strategy with such a high conditional accumulation phase length is the quadratic loss (2) strategy, indicating why these

Table 13: Results conditional on not being in the 'care free' or 'below minimum' groups. Freuency denotes the number of times a pension scheme met the described criteria divided by the total number of simulations.

	Frequency	APL	Premium	Lifecycle*	Mean
Strategy 1	0,50	45,63	11,75	13,96	0,89
Strategy 2	0,50	44,55	11,80	13,47	0,83
Strategy 3	0,40	43,42	11,71	14,29	0,79
Strategy 4	0,44	43,53	10,76	14,27	0,81
Strategy 5	0,49	44,92	12,00	13,90	0,99
Strategy 6	0,66	45,54	12,80	13,74	1,76
Strategy 7	0,62	45,33	12,20	13,57	1,18

strategies have high results for low percentiles. The utility curve (7) strategy has the lowest frequency besides these two but has an average accumulation phase length which is much lower, therefore one might argue that it does better than those two strategies in bad states of the world. Premia are higher for all strategies compared to the care free pensions in the previous group as could be expected. Nevertheless this result provides confidence in the strategies. Another interesting result is the average lifecycle that is chosen. All strategies invest a considerably larger portion of their assets in risky assets. They do so because of the higher expected value for the returns they desperately need to avoid obtaining a pension far lower than expected and desired. The chance of obtaining such a low pension is the highest for the mean variance (4) and risk neutral (6) strategy. This implicates that the mean variance strategy is not suited as a pension strategy and confirms previous findings for the risk neutral strategy.

The final group of results contains all results not included in the previous two groups. These results can be found in Table 13.

This category contains almost two thirds of the risk neutral (6) probability mass and has a mean even higher than the care free pension results. The category also contains a lot of probability mass for the utility curve (7) strategy. For this middle category the utility curve strategy and the percentiles (5) strategy have a much higher mean than the loss aversion strategies with no extreme differences in premia or the average accumulation phase length, stressing the importance not just punishing losses but also rewarding wins.

6.5   Final model

To make the model more realistic a wage growth path was included that mimics the empirically found wage increases, on top of the compensation for inflation, in the health and wellbeing sector. The next extension of this model included a risk factor on this wage growth to include the insecurity and variation in the wage growth. Elaborate results for both inclusions can be found in Appendix C starting in paragraph 9.2.1. These results can be summarized by stating that including wage growth severely reduces the mean as well as all the low

percentile results. The premium paid in the early phase of the accumulation phase is about twice as low compared to the same percentage over the final wage. In the previous model the wage, and thereby a premium of x percent, was assumed to be constant. Since the pension result is calculated over the final wage, the expected pension replacement ratio suffers from high wage growth. The inclusion of risky wage growth slightly increased the mean but reduced low percentile results, since it allowed for more extreme wage developments and is an extra risk factor. The marginal increase in the mean is more difficult to grasp. There are now low final wages, for which it is more easy to obtain a high pension replacement ratio, and high final wages, for which more risk, working years and premium is needed driving up the mean return. This phenomenon slightly lifts the mean pension replacement ratio.

The final model that is considered includes three additional changes that aim to make the model more realistic. The first change is the inclusion of an extra amount that should be payed when a participant chooses to retire before the standard accumulation period length of 45 years. This is roughly comparable to the period that should be bridged if a person chooses to retire prior to the year in which the person is eligible for a first pillar pension. The cost of bridging one year is fixed at 14.000 euros multiplied with the realized inflation, which is grosso modo the AOW provided by the government. Undoubtedly this will reduce the frequency of participants retiring before the standard accumulation period length.

The second extension is the risk of losing ones job. This risk is fixed at five percent throughout the whole accumulation period, with a re-entry chance equal to 66 percent. On average a participant will therefore miss about three years of income, obviously affecting the pension accumulation. It is interesting to look at the behaviour of the different strategies conditional on a participant having no income.

The third and final alteration is the target used in the absolute loss function and the quadratic loss function. A person aiming to maintain his current level of consumption can subtract the premium percentage from his target pension income, since the participant does not consume that part. The pension replacement ratios divided by this smaller amount are provided in Table 15. It is important to keep in mind that strategies can increase this second ratio easily by increasing the premium close to retirement, therefore for the other strategies the first pension replacement ratio is still used in the utility functions.

Table 14 provides the results for the final model with the above described extensions.

As expected all strategies suffer from the inclusion of job risk into the model, which is visible in the mean among other indicators. Including the costs of bridging the time until the participant is eligible for the first pension pillar is the main reason why the average accumulation phase is now longer for all strategies. Besides these changes the differences between strategies and between managed and unmanaged schemes remained quite the same. Strategies 1, 2 and 7 show the best balanced results in which bad states of the world are important. Strategy 6 performs the best when the only indicators are the amount of premia

Table 14: Table containing results for the final model using the basic economic scenario parameters, with a fixed domain for premia and no restriction on lifecycles. Lifecycle* denotes the mean for the first 30 years in which the lifecycles are constant.

	Mean	Median	prctile 5	prctile 1	Premium	Lifecycle*	APL
Strategy 1	0,93	0,90	0,47	0,34	11,94	11,83	45,77
Strategy 2	0,89	0,84	0,46	0,33	12,23	12,46	44,81
Strategy 3	0,84	0,76	0,40	0,28	12,18	14,20	44,06
Strategy 4	0,81	0,77	0,36	0,20	11,36	13,91	44,27
Strategy 5	0,95	0,90	0,40	0,27	12,14	13,87	44,92
Strategy 6	0,98	0,84	0,27	0,16	12,43	15,77	44,48
Strategy 7	0,98	0,87	0,44	0,34	12,37	11,86	45,34
Unmanaged 1	0,90	0,80	0,42	0,31	13	15	47
Unmanaged 2	0,38	0,33	0,13	0,09	9	15	43
Unmanaged 3	0,87	0,78	0,42	0,32	13	13	47
Unmanaged 4	0,36	0,32	0,14	0,09	9	13	43

paid, the accumulation phase length and the mean pension replacement ratio. The cumulative distribution functions of these strategies are compared against those of the first two benchmarks in Figure 13.
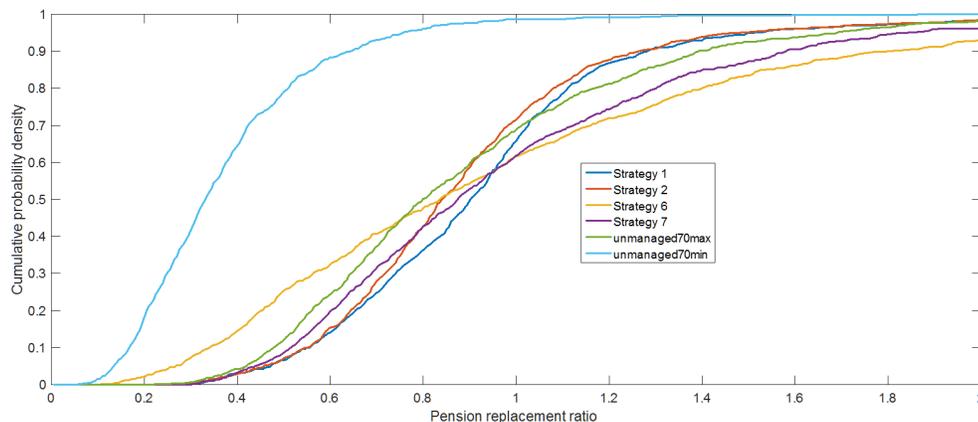

Figure 13: Plot of the cumulative distribution function for selected strategies.

The figure shows that every strategy is first order stochastic dominant (FOSD)[19] over the benchmark that chooses the lowest premium and accumulation phase length. This is no surprise since there is a large difference in the average premium and accumulation phase length. Strategy 7 however is also FOSD over the first benchmark, which chooses the maximum premium and accumulation phase

---

[19]Choice A is FOSD over B if for the chance of receiving x holds $P(A \geq x) \geq P(B \geq x)$ for all x, and for some x, $P(A \geq x) > P(B \geq x)$.

Table 15: Table containing results for the final model using the basic economic scenario parameters, with a fixed domain for premia and no restriction on life-cycles. Pension replacement ratios based on the final wage minus the final premium.

	Mean	Median	prctile 5	prctile 1
Strategy 1	1,03	1,01	0,54	0,39
Strategy 2	1,02	0,96	0,53	0,38
Strategy 3	0,96	0,88	0,46	0,32
Strategy 4	0,93	0,89	0,41	0,23
Strategy 5	1,09	1,03	0,46	0,31
Strategy 6	1,13	0,96	0,31	0,18
Strategy 7	1,11	1,01	0,51	0,39
Unmanaged 1	1,05	0,94	0,49	0,37
Unmanaged 2	0,44	0,39	0,16	0,11
Unmanaged 3	1,00	0,90	0,48	0,36
Unmanaged 4	0,41	0,37	0,16	0,11

length by default. Thus, if the economic scenario generator is completely correct no rational person would favour the unmanaged benchmark over the managed strategy 7. Strategy 1 is FOSD over strategy 2 but at the cost of one extra year of work on average, albeit that the average premium for strategy 1 is lower. Up until the median these strategies seem the most attractive. For participants who also care about results in good states of the world, or do not suffer heavily from poor results, strategy 7 becomes more attractive. An extreme variant is the risk neutral participant. Strategy 6 clearly shows that results could be disastrous but that the risk neutral strategy also offers the highest probability for high pension replacement ratios. Depending on the participants' preferences one of these four managed strategies seems to be the best choice.

A person who thinks low percentile results are very important chooses strategy 1 or 2 depending on whether the participants cares more about extra years of work or a higher premium. A risk neutral person chooses strategy 6 and a more balanced choice is strategy 7.

When compared to the consumed part of the wage a premium between 9 and 13 percent with an accumulation phase between 43 and 47 years is sufficient in this model to obtain a mean and a median around 100 percent, for the best performing strategies. The worst case scenarios would amount to obtaining about 40 percent of the final consumption during the retirement excluding the first pillar/franchise. For people whose income is relatively low, not much higher than the franchise/AOW, this percentage is increased considerably by the fixed first pillar pension. For example a person earning double the franchise/AOW benefits would receive 70 percent of the final consumed income as a pension in the first percentile. Which is a large shock towards the personal consumption level, but it is a rare scenario and it should not be a insurmountable shock.

The distribution of the pension replacement ratio has been shown to differ

among strategies based on 1000 scenarios. However the difference in individual scenarios is also of interest. Since it is relevant to know if strategies, in the same economic scenario, show differences in the pension replacement ratio's which are comparable to the differences in the aggregated results above or if differences can be much larger.
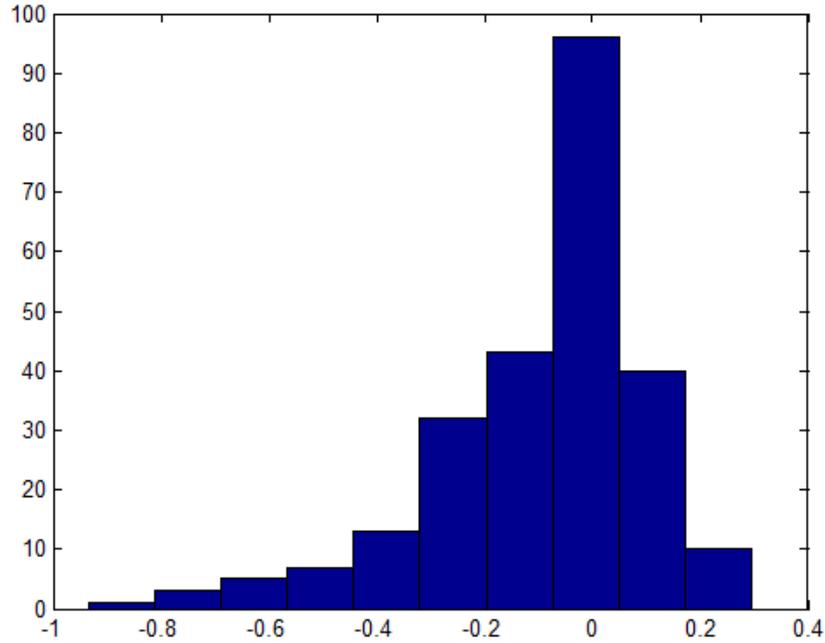

Figure 14: Histogram of the differences between the pension replacement ratio of strategy 1 minus that of strategy 7. Based on 250 simulations using the default economic scenario generator.

Figure 14 shows that differences in pension replacement ratio's can be large between different strategies in the same economic scenario. Unsurprisingly the distribution is heavily negatively skewed, since strategy 7 is more keen on obtaining high pension replacement ratio's where strategy 1 focuses on obtaining a minimum. The more conservative approach from strategy 1 occasionally results in a higher pension replacement ratio, ergo strategy 7 does not always outperform strategy 1 in terms of the pension. The results for strategy 2 are comparable to strategy 1, whereas strategy 6 is comparable to strategy 7.

Strategy 1 and 2 have comparable pension replacement ratio distributions, however they use different means to obtain said ratio as Figure 15 shows. A participant who has chosen strategy 2 retires earlier as can be derived from the means, but in some scenarios the difference can be 2 years or more. Of course a participant is not guaranteed to retire earlier when choosing strategy 2 and more premium is used to obtain these results, but the differences between strategies

44

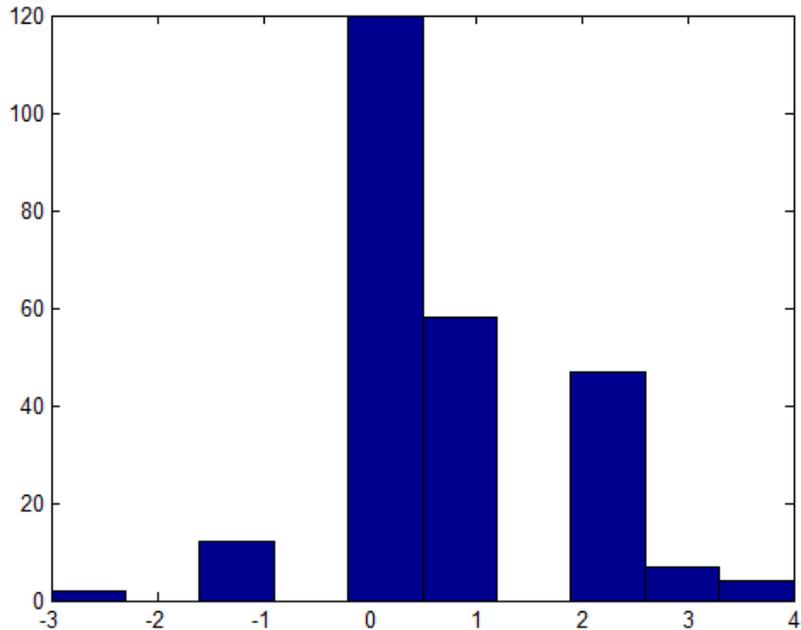are considerable, reflecting the importance of the personal preferences.

Figure 15: Histogram of the differences between the accumulation phase length of strategy 1 minus that of strategy 2. Based on 250 simulations using the default economic scenario generator.

6.5.1 Fully relaxed parameter space with risky wage, job risk and stability based domain for premia and strategy

The previous model can raise some practical issues. The flexibility in the choice of lifecycles may lead to undesirably high transaction costs, therefore the yearly change in lifecycles is restricted to at most two. This means that a scheme holding lifecycle 19 (90 percent equities) in year 1 can reduce the equity percentage to at most 80 percent or lifecycle 17 in year 2. Another issue is the uncertainty about next year's premium. The previous model made use of a small domain in which premia could change four percent on a year to year basis. To guarantee stability for the participant a maximum change of two percent is permitted. This restriction imposes a flexible domain of five premium percentages, enabling us to broaden the domain whilst maintaining the same computation time. The domain will range from zero percent up to 25 percent allowing schemes to stop paying premia or pay a high premium when necessary. Results can be found in Table 16.

These results indicate that for poor performing schemes a stability based domain is able to mitigate the harm by raising premia, thanks to the larger

Table 16: Table containing results for the final model using the basic economic scenario parameters, with a stability based domain for premia ranging from 0 to 25 percent and a stability restriction on lifecycles. Pension replacement ratios are based on the final wage. The starting premium is 15 percent and the starting lifecycle is 15.

	Mean	Median	prctile 5	prctile 1	Lifecycle*	Premium	APL
Strategy 1	1,10	1,04	0,71	0,59	13,2	0,14	44,8
Strategy 2	1,10	0,97	0,65	0,53	12,3	0,18	43,4
Strategy 3	1,04	0,92	0,56	0,46	13,3	0,16	43,2
Strategy 4	0,89	0,78	0,50	0,35	13,7	0,14	43,5
Strategy 5	1,23	1,18	0,61	0,47	12,9	0,16	44,6
Strategy 6	1,55	1,53	0,63	0,41	16,1	0,20	45,2
Strategy 7	1,38	1,30	0,68	0,55	13,3	0,17	45,0
Unmanaged 1	1,02	0,91	0,48	0,35	15,0	0,17	47,0

Table 17: Table containing results for the final model using the basic economic scenario parameters conditional on the pension replacement ratio being lower than 60 percent. The stability based domain for premia ranges from 0 to 25 percent and a stability restriction on lifecycles is imposed. Pension replacement ratios are based on the final wage. The starting premium is 15 percent and the starting lifecycle is 15.

	Frequency	APL	Premium	Lifecycle*	Mean
Strategy 1	0,01	47,00	0,20	14,18	0,55
Strategy 2	0,03	45,60	0,20	14,48	0,55
Strategy 3	0,09	44,22	0,18	13,94	0,54
Strategy 4	0,15	44,45	0,15	14,64	0,51
Strategy 5	0,05	44,91	0,16	14,50	0,52
Strategy 6	0,05	43,29	0,20	16,57	0,48
Strategy 7	0,02	45,57	0,19	14,78	0,54

domain for premia. The average premium chosen is higher than was possible in the fixed domain for all strategies. These extra premia allow the strategies to retire earlier and spectacularly increase the mean, especially for the wealth based strategies 5 to 7. The unmanaged benchmark premium has been increased to 17 percent to fairly compare results. The risk neutral strategy (6) invests most premium and chooses the riskiest portfolio since the expected returns are higher than the inter-temporal discount rate. To further investigate this phenomenon three subgroups are considered, of which one is provided below in Table 17 and the rest can be found in Appendix C in tables 42 & 43.

Conditional results for the obtained pension replacement ratio being lower than the minimum of 60 percent show that the average premium is higher for this subgroup. Thus, strategies signal the poor climate in an early phase and put in a high premium to prevent a very poor pension replacement ratio. When

comparing these results to the percentile results for the unmanaged benchmark a large performance gap is visible in favour of the managed strategies.

7 Final model scenario analysis

Until now the research has concentrated on a single set of economic parameters. To understand how changes in the global economic climate affect the behaviour and results of managed DC schemes, two additional scenario generators are considered. The global economic climate is influenced by a lot of different parameters, some latent, others observable and some even predictable. The long run mean for the risk free rate and the average expected geometric stock return, both important parameters in the economic scenario generator, are closely related to this global economic climate. The difference between these two means can be interpreted as the 'equity premium', which compensates investors for the additional risk associated with investments in equities. According to economic theory this premium should always be positive. The premium should also be relatively stable since long term expectations for the risk free rate mean and the mean of equity returns are seldom subject to large shocks. Figure 16 provides insight into these time series using a moving window of 50 years. This period is chosen to eliminate short term contaminations of the market such as asset bubbles or government interference. A good example of the latter is the quantitative easing programs in the US and EU, pushing down interest rates. The same time series for a shorter moving window of 20 years can be found in Appendix C.
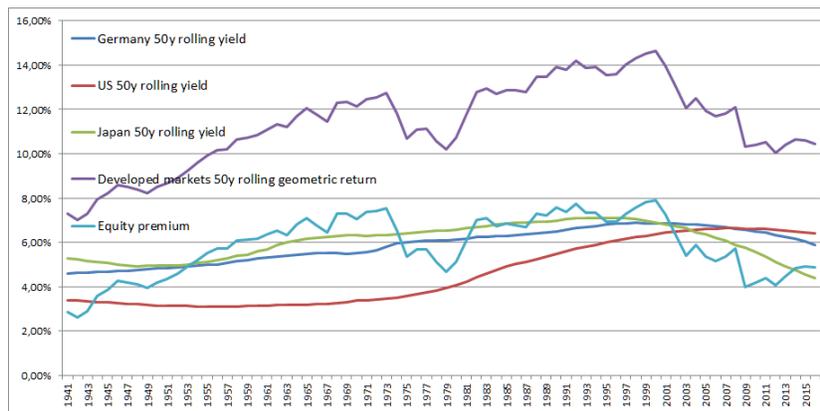

Figure 16: Geometric returns for time series calculated over a moving window of 50 years, with a yearly frequency. The equity premium is calculated by substracting the equally weighted mean of the bonds from the developed markets equities.

Figure 16 shows a decline in all time series following the crisis of 2008.

A pessimist can bring forward many arguments as to why a declining trend will persist. The hampering effect that the prevention or mitigation of climate change has on economic growth due to limitations on fossil fuel use and other environmental requirements driving up production costs is a well known threat. Another threat is the negative sentiment against free trade zones or globalism. The 'Brexit' referendum, the US election results and civilian protests against TTIP or CETA are indicative of this sentiment. Important non-western countries and governments such as China or Russia have little ambition to change this sentiment, since their cheaper labour currently gives them a significant advantage. These opposed interests are detrimental to geopolitical stability and thereby the global investment climate. Naturally states are free to act in their best interest, but their interest probably will not coincide with the global interest. Protectionism hurts the global economy by definition since it diminishes the importance of free market virtues such as competition and freedom to make mutually beneficial exchange happen. A pessimist might also argue that the aging of the population forms a problem for the economic climate. In most western countries, including the Netherlands, the birth rate is below 2 adding to the demographic problem. In Japan these demographic trends were present earlier and are often cited as one of the reasons for their persistent low interest rate. The reasoning behind this is that the countries' workforce decreases, both absolute and relatively.

An optimist might argue that a technological revolution in the energy field enabling the world to use renewable and relatively clean energy would solve the first problem. The protectionism problem, the optimist will argue, is of all times. Revolutionary technology such as the internet and increasing access to traveling means reduce the importance of physical borders and differences between nations, gradually harmonizing the public, thus reducing the need of protectionism. The optimist will also bring under attention that the demographic trends of the world as a whole are vastly different compared to the trends in western countries.

To investigate how managed DC schemes perform in the world of both a pessimist and an optimist two scenarios are considered. From Figure 16 one can tell that the equity premium in the default scenario, 2.5 percent, is relatively low historically. In the optimistic scenario the equity premium is increased to 4.5 percent by increasing the expected equity returns to 8.5 percent. This value of 8.5 percent would not be a remarkable value historically as described in section 4.1. In the pessimistic scenario the equity premium is again increased to 4.5 percent, this time by decreasing the long run interest rate mean to two percent. The pessimistic scenario would result in a historically low 50 year rolling yield and a historically low 50 year rolling geometric equity return. This does not mean that the pessimistic scenario is improbable given that the interest rates are currently at a historically low level well below two percent.

Table 18: Table containing results for the final model using the pessimist economic scenario parameters, with a fixed domain for premia and no restriction on lifecycles.

	Mean	Median	prctile 5	prctile 1	Premium	Lifecycle*	APL
Strategy 1	0,66	0,61	0,29	0,20	12,54	14,53	46,63
Strategy 2	0,64	0,60	0,29	0,20	12,56	16,27	45,70
Strategy 6	0,54	0,43	0,15	0,09	12,28	17,75	43,61
Strategy 7	0,64	0,56	0,29	0,21	12,57	15,05	45,58

Table 19: Table containing results for the final model using the optimist high equity economic scenario parameters, with a fixed domain for premia and no restriction on lifecycles.

	Mean	Median	prctile 5	prctile 1	Lifecycle*	Premium	APL
Strategy 1	1,12	1,02	0,54	0,38	15,4	0,113	45,0
Strategy 2	1,08	0,98	0,53	0,39	14,7	0,117	44,2
Strategy 6	1,32	1,09	0,34	0,20	17,8	0,125	44,8
Strategy 7	1,25	1,14	0,52	0,38	15,0	0,120	45,1

7.1 Pessimist low interest rate mean scenario

An increase in the inflation rate or a decrease in the interest rate, i.e. a decrease in the real interest rate, is detrimental for the performance of the pension schemes. High inflation means that a high return is necessary to compensate for this inflation, whereas a low interest rate entails lower returns on the government bonds. Most importantly, a low real interest rate drives up the cost of the annuity bought at the date of retirement. The results for the pessimistic scenario are provided in Table 18.

The difference with the default scenario set is large in all indicators. The lower interest rate shifts the portfolios to a higher equity weight to compensate for the lower level of returns for bonds. In the pessimist scenario set the annuity costs are much larger, since the long term interest rate is only slightly higher than the long term inflation rate, the pension one receives for a certain nominal value is only marginally higher than this value. The average premia and accumulation phase length reflect the difficulties the pension schemes have in the pessimist scenario.

7.2 Optimist high equity return scenario

The results for the optimistic scenario are provided in Table 19.

The optimist scenario parameters unsurprisingly result in a better performance for the managed strategies. The higher expected returns on equities induce the strategies to invest more heavily in equity. Interesting is that the percent wise increase in the mean is higher than the increase in the first percentile results. This means that scenario's in which the investment climate, and

thereby the accumulation of pension capital, is relatively good, benefit more from the higher expected equity returns since they are able to take more risks. The average premia are a little lower compared to the default scenario and the same goes for the accumulation phase length, which are both unsurprising results.

8 Sensitivity analysis

After obtaining promising results for the managed DC strategies it is important to know how robust these results are. Many assumptions are needed to model economic processes. The effect of 'black swans'[20], which are not present in the economic scenario set, are investigated using deterministic scenarios. The sensitivity towards individual parameter values is researched by changing a single parameter ceterus paribus. The consequences of using wrong economic parameters to simulate scenarios are discussed and the sensitivity of the managed DC strategies to their parameters is investigated. Only the results for the selected strategies from chapter 7 will be provided to ensure clarity.

8.1 Sensitivity of the strategy parameters

To get a clearer vision of how each strategy behaves and why it behaves the way it does, it is very helpful to investigate the individual effects of the parameters used in the strategy. Furthermore the aim of this section also is to shed light on how different individuals with different subjective preferences regarding premia, retirement and wealth levels should set their strategy parameters. The final model with the stability based domain for both premia and lifecycles is chosen because it allows for larger differences between strategies. Therefore possible differences should be clearer in the results.

8.1.1 Strategy 1 the minimum strategy

To investigate the sensitivity of strategy 1 to some important parameters used in the utility function some variants are considered. Results for these variants can be found in Table 20.

Table 20 shows that if a higher minimum needs to be achieved with 95 percent certainty, more premium is needed. The use of more premium leads to a higher mean and indeed to a higher pension replacement ratio that can be achieved with 95 percent certainty. Interesting is that strategies with a higher minimum invest more conservative. This can be explained by looking at the construction of the strategy. As long as the minimum can not be achieved with enough certainty, the lifecycle that results in the best fifth percentile result is chosen. This is mostly a lifecycle with little invested in risky equities resulting in a lower average lifecycle. The average accumulation phase length is slightly

---

[20]Unprecedented events, such as a stock crash higher than the beurskrach in 1928, 'black monday' or the 2008 crisis.

Table 20: Results for strategy 1 in the final flexible model with stability based domains for lifecycles and premia. The minimum is the pension replacement ratio the strategy targets to achieve with 95 percent certainty. Based on 250 scenarios.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Default (min.=0.6)	1,08	1,01	0,74	0,54	13,56	0,15	44,88
Minimum = 0.7	1,22	1,15	0,77	0,58	11,97	0,17	45,15
Minimum = 0.5	0,91	0,85	0,56	0,51	14,02	0,12	44,91

Table 21: Results for strategy 2 in the final flexible model with stability based domains for lifecycles and premia. Phi denotes the risk aversion with 10 used as default throughout the research and Ksi denotes the willingness to continue working for which 0.05 is used as default. Based on 250 scenarios.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Phi = 15	1,15	0,99	0,69	0,62	12,28	0,18	43,48
Default	1,10	0,94	0,66	0,60	12,51	0,17	43,49
Phi = 5	1,00	0,85	0,59	0,51	12,98	0,15	43,47
Ksi = 0.1	1,17	0,99	0,64	0,54	12,64	0,18	43,22

higher for the highest minimum, which is logical since the participant continues working until the minimum is reached.

8.1.2 Strategy 2 the quadratic loss strategy

To investigate the sensitivity of strategy 2 to some important parameters used in the utility function some variants are considered. Results for these variants can be found in Table 21.

The results for the variant with the highest loss aversion are provided in the top row of Table 21. Since the loss aversion is higher the strategy chooses to put in a higher premium compared to the less loss averse strategies and also chooses a slightly lower lifecycle on average. The extra premium results in a slightly higher mean and also in higher results in the lower percentile. This illustrates the effect the extra strong loss aversion has on the choices made by the strategy and that the parameters in the model have their desired effect. The accumulation phase length is virtually unaffected by changes in the loss aversion, but is affected when the willingness to work an extra year is changed. The bottom row of Table 21 shows results for strategy 2 when the border value of the gain from working an extra year is twice as high as was the case in the default strategy. Now the participants is eager to shorten the accumulation phase. It therefore puts in a bit more premium compared to the default case resulting in a high mean, but often chooses to stop working at the earliest possible time which in turn results in poorer low percentile results.

Table 22: Results for strategy 6 in the final flexible model with stability based domains for lifecycles and premia. q denotes the fraction of the final income which should be earned as return on the pension capital to continue working. Based on 250 scenarios.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
q = 1	1,55	1,44	0,73	0,46	16,18	0,18	45,84
default(q=1.5)	1,53	1,45	0,61	0,38	16,23	0,19	45,21
q = 0.5	1,57	1,44	0,76	0,53	16,05	0,17	46,29

Table 23: Results for strategy 7 in the final flexible model with stability based domains for lifecycles and premia. Epsilon denotes the risk aversion coefficient, with 0 being the risk neutral case.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
epsilon = 1.5	1,68	1,52	0,80	0,67	13,26	0,23	45,08
epsilon = 3	1,43	1,30	0,63	0,57	13,95	0,17	45,48
Default (eps.=2)	1,40	1,29	0,63	0,57	13,96	0,17	45,21

8.1.3 Strategy 6 the risk neutral strategy

To investigate the sensitivity of strategy 2 to some important parameters used in the utility function some variants are considered. Since the risk neutral strategy contains little parameters, only the willingness to continue working is researched. Results for these variants can be found in Table 22.

The default strategy requires an expected return equal to 1.5 times the last earned income on the pension capital to continue working. For good investment climates this is possible as the accumulation phase length is much higher than the minimum. If this border is reduced obviously the risk neutral strategy chooses to work longer as is reflected by the higher accumulation phase lengths for the variants. The most notable increase happens in the lower percentiles, however this could be partly due to the lower number of scenario's used for the results.

8.1.4 Strategy 7 the power utility function

To investigate the sensitivity of strategy 7 to some important parameters used in the utility function some variants are considered. Results for these variants can be found in Table 23.

The differences between the bottom two rows in Table 23 is quite small. The strategy with the lower epsilon in the top row does show large differences. The expected return on the premium is higher than the inter-temporal discount rate and the lower epsilon means that the marginal utility is higher for large pension replacement ratio's. Therefore the participant keeps putting in a high premium even when the expected pension replacement ratio has long surpassed 100 percent. This is reflected in the higher average premium and the higher

Table 24: Results for the final model using default parameters, with a deterministic stock crash in year 20.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,76	0,74	0,38	0,30	11,84	12,26	46,34
Strategy 2	0,73	0,70	0,38	0,30	12,50	12,35	45,34
Strategy 6	0,66	0,51	0,17	0,13	15,75	12,37	44,10
Strategy 7	0,79	0,69	0,38	0,31	12,10	12,53	45,77

results for the pension replacement ratio distribution.

8.2 Scenario analysis

The antithesis of a stochastic scenario is a deterministic scenario. A deterministic scenario uses preset values to calculate how a pension develops opposed to the stochastic, uncertain, movements apparent in a stochastic scenario set. This section mostly uses a hybrid variant in which a large part of the model is stochastic but some events are deterministically plugged in to see what effect they have on the results.

8.2.1 Stock crash

Perhaps the most well known event that can have a detrimental effect on ones pension is a large stock market crash. Such a crash can be caused by a financial crisis, an asset bubble or simply panic. The oldest bubble dates back to 1637. During the Dutch golden age recently introduces tulip bulbs made a spectacular increase in price, only to abruptly lose 95 percent of their value within a couple of months. The largest negative return since the start of the index in 1891 for the developed markets equities is -37.5 percent in 2008. During the period 1929-1931 the index lost just over half of its value. The negative skewness in the economical scenario generator already accounts for the possibility of such negative returns. The deterministic stock crash that is researched consists of a -60 percent return on equities in a single year, namely year 20 of the accumulation phase. Year 20 is chosen since strategies may be fully invested in equities at this time, but also have some time to react on the stock crash. It is important to understand how and why the strategies react on such a crash.

Table 24 shows that the pension replacement ratio of strategy 6 suffers the most from including the deterministic stock crash, which has to do with the higher percentage invested in equities by this strategy. The other strategies also show a sharp decrease between 20 and 25 percent as a result of losing 60 percent of the capital invested in equities which is on average around 55 percent. The average accumulation length is increased and the same goes for the premium, which is a logical consequence of a large loss of pension capital. The choice in lifecycles seems relatively unaffected by the stock crash. Figure 17 compares the choice of lifecycles for strategy 7 for the stock crash scenario to the default

scenario, these results are exemplary for the other strategies which are omitted for clarity.
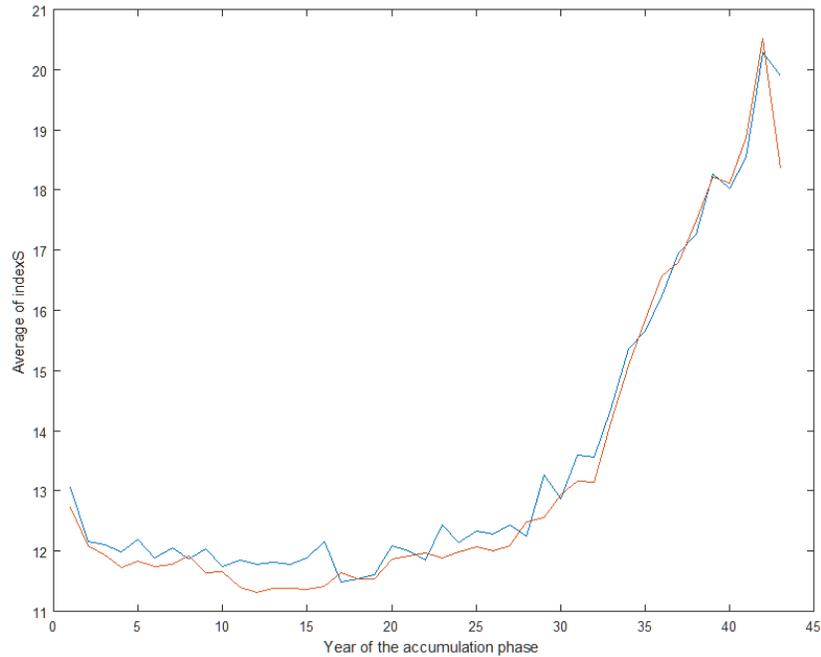

Figure 17: Plot comparing the average chosen lifecycle for strategy 7 in the scenario with the deterministic stock crash denoted by the blue line and the default scenario denoted by the red line.

In the economic scenarios there is no autocorrelation between equity returns, thus equities remain as attractive after a stock crash as they were before. The most important parameter for deciding which lifecycle to choose is the interest rate level, which is an indicator of the global economic climate. This is why no large changes in the choice of lifecycles is observed following a market crash. Small changes might occur depending on the resulting expected pension. One could decide to cut down the risk to avoid high losses or decide to take more risk and try to reach a target, but the premium and the accumulation phase length are the most important steering-gear.

8.2.2   Macro longevity risk

Macro longevity risk can be described as the risk a pension fund or the issuer of the annuity bought at the date of retirement faces regarding the future mortality trends. In the managed DC framework assumed in this research the participant buys an annuity at the date of retirement, after which the risk is transferred to the issuer of the annuity. The longevity risk faced by the participant consists

Table 25: Results for the final model using default parameters, with macro longevity shocks.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,80	0,78	0,37	0,27	11,78	12,17	46,19
Strategy 2	0,75	0,72	0,36	0,26	12,42	12,31	45,09
Strategy 6	0,78	0,63	0,20	0,13	15,76	12,38	44,10
Strategy 7	0,81	0,72	0,37	0,26	11,84	12,46	45,46

of the increase in life expectancy during the accumulation period, driving up the length of the decumulation period and hence the price of the annuity. In the Netherlands the first pension pillar is connected to the life expectancy with persons becoming eligible for these benefits 20 years prior to the average life expectancy for their age group. This link also solidifies the relation between the second pillar decumulation period and the life expectancy. Firstly the default option is linked with the life expectancy and secondly a pension gap needs to be overcome if a participant chooses to retire earlier. The macro longevity scenario considered in this research will consist of a positive shock to the life expectancy of 1.5 year every 10 years, which is a stronger ascension than the life expectancy has had for the past decades. The chance of 'disruptive' changes to the life expectancy which would make it possible for a human to live 130 years or longer are not taken into account. The defense for these choices can be found in Appendix A.2. Whereas results are provided in Table 25.

The macro longevity shocks reduce the pension replacement ratio's by around 20 percent. The choice in lifecycles is virtually unaffected by including macro longevity shocks and the same goes for the average premium. An increase is apparent however in the accumulation phase length. The decumulation phase at the date of retirement is 6 years longer or 30 percent longer (for the default default accumulation phase length of 20 years and accumulation phase length of 45) than expected in year 1. This 30 percent increase in the decumulation phase does not mean a 30 percent decrease in the pension replacement ratio, since the pension received in years 21 to 26 of the decumulation phase is relatively cheap compared to the early years. This, because the issuer of the annuity can get a risk free return by investing in bonds with a higher return than the inflation. The issuer of the annuity discounts the cash flow in year 21 with the risk free real interest rate, which is most often positive.

Shocks to the macro longevity are thus difficult to mitigate in a small premia domain and via lifecycles, especially shocks close to the end of the accumulation phase. The most straightforward solution is to continue working. For 20 years of decumulation there are 45 years of accumulation in the default scenario. From Figure 26 in Appendix C one can tell that an extra accumulation year increases the pension replacement ratio by around 5 percent. In casu the pension replacement ratio's are 20 percent lower. Working 3 years longer in a world where the life expectancy is 6 years higher would suffice to maintain the average pension replacement ratio, since the decumulation phase is now only 3 years or

15 percent longer and the accumulation phase is 3 years longer. A participant in a pension scheme would thus be able to enjoy a similar pension to a world without macro longevity shocks at the cost of working 3 years longer, but with the benefit of 3 extra years of pension and 6 years of life extra.

8.2.3 Mortality rates

As described in the macro longevity shocks section, years further away in the future are cheaper to acquire a pension for compared to years closer in the future. This research works with a fixed remainder of the life of 20 years after the default accumulation phase length of 45 years. Including mortality rates would make the model slightly more realistic. However, since the model can not 'manage' in the decumulation phase, since the annuity is already bought the extension would provide very limited extra insights.

Consider 3 participants with the same pension who retire after the default accumulation phase length of 45 years. In the current model they all die, or stop receiving a pension, after 20 years. Suppose that in a model with mortality rates the first would die after 19 years, the second after 20 years and the last after 21 years. In both models the average life expectancy after retirement is 20 years. The only differences from the perspective of the issuer of the annuity are the cash flows concerning participant 1 in year 20 and participant 3 in year 21. The issuer can pay the same inflation corrected cash flow a year later. If a positive real rate is assumed, which is realistic and on average 2.25 percent in this model, the issuer needs less money to pay the pensions. Thus when the average life expectancy stays equal but more variation in life expectancies is observed, annuities would be slightly cheaper.

With a constant real risk free rate of 2.25 percent, 2 annuities of 20 years would be 1.1 percent more expensive than an annuity of 15 years and an annuity of 25 years[21]. These differences are so small that including mortality rates would show little to no changes. In practice the profit and risk margins required by the issuer of the annuity dwarfs the price benefit from including mortality rates. Therefore including more sophisticated mortality rates is considered irrelevant for the purposes of this research.

8.3 General sensitivity

8.3.1 Mean reversion

Mean reversion allows managed DC strategies to anticipate changes in the interest rate and the inflation rate. This is an obvious advantage over the unmanaged schemes holding a fixed mix of equities and bonds. The degree of mean reversion in these rates is crucial. Very strong mean reversion would almost provide certainty about the interest rate process, whereas no mean reversion at all would be an unpredictable random walk. Since the managed DC strategies benefit from

---

[21] $\sum_{N=1}^{20} \sum \frac{1}{(1+0.0225)^{N-1}} / (\sum_{N=1}^{25} \sum \frac{0.5}{(1+0.0225)^{N-1}} + \sum_{N=1}^{15} \sum \frac{0.5}{(1+0.0225)^{N-1}}) - 1 = 0.011$

Table 26: Results for the final model using default parameters. With the exception of the mean reversion for the inflation and interest rate, both changed to 5 percent.

	Mean	Median	prctile 5	prctile 1	Lifecycle*	Premium	APL
Strategy 1	1,01	0,91	0,32	0,22	12,74	11,97	45,49
Strategy 2	0,94	0,82	0,32	0,22	13,41	11,99	44,87
Strategy 6	1,14	0,82	0,17	0,10	14,62	12,21	44,69
Strategy 7	1,05	0,85	0,33	0,22	12,81	12,22	45,17
Unmanaged 1	1,00	0,80	0,30	0,21	15,00	13,00	47,00
Unmanaged 2	0,42	0,33	0,09	0,05	15,00	9,00	43,00

Table 27: Results for the final model using default parameters. With the exception of the mean reversion for the inflation and interest rate, changed to 1 percent, and the standard deviation for the inflation and interest rate, changed to 0.02 percent.

	Mean	Median	prctile 5	prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,65	0,64	0,36	0,29	12,01	11,98	45,57
Strategy 2	0,72	0,76	0,36	0,27	15,07	12,43	45,97
Strategy 6	0,63	0,50	0,19	0,14	19,34	12,26	43,18
Strategy 7	0,82	0,73	0,37	0,29	13,76	12,58	46,94
Unmanaged 1	0,80	0,73	0,36	0,28	15,00	13,00	47,00
Unmanaged 2	0,33	0,29	0,12	0,09	15,00	9,00	43,00

high mean reversion, this paragraph will look into what happens when mean reversion is less strong. The default mean reversion was ten percent which will be changed to five percent ceterus paribus. Finally a mean reversion of only one percent is considered. To ensure that interest rates and inflation rates remain within a reasonable domain, the standard deviation is reduced from 0.7 to 0.2 percent, in the latter case.

Table 26 shows that the results have changed dramatically. For all strategies the mean pension replacement ratio has increased significantly. This is in sharp contrast with the decrease of the lower percentile results. Since there is less mean reversion, interest rate and inflation rates can take more extreme values increasing the annuitization risk. Unsurprisingly in some cases this results in a poor pension replacement ratio. The differences between the managed strategies and the unmanaged benchmarks remained more or less the same, albeit that the results for the first unmanaged benchmark are a little closer to the best performing managed strategies. Managed DC strategy 7 remains FOSD over all benchmarks, using less premium and retiring almost two years earlier on average compared to benchmark 1.

Table 27 shows that the smaller mean reversion has an even more dramatic effect on the results. The absence of strong mean reversion makes bonds less predictable and hence less attractive, this explains why more equity heavy life-

cycles are chosen. Another driver of lower results is the lower standard deviation of the interest rate, which is most clearly visible in the unmanaged benchmark results. Since an unmanaged portfolio rebalances on a yearly basis, the unmanaged benchmark profits from mean reversion combined with volatility. Consider owning 100 percent of a bond bought in year 1 and the rest of the portfolio obtaining a certain return of zero. When the interest rate drops the bond becomes more valuable, therefore the unmanaged portfolio will sell a fraction of its bond in year 2. Since the interest rate has gone up and is above its long run mean, the interest rate and hence the value of the bond is expected to drop. If the interest rate would return to its mean in year 3, a rebalancing portfolio would have obtained a higher return compared to a portfolio that would not have sold part of the bond to rebalance. In a similar way the unmanaged benchmark profits for a rise and a subsequent drop in the interest rate. This profit is higher when mean reversion is stronger and when volatility is higher, explaining the difference for the unmanaged benchmark with the basic scenario results.

For the managed schemes it holds that there is much less mean reversion, which enabled them to predict the movements to a certain extent. The average lifecycle chosen by managed schemes contains much more equity. The difference for the risk neutral strategy 6 indicates how often returns for bonds are more attractive due to mean reversion, with little mean reversion almost nothing is invested in bonds. Due to the volatility being lower, extreme rates are still as common as they were in the basic scenario generator, therefore the positive effect on the mean seen for the five percent mean reversion is absent. This leads to much lower results for managed schemes. The fact that strategy 7 still outperforms the unmanaged benchmark with a premia about three percent lower under these extreme settings means that it is robust with respect to mean reversion and volatility of the interest rate. Strategies 1 and 2 are able to obtain the same results in lower percentiles, but put in less years of work and premia which has its effect on the means and medians. Since it is very difficult to obtain high results the risk neutral strategy decides to quit working early, explaining the very low percentile results.

Mean reversion in the economic scenarios is one of the most important sources of return for the managed DC schemes and to a smaller extent also for an unmanaged DC scheme. All managed strategies, with the exception of the risk neutral strategy, provide decent results in terms of the lower percentiles. Strategies 2 and 7 obtain a decent median relative to the input of the unmanaged benchmark. Strategy 1 rarely obtains a high pension result, which is due to the difficulty and risk averseness associated with reaching the minimum pension replacement ratio of 60 percent. A shift in the premium domain to higher premia would provide a pension distribution that is similar to the basic scenario.

8.3.2 Moving long term interest rate mean

On obvious advantage for every managed strategy is that the mean to which the interest rate reverts, is known. This paragraph investigates the effect of including randomness in the long term mean to which the process reverts. The

Table 28: Results for the final model using default settings. With the exception of the inclusion of uncertainty around the long term interest rate mean.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,93	0,91	0,47	0,32	11,89	11,90	45,69
Strategy 2	0,88	0,82	0,45	0,29	12,42	12,14	44,63
Strategy 6	1,02	0,87	0,24	0,15	15,61	12,43	44,60
Strategy 7	1,00	0,90	0,44	0,30	11,91	12,35	45,45

long term interest rate mean is assumed to start at 4 percent, as in the basis scenario. During the 25 years that serve as a burn in period and the accumulation phase this mean can either go up or down with 10 basis points every year with equal chance. The long term interest rate thus has small, unpredictable shocks to its mean. Using a burn in period of 25 years and a accumulation phase of 47 ensures that the long term means keep within reasonable bounds, as Figure 18 shows.
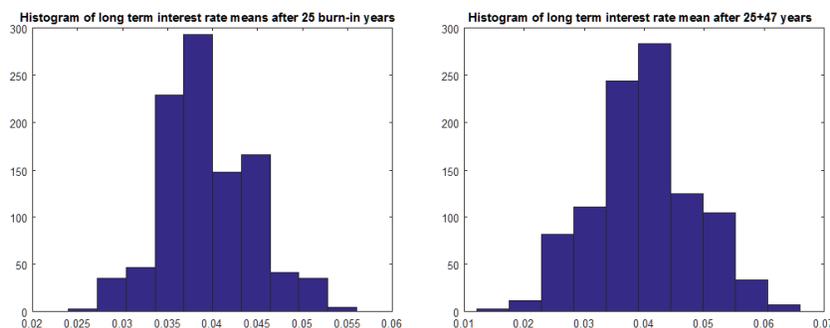

Figure 18: Histograms comparing the distributions of the long term interest rate mean after the burn-in period and after the entire simulation.

The figure shows that the property that the uncertainty is larger for longer periods is nicely incorporated. The 1000 paths that are simulated at each point in time for each scenario take a quit naive approach towards predicting the 'current' long term interest rate mean. Each path simply starts at 4 percent, but does account for the future uncertainty or movements in the mean. Note that this framework also allows to obtain 1000 starting values drawn from the relevant distribution of means or that a rolling window of past interest rates could help predict the current long term mean. Such a confidence interval around parameters could e.g. also be included around the mean equity return, but these extensions are outside the scope of this research. Table 28 contains the results for the moving long term interest rate mean extension.

The differences with the results in Table 14, containing the results for the final model using the default settings, are minimal. The mean, median, average paid premium and accumulation phase length are slightly better for the model

Table 29: Results for the final model using default settings. With the exception of the lifecycles which cut down the percentage in equities to half of the original weight instead of zero.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,90	0,78	0,46	0,34	12,29	11,48	44,61
Strategy 2	0,93	0,86	0,48	0,34	12,72	12,15	44,89
Strategy 6	1,11	0,89	0,28	0,17	16,03	12,49	44,85
Strategy 7	1,15	1,05	0,47	0,35	12,28	12,34	46,72

with the interest rate mean extension. Interest rates now take on more extreme values, large positive values are very beneficial for the managed schemes, whereas very low values provide an incentive to invest more heavily in equities which have a high expected return but are also risky. This risk can be seen in the lower percentile scores which are marginally lower compared to the results of the final model. The average lifecycle chosen shows that bonds on average are still about as attractive as they are in the default model. Thus a naive approach towards dealing with uncertainty in the long term interest rate mean is fairly robust. This indicates that the benefit from managed strategies is for a substantial part investing a smaller portion in bonds when interest rates are lower, rather than only profiting from predictable mean reversion.

8.3.3 Lifecycles cutting down the percentage invested in equities by half the original percentage

In the light of recent changes in Dutch law, enabling pension schemes to continue investing a part of the pension capital after retirement, lifecycles that do not reduce their portfolio weight in equity towards zero are investigated. Instead lifecycles in this extension reduce their weight in equity to half of the original weight. Lifecycles still start reducing their weight in equities 15 years prior to the expected retirement year. Thus a lifecycle may start with 80 percent invested in equities and will start linearly reducing this percentage towards 40 percent, 15 years prior to retirement. Of course this would have also been possible without the change in law. Nevertheless these lifecycles could straightforwardly and fluently be continued after the date of retirement by either remaining at the level of equities reached at retirement, e.g. the 40 percent in the above mentioned example, or by reducing it towards zero in another 15 years by continuing the linear trend.

Table 29 provides one stunning result. The accumulation phase length of the minimum strategy (1) is reduced by more than a year and the average premium is reduced by almost a half percent. The median however is severely reduced but the mean and the low percentile results suffer little. The average lifecycle chosen for strategy 1 in the early accumulation phase is higher, suggesting that it is now easier to obtain the minimum with sufficient certainty enabling the strategy to choose higher equity percentage. Thus higher expected returns are chosen,

Table 30: Comparison of the lifecycles chosen in the last 15 years prior to retirement between the default model with the default lifecycles and the model with lifecycles that reduce the equity percentage less.

	Mean default lifecycles	Mean less reduced lifecycles
Strategy 1	15,88	16,60
Strategy 2	16,69	17,85
Strategy 6	11,01	11,08
Strategy 7	16,12	17,10

rather than having to choose, by construction, a fairly low risk portfolio which provides a decent pension replacement ratio in the fifth percentile. Another benefit is that strategy 1 can still invest in equities instead of being forced to invest in bonds, when the minimum is not yet reached. The other strategies, especially strategy 7, show that the forced reduction in equities present in all default lifecycles considerably hurts performance without raising low percentile results. A portfolio invested for 50 percent in equities might lead to high shocks prior to retirement, but there is no rational ground to limit this possibility a priori and the strategies factor in the possible losses that go with such a position. Reducing lifecycles towards zero can therefore be concluded to be too restrictive since it harms performance.

Table 30 compares the lifecycles chosen in the final stage of the accumulation phase between the default model and the model with the variant on the lifecycles described above. The table shows the hunger for risk present in the strategies since the average lifecycle is higher than during the early phase for most strategies. For the risk neutral strategy (6) this is not the case. The result is close to the average of the highest (21) and lowest (1) lifecycle. The risk neutral strategy, by its nature, tends to invest either everything or nothing in equities. Over a very long period equities are expected to outperform bonds, however over a shorter period this is less obvious and, dependent on the level of the interest rates, bonds might have a higher expected return when the level is above the long term mean. The other strategies slightly increase their chosen lifecycle signaling that there is a need to keep investing in equities. Furthermore it has a stabilizing effect on the pension replacement ratio when not the entire portfolio is invested in bonds. The returns of these bonds are namely closely related to the changes in the cost of the annuity bought at the date of retirement. Investing a portion of the portfolio in equities therefore also has a diversification benefit, besides the high expected return. The investor can hedge the changes in the annuity costs by investing in bonds, but benefits from the diversification benefit and the higher expected return on equities.

8.3.4 Duration investing versus nominal matching for bonds

The investments in bonds are assumed to be nominally matched in this research. If a portfolio would consist only of bonds, the corresponding cash flows generated

Table 31: Results for the final model using default settings. With the exception of the investments in bonds, which are not nominally matched but invested in a bond with a 10 year duration.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,66	0,65	0,38	0,28	14,51	12,15	45,46
Strategy 2	0,73	0,76	0,37	0,28	18,10	12,53	45,85
Strategy 6	0,61	0,50	0,19	0,13	19,62	12,27	43,13
Strategy 7	0,81	0,73	0,38	0,28	15,17	12,66	46,96

by this portfolio would result in a nominal fixed pension. During the early phase of the accumulation period this means that the strategies invest in bonds with maturities as high as 60 years. In practice bonds with such long maturities are illiquid or simply unavailable. A more practical approach would be to invest in bonds with a maturity of 10 years. Table 31 provides the results for the final model, with the more practical approach for investing in bonds.

The benefits of the mean reversion or predictability present in the interest rate are much smaller when only bonds with a duration of 10 years are available. A contract providing an interest rate of 5 percent for the next 50 years has a higher return than the same bond providing the same interest rate for the next 10 years when the interest rate drops. This phenomenon is clearly visible in the pension replacement ratio results, which are significantly lower. Furthermore bonds are far less attractive which is reflected in the much higher average lifecycle. It is well known that for a long term investor rolling over short bonds is riskier than buying a single bond with a longer maturity. The impact on the pension replacement ratio's is considerable, therefore pensions funds could try to create a market for longer maturities enabling them to buy bonds that match their investment profile better.

8.3.5   Correlations

In an efficient market equity returns must rise or fall with bond returns, since the price of one asset category should drop when the other asset category becomes more attractive. Thus when interest rates go up the prices of both bonds and stocks should go down. However in practice the movements in the interest rate might have a positive relation with equity returns, though the empirical evidence is scarce. Since interest rates are expected to rise when investment opportunities in equities are good. Such a relation would stabilize the equity premium. To investigate how the managed schemes would perform if such a correlation would exist, the average correlation between the equity returns and both the inflation and the interest rate is set at 20 percent.

Due to the correlation with bond movements, equity becomes slightly more attractive as is demonstrated by the higher average lifecycle. The mean pension replacement ratio is lower, but the lower percentile results are higher for all strategies except the risk neutral strategy 6. The positive correlations thus pull

Table 32: Results for the final model using default settings. With the exception of the correlations between shocks in equity returns and inflation and shocks in equity returns and the interest rate both set at 0.2.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,88	0,87	0,48	0,35	11,95	11,91	45,88
Strategy 2	0,86	0,83	0,50	0,33	12,57	12,24	44,95
Strategy 6	0,95	0,79	0,27	0,15	15,77	12,41	44,48
Strategy 7	0,95	0,87	0,46	0,35	12,22	12,39	45,53

Table 33: Results for the basis scenario, but with the assumed long term interest rate mean equal to 4 percent, whereas the true value is 3 percent.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,80	0,77	0,36	0,25	14,04	12,34	46,29
Strategy 2	0,75	0,71	0,34	0,25	15,07	12,37	45,04
Strategy 6	0,66	0,52	0,18	0,12	18,00	12,29	43,54
Strategy 7	0,77	0,69	0,35	0,25	14,13	12,47	45,07

the pension replacement ratio's towards the middle. This could be expected since returns in both asset classes are now positively related. The idiosyncratic shocks have a positive correlation, thereby making it less probable that an unfortunate investor would keep investing heavily in the asset category with a poor idiosyncratic shock in the next period. The overall effect of the introduced correlations is relatively minor.

8.4 Wrong assumptions

The importance of mean reversion for the managed DC strategies has already been uncovered. What might be even more detrimental to the performance is when the assumed mean differs from the true mean. This is investigated by simulating the basis paths in step one of the algorithm using the true mean, but simulating the paths in step two of the algorithm using an assumed mean which differs from the true mean. Since there is no way of determining the mean towards which the time series revert, although experts agree on a long term mean and a certain degree of mean reversion, this is a relevant question. The model needs a long term mean for the interest rate, inflation rate and equity returns to function.

8.4.1 Wrong interest rate mean

The first situation that is researched is when the long term mean is assumed to be 4 percent, the default long term mean, but where the true long term interest mean is 3 percent. Table 33 provides the results for these specifications.

Bonds are specifically attractive to investors when the current level is above the long term mean perceived by the investor. An expected drop in the interest

Table 34: Results for the basis scenario, but with the assumed long term interest rate mean equal to 3 percent, whereas the true value is 4 percent. Based on 250 scenarios.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,94	0,90	0,46	0,39	10,36	11,97	45,73
Strategy 2	0,89	0,84	0,46	0,40	11,08	12,18	44,57
Strategy 6	1,03	0,82	0,27	0,18	14,13	12,40	44,56
Strategy 7	1,01	0,88	0,44	0,37	10,47	12,37	45,47

rate would mean earning an additional return besides the interest level. However the strategies perceive the mean to be higher than the true value, therefore bonds are much less attractive because of the faulty assumption. This is clearly visible in the high average lifecycle chosen by the strategies. The true investment climate is worse than in the default scenario, therefore it is not a surprise that the premia are higher for strategies 1, 2 and 7. Strategy 6 puts in less premium since bonds are less attractive and invests on average 90 percent in equities when given the choice. Because the true interest rate mean is lower than in the default scenario, pension replacement ratio's are lower due to the higher annuity costs. Paradoxically, assuming a too optimistic mean for the long term return level of bonds results in investing a smaller portion in bonds.

Results for the other way round, when the true long term mean is 4 percent whereas the assumed mean is 3 percent, are provided in Table 34.

Since the strategies assume a lower value for the long term interest mean that the true value, bonds are more attractive. This can be distilled from the lower average lifecycles chosen by the different strategies. The premia are practically equal to the premia used in the default final model and the same holds for the accumulation phase lengths. The means are slightly higher and the same holds for the lower percentile results. This could be due to the increased investments in the safer bonds, however the differences are very small and could also be a result of the fewer scenarios these results are based on. The faulty underestimation of the long term interest mean can be concluded to have very limited negative consequences.

8.4.2 Wrong equity return mean

Another interesting situation is when the assumed equity returns mean differs from the true mean. Table 35 provides results for a true equity mean of 5.5 percent where the assumed mean is equal to the default mean of 6.5 percent.

The assumed investment climate is exactly equal to the default scenario. Since the equity returns are unpredictable and uncorrelated with bonds, investors keep assuming that they are right and choose practically the same lifecycles as they did in the default model. Some minor changes could occur in strategies 1, 2 and 7 based on their current expected pension replacement ratio's , but equity remains as attractive as it was and investors blame the disappointing results on bad luck. The premia are slightly higher which is explained by

Table 35: Results for the basis scenario, but with the assumed equity mean equal to 6.5 percent, whereas the true value is 5.5 percent.

	Mean	Median	prctile 5	prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,85	0,84	0,42	0,31	11,8	0,121	46,1
Strategy 2	0,82	0,79	0,42	0,31	12,5	0,123	45,1
Strategy 6	0,85	0,71	0,23	0,13	15,8	0,124	44,3
Strategy 7	0,88	0,79	0,41	0,31	11,9	0,124	45,4

the need to compensate disappointing returns and the same goes for the accumulation phase length. The pension replacement ratio distribution is about 10 percent lower for strategies 1, 2 and 7. For strategy 6 this difference is larger since its pension replacement ratio depends more heavily on equity returns, as is reflected by the higher lifecycle.

Assuming a equity mean of 5.5 percent, whilst the true value remains 6.5 percent, yields results that are very similar to the second table in the previous subsection since the assumed and the true equity premium in both cases are the same. Therefore bonds are favoured more by the strategies with only limited effect on the results.

A higher true interest rate mean than the assumed mean of 4 percent would result in portfolios heavily invested in bonds, anticipating a future drop. However if the bonds were not to drop strategies would still make a comfortable return close to 4 percent and would increase the position in bonds. Thus the strategies would make a comfortable return in any case, making this faulty assumption a less interesting case. The same reasoning explains why a true equity returns mean higher than the assumed mean is less relevant.

9   Conclusion

The models developed in this paper bring forward two main conclusions. The first conclusion is that it makes sense to differentiate in the asset portfolios, premium and accumulation phase length based on an individuals' risk attitude. The four selected managed strategies each outperform the other strategies in one field. The most obvious example is the risk neutral strategy (6) which has the highest average pension replacement ratio due to a high allocation towards equities and high premia in a good investment climate. The power utility function strategy (7) provides perhaps the most balanced strategy, averting low pension replacement ratio's but also valuing high pension replacement ratio's. The minimum strategy (1) is focused on reaching a minimum target, thus preventing poor pension replacement ratio's. The same holds for the quadratic loss strategy (2), only the minimum strategy takes an approach with a small premium and continues working until the minimum is met, whereas the quadratic loss function uses a higher premium enabling a participant to retire earlier. Since such large differences occur in the resulting pension replacement ratio based on

the chosen strategy, it makes sense to choose a strategy that matches the personal preferences regarding the pension. The same holds for parameters within strategies.

The second conclusion is that a personal pension strategy should take the investment climate into account when selecting a lifecycle. The naive benchmark strategy holds the same asset mix regardless of the current level of the interest rate relative to the perceived long run mean. For very low levels of the interest rate it would be ill-advised to maintain a high allocation towards bonds, since the return gained on these bonds is very low and an increase which is to be expected would deteriorate these returns even further. Depending on the risk attitude associated with the scheme a portion of bonds is still maintained to stabilize the expected pension replacement ratio. The opposite is true for interest rates well above their perceived long run mean. The research has shown that this tactic is robust for wrong assumptions about the perceived mean or mean reversion, indicating how much is to be gained from taking the investment climate into account.

Other results worth mentioning are firstly the benefits associated with being able to invest in bonds with a very long maturity. This suggests that pensions funds will benefit from the creation of a (larger) market in such bonds. Secondly the freedom to keep a portion of the pension capital invested after retirement and using lifecycles that shrink the percentage in equities less prior to retirement support the chance of obtaining the desired pension. Finally a large stability based domain for premia can relatively confident provide a decent pension, the uncertainty faced by participants about changes in the premium can be maximized without severely hurting performance.

9.1 Practical issues

The nemesis of theoretical economic models has always been the irrationality apparent in consumer behaviour. Following a stock crash or sudden drop in the interest rate or a period of euphoria beliefs about parameters, vital to the economic scenario generator, can suddenly change. Such a change may be partially justified, nevertheless an overreaction resulting in a boom-bust-boom-bust cycle seems interwoven with human nature. Therefore the executor of the pension scheme should hold firm beliefs based on fundamental indicators and act on these beliefs.

In the very individual managed DC schemes suggested in this research the participants will show more 'anxious vigilance' resulting in an increased control on the performance of the executor. An important part of the executor's job would be to persuade participants to follow the course suggested by the executor's expert opinion or at least debate the relevant beliefs, increasing the level of accountability hence forcing the executor to think more deeply about the beliefs. The executor's interests do not conflict with those of the pension grantee in a DC scheme, therefore the executor can provide the pension grantee with an unbiased and trustworthy advice.

The main advantage a DB scheme has over a managed DC scheme is solidarity. Solidarity between participants is not researched in this paper, but can be incorporated relatively easy. An example would be to reallocate a part of very high pension replacement ratios to very low pension replacement ratios. Generational solidarity may seem impossible to incorporate in the individual managed DC scheme. However, the government would still be able to enforce generational solidarity via the first pillar pension. A generation that accumulated a low pension in a difficult economic climate but laid the foundations for future prosperity could be rewarded via a link between the first pillar pension and the economic growth as a result of their efforts.

An important drawback of DB schemes over managed DC schemes is the inability to take individual preferences into account. Because very different pension distributions can be achieved via different strategies, the default fit of a DB scheme can not compare to the tailor made DC strategies. Due to the size and importance of large DB funds there are legal restrictions, which are always bad for performance. The best example is the obligation to invest in bonds when funds are in distress. Pension funds that suffer as a result of the low interest rate, through the annuity costs that affect their coverage rate, are forced to invest in very low interest paying bonds that decline sharply in value when the interest rate starts rising again.

The main advantage of the DC schemes is thus the freedom to fit the plan and investment policy to personal needs. Sigmund Freud observed the following: "Most people do not really want freedom, because freedom involves responsibility, and most people are frightened of responsibility." A switch to a DC system would mean that pension fund managers would have to make more tough choices, e.g. total freedom in the allocation to bonds in low interest rate environments, and would no longer be able to blame government legislation for their poor performance. Time will tell whether individuals and pension funds are willing and able to obtain the freedom to choose and provide a personal pension plan, or that the DB schemes with a default fit are considered good enough and less frightening.

9.2 Limitations of the research

The research is able to answer the central question, namely that a managed DC scheme indeed leads to welfare gains for participants. In order to implement a managed DC scheme in practice however, several questions remain. Firstly the list of utility functions considered is not exhaustive. Secondly it can be hard for a participant or pension fund to estimate the relevant utility function and the parameters associated with it. Furthermore the model in this research incorporates the first pillar income and human capital risk, but no third pillar pension and possible risks faced on this capital. Modeling the third pillar capital for a participant and the correlations between e.g. the real estate holding this third pillar capital and the managed DC capital would be a relevant extension.

The most important pitfall of a simulation study is reaching a conclusion that follows from assumptions used whilst modeling. In this research the mean

reversion present in the interest rate process accounts for a substantial part of additional returns by managed strategies. A robust and diverse economic scenario generator therefore is essential when using a managed DC scheme. Of course one can never be certain about a latent process, but it is important to keep investigating the sensitivity of the scheme to assumptions while operating a managed DC scheme.

The model thus has to be refined before implementing it in practice and some difficult challenges lay ahead. However the welfare gains are promising enough to invest in developing a managed DC scheme.

References

Merton, R. C. (1969). Lifetime portfolio selection under uncertainty: The continuous-time case. The review of Economics and Statistics, 247-257.

Vigna, E. (2014). On efficiency of mean–variance based portfolio selection in defined contribution pension schemes. Quantitative finance, 14(2), 237-258.

Cairns, A. J., Blake, D., & Dowd, K. (2006). Stochastic lifestyling: Optimal dynamic asset allocation for defined contribution pension plans. Journal of Economic Dynamics and Control, 30(5), 843-877.

Blake, D., Wright, D., & Zhang, Y. (2013). Target-driven investing: Optimal investment strategies in defined contribution pension plans under loss aversion. Journal of Economic Dynamics and Control, 37(1), 195-209.

Vigna, E., & Haberman, S. (2001). Optimal investment strategy for defined contribution pension schemes. Insurance: Mathematics and Economics, 28(2), 233-262.

Battocchio, P., & Menoncin, F. (2004). Optimal pension management in a stochastic framework. Insurance: Mathematics and Economics, 34(1), 79-95.

Chang, S. C. (1999). Optimal pension funding through dynamic simulations: the case of Taiwan public employees retirement system. Insurance: Mathematics and Economics, 24(3), 187-199.

Blundell, R., Meghir, C., & Smith, S. (2002). Pension incentives and the pattern of early retirement. The Economic Journal, 112(478), 153-170.

Bodie, Z. (1989). Pensions as retirement income insurance.

Björk, T., Murgoci, A., & Zhou, X. Y. (2014). Mean–Variance Portfolio Optimization with State-Dependent Risk Aversion. Mathematical Finance, 24(1), 1-24. Deelstra, G., Grasselli, M., & Koehl, P. F. (2004). Optimal design of the guarantee for defined contribution funds. Journal of economic dynamics and control, 28(11), 2239-2260.

Gomes, F. J., Kotlikoff, L. J., & Viceira, L. M. (2008). Optimal life-cycle investing with flexible labor supply: A welfare analysis of life-cycle funds (No. w13966). National Bureau of Economic Research.

Blake, D., Cairns, A. J., & Dowd, K. (2001). Pensionmetrics: stochastic pension plan design and value-at-risk during the accumulation phase. Insurance: Mathematics and Economics, 29(2), 187-215.Blake, D., Cairns, A. J., & Dowd, K. (2001). Pensionmetrics: stochastic pension plan design and value-at-risk during the accumulation phase. Insurance: Mathematics and Economics, 29(2), 187-215.

Bodie, Z., Merton, R. C., & Samuelson, W. F. (1992). Labor supply flexibility and portfolio choice in a life cycle model. Journal of economic dynamics and control, 16(3), 427-449. I

Samuelson, P.A. (1969): 'Lifetime Portfolio Selection by Dynamic Stochastic Programming', The Review of economics and Statistics, Vol. 51, No. 3, pp. 239-246.

Maccheroni, F., Marinacci, M., Rustichini, A., & Taboga, M. (2009). Portfolio selection with monotone mean-variance preferences. Mathematical Finance, 19(3), 487-521.

Appendix A (Theory)

Appendix.A.1 Economic substantiation of the pension strategies

The aim of this section is firstly to provide intuition regarding the different strategies used in this research for readers who are less comfortable with the technical explanation provided in the methodology. Secondly the choices regarding the parameters are laid out. Thirdly these choices regarding the parameters used to compare premia, accumulation phase length and the pension replacement ratio in terms of utility will be discussed and explained. Ideally, after reading this section, a participant in a DC pension scheme would be able to choose the strategy that fits their personal situation best as well as the parameters that closely mimic their personal utility function.

All strategies will use the same inter-temporal discount factor δ of five percent and the demanded expected increase in the pension on the relevant interval to continue working, ξ, is five percent as well.

The first managed strategy considered in this research is focused on obtaining a certain minimum pension replacement ratio and can be found in equations 22 and 23. The value of α used as default is five percent to ensure a 95 percent probability of reaching the minimum is needed to switch to choosing the highest median. The default value for the minimum pension replacement ratio needed is 0.6. This value is both realistic for good scenarios as high enough to prevent poor pension replacement ratios.

The second managed strategy, from equation 19, considered in this research is the quadratic loss strategy. The tuning parameter φ is equal to 0.1. For the third managed strategy, the absolute loss strategy from equation 18, The tuning parameter φ is equal to 0.5. For the absolute loss strategy this tuning parameter is not multiplied with the retirement utility.

The fourth managed strategy is the mean variance strategy stated in equation 16. As in Maccheroni (2009) the results are capped on the economic meaningful interval. Above the required target this cap will enforce NaN values for both the expected value and variance of the pension. The risk aversion parameter γ is 2 by default.

The fifth managed strategy is referred to as the percentiles strategy and can be found in equation 21. Again a cap is used to ensure economic meaningful choices. For the wealth based strategy 5, compared to the loss aversion based strategies 1 to4, this cap is placed at a pension replacement ratio of 1.5. ϑ is equal to the sum of the (capped) results in the five different percentiles divided by the (capped) mean result.

The sixth managed strategy is the risk neutral strategy from equation 17. The strategy chooses to continue working if the discounted expected pension increase is equal to 1.5 times the final wage earned. Therefore ξ is set to 1.5/disc denoted as q for this strategy.

The seventh and final managed strategy is referred to as the power utility strategy or -1 over x strategy, which is the simplification for the default η of 2. The mathematical representation can be found in equation 20 and uses a λ

equal to 0.3. Below this pension replacement ratio the utility value is equal to an enormous negative constant of -100. A pension ratio of 0.3 may be interpreted as a zero, since λ shifts the whole utility curve to the right. Values below this ratio are very rare therefore a constant below this level is considered sufficient. This constant is needed since the default η for the power utility function used in this research is 2, which could result in dividing by zero or negative values for a pension ratio of 0.3 or lower.

Strategy 1 (minimum/median function)

The strategy denoted as strategy 1 in this research is mathematically stated in equation 22. The main focus of this strategy is to obtain a pension replacement ratio higher than a minimum specified by the participant with a certain level of confidence. For the combinations of lifecycles, premia and accumulation phase length that exceed the minimum with a value higher than the required confidence level this strategy maximizes the median result. Choosing a strategy based on the highest median value is 'safer' or more risk averse compared to selecting the highest mean, since the median does not take extremely large or small values into account. Nevertheless maximizing the median when confident of reaching the minimum is a strategy which aims to take responsible risks and tries to profit from market timing. If one would prefer maximizing the mean or mode, the strategy could easily be changed towards optimizing these values.

When all possible premia, ages of retirement and lifecycles are unable to reach the minimum with the required level of confidence the strategy automatically picks the highest possible premium and accumulation phase length because obtaining the minimum pension replacement ratio is most important. Since the highest premium logically results in the highest chance of reaching this minimum and increasing the accumulation phase length is also expected to increase this chance, both will be maximized. The lifecycle will be chosen that has the highest pension replacement ratio for the specified level of confidence, which will thus always be lower than the required minimum.

Since the participant chooses to retire at the earliest possible point in time when the minimum is reached and chooses to continue working as long as the minimum is not reached the accumulation phase length is not included directly in the utility function. The premia are taken into account when the minimum can be reached with enough confidence. The differences in the premia are compared to the extra pension cash flows they will result in. All cash flows are discounted using the discount rate to take into account the time value of money. The strategy will put in a higher premia when the return this extra deposit is expected to make during the remainder of the accumulation phase is higher than the discount rate in the median. Thus for this strategy the utility function is able to compare an additional premium to the expected pension using only a discount rate.

In the Netherlands the vast majority of the population is eligible to receive the first pillar of the pension. For these people it is less obvious to assign so much weight towards obtaining a minimum, since the government benefits

already provide a reasonable minimum allowance. For participants only eligible for a part of this first pillar allowance or participants with large fixed costs implementing a minimum is an interesting option.

The most important parameter in this strategy is the minimum pension replacement ratio. A plain choice for the minimum is the allowance which is needed to cover all fixed costs. The extra pension the participant receives can then be used for leisure purposes. It is important to know that a irrealistic or high minimum might hurt performance. The strategy selects a lifecycle with a high result in a low percentile of the distribution, which favours more risk averse lifecycles. Choosing risk averse lifecycles might further thwart the chance of reaching the minimum and the expected value of the pension. Another parameter that needs to be set is the required confidence level. A value of 5 percent is commonly used in econometric literature, however any value is possible. There also is a clear interaction between the minimum and the confidence level. E.g. a high minimum with a low confidence level might be equivalent to a lower minimum with a higher confidence level.

Strategy 3 (absolute loss function)

In strategy 3 the research makes use of a absolute loss function, for which the mathematical representation is provided in equation 18. The intuition related to this function is fairly simple. The participants values each unit the same until a certain target is met. Since risk averseness is assumed, the loss aversion coefficient φ should be larger than one. The value used throughout this research for φ is two. This means that a participant is willing to pay a discounted extra premium of 1 in order to avoid a pension allowance which is 0.5 lower if the target is not met or a 50 percent chance of the pension allowance being 0.5 below target.

To understand this utility function, consider a person who's basic needs are all fulfilled by the first pillar allowance. This person can only derive extra utility from time spent in a safari park. The park is open 40 hours a week and the person derives the same utility from the first minute in the park as from the last minute or any other minute, assume that the person pays the safari park for the total time spent in the park. If his pension provides him with enough money to spend 42 hours in the park on a weekly basis, he would have the same utility as he would have if his pension only provided enough money for 40 hours per week since he cannot derive any utility from the money he does not spend. The difference in utility between spending 0 or 5 hours in the park or 20 and 25 hours in the park is the same, since he gets to spend 5 additional hours in the park in both cases.

The absolute loss function is an oversimplification for the pension allowance. No matter how much money one already possesses extra money always provides a humble increase in utility. The constant slope of the utility might also be too restrictive. Consider the above example again, most people would derive most utility from the first moments in the park with the marginal utility gradually declining. The latter part can be incorporated in the utility function via the

means of a quadratic utility function.

The loss aversion coefficient can be estimated by doing a thought experiment how much one would be willing to pay to gain 1 in a bad state of the world in which the target is not met. This coefficient thus has a lower bound of 1 if risk averseness is assumed. The absolute loss function seems suited for participants who have their fixed costs covered via other pillars and use their pension for leisure. They are not risk neutral since the loss aversion coefficient is higher than 1 and they have a utility cap.

For an extra year of work the participant with an absolute loss function needs to have an expected increase of 5 percent in the pension replacement ratio on the relevant interval below his target.

Strategy 2 (quadratic loss function)

The quadratic loss utility function is presented in equation 19. The intuition behind the function is related to the absolute loss function. Above a certain target there is no more increase in utility. However quadratic loss utility means a very high marginal utility for very poor pension replacement ratios. This marginal utility declines sharply when the replacement ratio comes closer to the target. In other words a person who has lost his entire pension will value an increase of 1 unit (much) more than a person who has lost 10 percent of his pension target. The coefficient for the quadratic loss aversion phi is equal to 10. If the pension replacement ratio is 90 percent the loss would be 10 percent. $10*(0.1)^2=0.1$ ergo a participant with this loss aversion function is indifferent between the loss associated with not reaching the pension target and e.g. paying additional premia. For a loss of 20 percent however, the participant is willing to pay an amount of 4 to gain 1 in this bad state of the world.

The quadratic loss function still has the first problem encountered in the absolute case, namely that it cannot account for extra utility when the target is reached. The quadratic loss function does have different marginal utilities, but these marginal utilities need not necessarily follow a quadratic pattern which might overweight extreme losses.

The nature of the quadratic loss utility function makes it well suited for participants who rely on (part of) their pension for basic expenses. If they had to skimp on these basic expenses they would suffer heavily. Once they have these basic expenses covered their marginal utility which they derive from 'luxury' articles is lower and again the utility is capped at a target.

The coefficient for postponing retirement is again 5 percent. For a pension replacement ratio of 90 percent an expected increase of about 5 percent would be needed to continue working, but for a pension replacement ratio of 80 percent an increase of only about 1.25 percent would suffice.

Strategy 4 (mean-variance function)

One of the most well known theories from portfolio management is the mean variance theory. This theory assumes that an investor is interested in a high

mean whilst minimizing the variance associated with the portfolio. All investors are assumed to be risk averse with an individual parameter gamma. The mathematical expression of these mean variance preferences is given in equation 16. The higher ones risk aversion parameter gamma is the more the variances decreases his utility. An infinitely risk averse investor would invest all money in the global minimum variance portfolio. A problem encountered in this research is directly related to this phenomenon. Raising the premium increases the variance and is therefore less attractive in this strategy in comparison to other strategies. High pension results also increase the variance, therefore mean variance preferences could lead to selecting a strategy which is strictly dominated by another strategy. To counter this the pension replacement ratios are truncated at the target ratio which is equal to maintaining the consumption level. The risk aversion parameter gamma is often put equal to 2, 5 or 10 describing a low, normal and high risk aversion respectively.

To convert the difference in premia to a common unit this strategy simply uses the discount rate to compare positive or negative sums of money in different time periods to one another. For one year of extra work the participant should be expecting an increase in his pension replacement ratio of at least 5 percent on the interval where it matters to the participant. An expected increase of 10 percent above the truncation border therefore will not result in the person postponing retirement.

Strategy 5 (percentiles function)

This strategy focuses on returns in bad states of the world. It sums over five different percentile results, that can be interpreted as the lower bounds for five confidence intervals. It bases its choice of strategy on the performances in these five bad states. The idea behind this strategy is that choosing investments that perform well in bad states of the world will always provide the participant with a reasonable pension. The return of premia is obviously lower than average in bad states of the world, but this is corrected by using the mean return. Therefore this strategy will often put in the maximum premium since the expected return in the mean is often higher than the discount rate. For one year of postponing retirement this strategy needs an expected increase of the pension of 5 percent.

This strategy is suited for participants with a conservative nature who like to have some confidence that they will not lose a large portion of their pension in a single year.

Strategy 6 (risk neutral function)e

When a person is risk neutral; gamma is equal to zero and the variance part is dropped. The participant is thus only interested in maximizing the expected pension replacement ratio. Therefore this person will invest heavily in the the asset classes with the highest expected value. Taking high risks has a down side of occasional catastrophic results. Due to this huge downside risk this pension strategy is only advisable for people who consider their pension as

a bonus additional to their first and third pillar. They can enjoy the high expected value that this strategy provides, but will not be devastated if returns are disappointing. This strategy will pay the maximum premium almost always, since the expectation of the premium invested in equities is larger than the discount rate. Incorporating the retirement age is difficult in this strategy is difficult. The option chosen is to work a year longer if the expected payoff is equal to 150 percent of the last earned salary, this might occur when the pension replacement ratio is already very high earning a return over a already large pool of money.

To convert the difference in premia to a common unit this strategy again simply uses the discount rate. For one year of extra work the participant should be expecting a compensation of over 1.5 times his last earned wage.

Strategy 7 (1 over x utility or power utility)

The concerns for both the absolute loss function and the quadratic loss function can be solved by using a power utility function for the pension utility, which simplifies to a -1 over x function for the default η of 2. This function is concave, meaning marginal utility decreases with a higher pension replacement ratio, and monotone increasing, meaning utility increases with a higher pension replacement ratio. This function can be tuned by shifting the curve to the right, which corresponds to setting a minimum below which utility can not be lower. This is useful since a -1 over x function can take extreme values when x is close to zero, one extreme value could dramatically influence the mean of all utilities on which the strategies are selected, therefore this is undesirable. In this research a pension replacement ratio of 30 percent is used as minimum below which utility is a large negative constant.

To convert the difference in premia to a common unit this strategy uses the discount rate to compare positive or negative sums of money in different time periods to one another. The utility derived from pension replacement ratios however is somewhere in between the quadratic loss function and the absolute loss function for the interval below the target. Above this target the marginal utility is still positive but declining. The accumulation phase length has again the coefficient of 5 percent but has the same interpretation problem as the premium. This strategy is thus highly arbitrary since it is difficult to compare additional premia paid and postponing retirement to arbitrary utilities derived from the pension but is nevertheless the closest one will get to ones 'true' utility function.

To acquire some intuition regarding the power utility function a numerical example is considered. Suppose the pension replacement ratio is equal to 70 percent if the participant chooses to retire . The utility associated with this pension replacement ratio is -1/(0.7-0.3) = -2.5. A utility value on average higher than -2.45 would stimulate the participant to continue working for an extra year. (-1/-2.45)+0.3= 0.7082, ergo an increase of 0.82 percent in the pension replacement ratio, with certainty, would suffice. Now, consider a risk of the pension replacement ratio falling to 60 percent with 20 percent probability.

The expected value for the remaining 80 percent should be higher than 0.75 in order for the participant to continue working in this case.

Appendix A.2 Market timing

An old well known trading adage is "Sell in May and go away, and come on back on St. Leger's Day." This saying reflects a belief that equity returns tend to be lower in the May-October period. There is empirical evidence that this adage has some truth in it. Albeit that there is no satisfying explanation why this phenomenon would still occur. At the time the adage came into fashion the seasonal moving to the countryside common among English aristocrats would lead in a slow period in which not much trading was done and little returns were to be made. This example is illustrative of why the efficient market hypothesis might be too strong of an assumption. The efficient market hypothesis states that asset prices perfectly reflect all available information. For equity returns however this research assumes the efficient market hypothesis to hold, since equity returns are unpredictable in the framework used in this research.

For the returns on bonds a degree of predictability is assumed, due to the mean reversion present in the interest rate model. In econometric literature the model used for the interest rate, and inflation rate, is known as Vasicek's model, known for capturing mean reversion. The idea behind the mean reversion is that interest rates can not rise indefinitely. Very high levels would increase the savings rate, in turn decreasing economic activity negatively affecting the interest rate. The theoretical lower limit for an interest rate is nil percent, with a practical bottom estimated at minus one percent due to the safety and service a bank provides. Since the interest rate moves within a limited domain it therefore can be expected to have a long run equilibrium value. A similar reasoning can be used for the inflation rate, since both hyperinflation and hyperdeflation are rarely observed and certainly not durable in a developed market.

Appendix A.3 Inter-temporal discount rate or rate of time preference

All consumers search for a balance between consuming their wealth now and saving for greater future consumption. There are two main reasons for a consumer to favour consuming immediately over postponing to consume the same unit. The first reason is rational. A consumer can consume now with certainty or postpone and face the risk of losing his wealth or becoming unable to consume. Death would be an extreme example of the latter. The second reason is irrational and has to do with the 'impatience' that is a fundamental attribute of human nature. Both reasons are subjective and can be combined into one subjective inter-temporal discount rate.

This discount rate should be higher than the average risk free rate, otherwise consumption would be postponed indefinitely. The discount rate should also be lower than the highest expected return. If the opposite were the case, a consumer would immediately consume all wealth as it becomes available. Throughout this research the inter-temporal discount rate is fixed at five percent which meets

the above criteria.

Appendix A.4 Regarding the choices on the macro longevity risk

This paragraph will start with explaining why certain scenarios are not considered and will end with defending the scenario that was chosen. First of all a significant decrease in the life expectancy only happened twice in the last century as a result of a world war. These wars however have a more limited effect on the life expectancy of a person conditional on that person reaching the age of 65. Apart from a major war, people becoming increasingly obese or a pandemic could threaten the positive trend in life expectancy. This research chooses to ignore these possibilities. Besides the problematic estimation of such risks, a decrease in the life expectancy would result in a cheaper annuity. Ergo, if these events occur, a participant receives a higher pension than expected.

Some gerontologists claim that developments in their sector could dramatically increase the life expectancy to ages such as 130. A Cambridge researcher even claims a 50 percent probability that the first human to live past a 1000 years is already born[22]. If these extravagant claims prove to be true the pension system would definitely be disrupted. However if people are able to live until 130 in relative good health the potential accumulation or productive phase of ones life is dramatically larger. In such an extreme scenario the retirement age will increase without doubt, but it would not threaten the foundation of a managed DC scheme. On the contrary, a managed DC scheme would require a smaller burden during the accumulation phase.

Now the focus is shifted towards the scenario that is used as a robustness test for the managed DC scheme[23]. Figure **??** shows the trend over the past decades which has been roughly an increase of one year in life expectancy per ten years for both men and women.

The increase in life expectancy is for a large part due to the reduction of the chance of dying 'young'. Figure **??** shows that this is also the main driver for future increases in life expectancy. The maximum age only minimally improves whereas a lot of probability mass is shifted to the right. This means that the increase in life expectancy conditional on having reached the age of 65 is a lot weaker. These changes anticipated by the Actuarieel Genootschap & Instituut implicate a reduction in the variance of longevities, which would enable issuers of annuities to reduce their premium.

To check the robustness of the managed DC scheme the effects of an unanticipated increase of 1.5 years in the participants' life expectancy in year 10, 20,

---

[22]De Grey, A., & Rae, M. (2007). Ending aging: The rejuvenation breakthroughs that could reverse human aging in our lifetime. Macmillan.

In my view this 50 percent probability for living past a 1000 years sounds more like it has been plucked out of thin air rather than it being based on any sound mathematical calculation of probabilities. However the size of the probability does not matter as is explained later in the paragraph.

[23]Both figures are taken from: Genootschap, A. (2010). Prognose-tafel 2010-2060. Utrecht: Actuarieel Genootschap & Actuarieel Instituut.

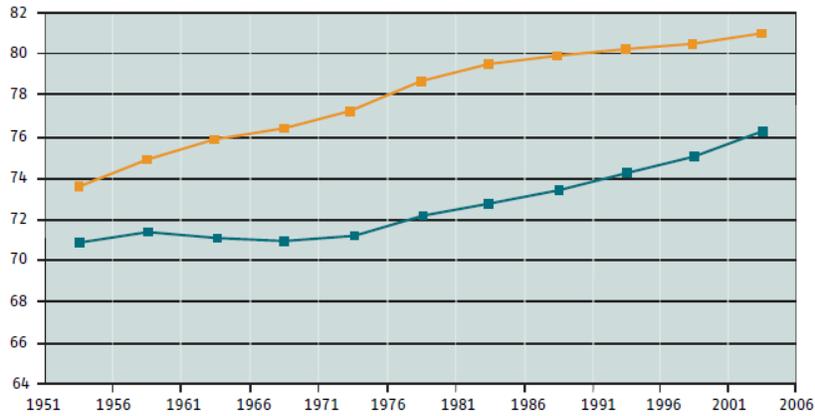This institute provides actuarial mortality tables for all Dutch pension funds.

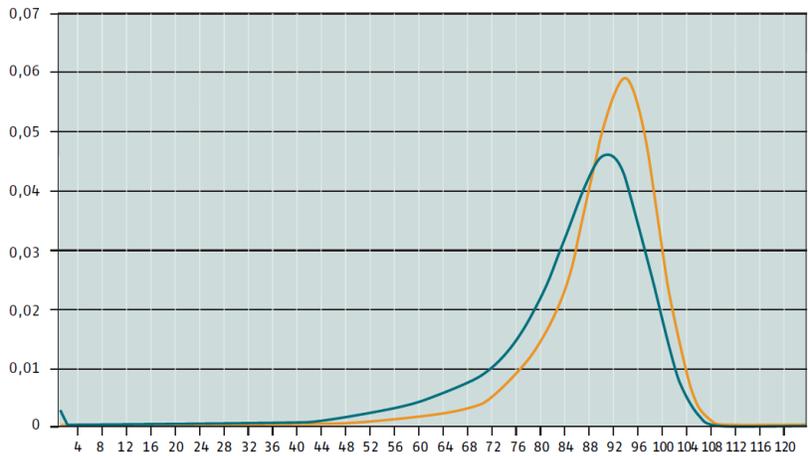Figure 19: Life expectancy at birth for women (orange) and men (green).

Figure 20: Probability density function of women's life expectancy. The orange line denotes a girl born in 2060 and the green line a girl born in 2010.

30 and 40 of the accumulation phase will be considered, which is a strong trend compared to the realized trend over the past decades. Keep in mind that these increases would also drive up the age on which a person becomes eligible for the first pillar pension provided by the government.

Appendix B (Other)

Fifteen years prior to the expected accumulation phase length the lifecycles start linearly reducing the percentage invested in equities towards zero. Thus,
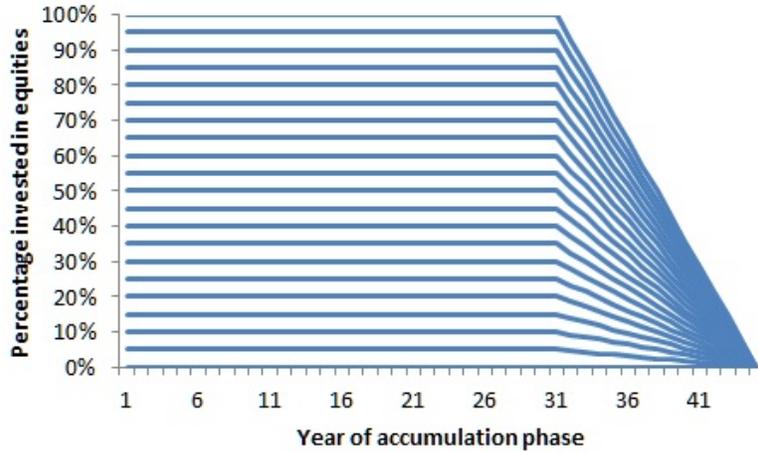
Figure 21: Overview of the 21 lifecycles considered for a fixed accumulation phase length of 45.

for different accumulation phase lengths the flat part of the lifecycles is either longer when the anticipated accumulation phase length is higher than 45 years or smaller when the anticipated accumulation phase length is lower than 45 years. The lifecycles do no allow a full or large investment in equities when close to the retirement. This is important to have in mind when interpreting results, especially behaviour of strategies close to retirement. Also note that an investment mix completely made up out of assets in year one consists of bonds during the last phase of the accumulation period and this is taken into account in the expected return.

Any monotone increasing utility function, as is provided in Figure 22, will derive extra utility from an increase in the pension replacement ratio. The concavity of the utility function guarantees that the marginal utility declines. Thus, the additional utility from the first extra unit is always higher than for the second extra unit. This is one of the characteristics of a risk averse preference set.

Appendix C (Additional results)

9.2.1 Unmanaged lifecycle investing

Figure 21 in Appendix B contains 21 different lifecycles. For computational feasibility only 21 options are researched and differences of five percent between the lifecycles are considered sufficiently small for the purposes of this research. When the retirement approaches, all equities will be phased out to avoid large losses close to the retirement date. Figure 23 shows results comparing fixed
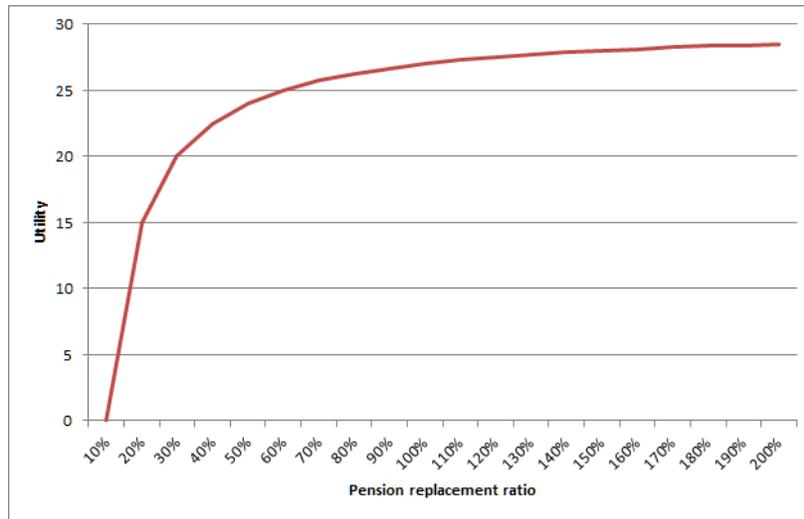
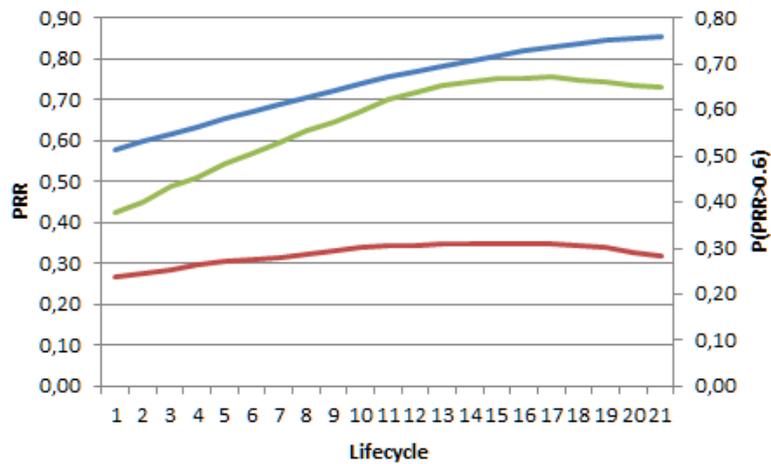Figure 22: Concave and monotone increasing utility function.

lifecycles.

Figure 23: Results for fixed lifecycles. The green line denotes the probability of meeting the target on the right axis. The red line denotes the fifth percentile result, whereas the blue line denotes the expected value, both on the left axis.

Economic theory states that investors require to be compensated for risk. Therefore investing in risky equities has a higher expected value compared to virtually riskless government bonds, due to the risk premium. The blue line

in Figure 23 denoting the expected value clearly shows a linear upward trend reflecting this relation. However, the other two lines show that there is also a downside to high equity mixes. The fifth percentile results, shown by the red line, are best for lifecycles starting with a equity percentage of around 60 or 70 percent, which is close to traditional values for static asset mixes. The green line denotes the chance of obtaining a pension replacement ratio higher than 60 percent. Again high equity mixes, which have a high expected value, are outperformed by more conservative mixes in terms of the chance of reaching a realistic target. Important aspects of individual pension investing are the result obtained in a bad state of the world, the chance of reaching a certain realistic target and obtaining high returns in general. Classic asset mixes often used equity percentages of around 70 percent for this type of investing, which is backed up by the results in Figure 23. This induces us to consider lifecycles 13 and 15, starting with 60 and 70 percent equities respectively, as benchmarks against which the managed DC strategies will be compared.

9.2.2 Unmanaged investing for different premia

Unmanaged investing using different premium percentages, ceterus paribus, is simply a matter of multiplication. In the individual DC framework a person receives double the pension if a double premium is payed throughout the accumulation phase. Therefore the expected value is also a simple linear function. For the chance of obtaining a certain target pension replacement ratio this relationship is non linear as can be seen in Figure 24:
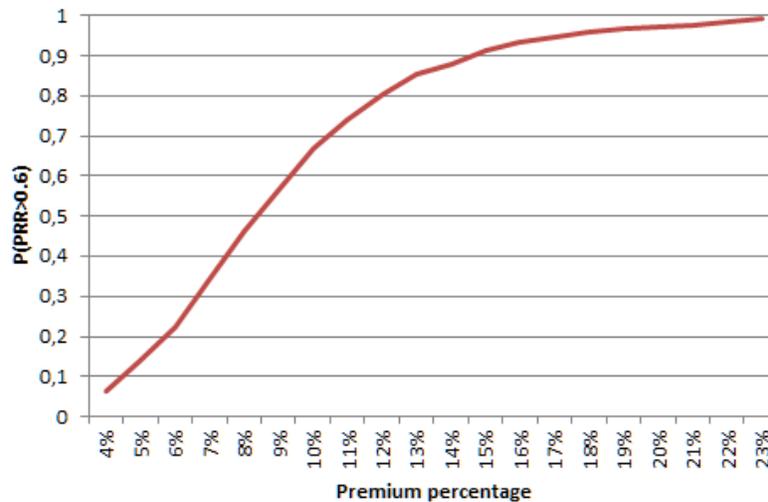

Figure 24: Results for fixed premia. The line denotes the probability of meeting the target.

Unsurprisingly the chance of reaching the pension goals increases when premia increase. The red line moves asymptotically towards 1, thus extreme scenarios still require a higher premia or smarter investing. The declining increase is logical since 1 is a natural boundary for a chance. Figure ?? provides a plot of the probability density function which illustrates why this increase declines as the distribution is shifted further towards the right.
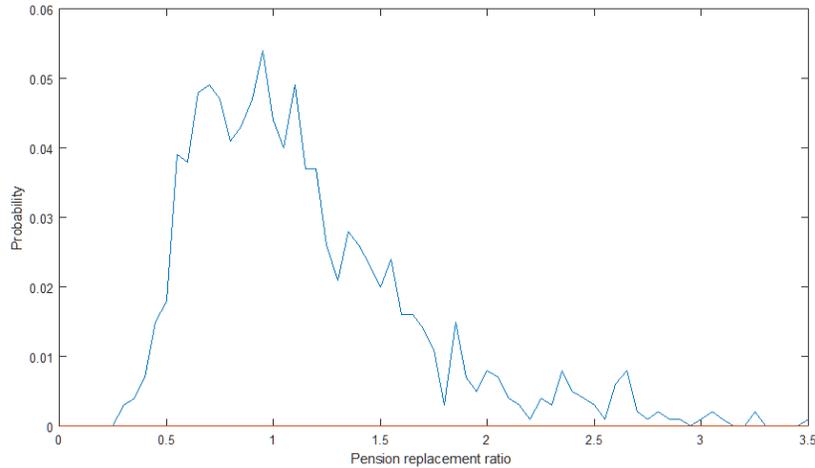

Figure 25: Probability density function for an unmanaged strategy with lifecycle 15, a premium of 11 percent and an accumulation length of 45 years.

Paying an extra premium multiplies every value in the probability density function, thereby shifting it rightward and flattening it. Ergo the probability mass covered by an extra percent of premium declines once the mode or the peak of the distribution is higher than the target.

9.2.3   Unmanaged investing for different retirement ages

Again the same pension scheme is considered, this time for different accumulation lengths. These results can be found in Figure 26.

The increase in the chance of obtaining the pension, denoted by the green line, shows a stronger trend towards 1 compared to the graph for the premia. This is due to the twofold effect an increase of the retirement age has on the chance of obtaining the pension goal. First the period over which the pension is invested becomes longer and an extra year of premia is added. Second the period over which the pension grantee receives the pension becomes, since the research works with a fixed life expectancy equal to 65 minus the accumulation phase[24], thus less capital is required to pay the pension goal. This twofold effect is also visible in the expected pension replacement ratio, the blue line, as well as

---

[24]A person that starts working/accumulating at the age of 21 thus has a life expectancy of 86 and a standard retirement age of 66.
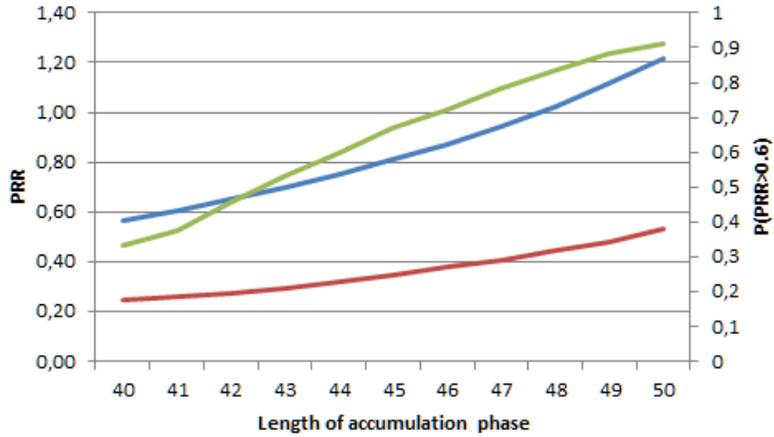
Figure 26: Results for fixed accumulation phase lengths. The green line denotes the probability on the right axis. The red line denotes the fifth percentile result, whereas the blue line denotes the expected value, both on the left axis.

the fifth percentile pension replacement ratio, the red line. Especially for higher lengths the second effect is large, since the extra year becomes larger relative to the period in which the pension is consumed.
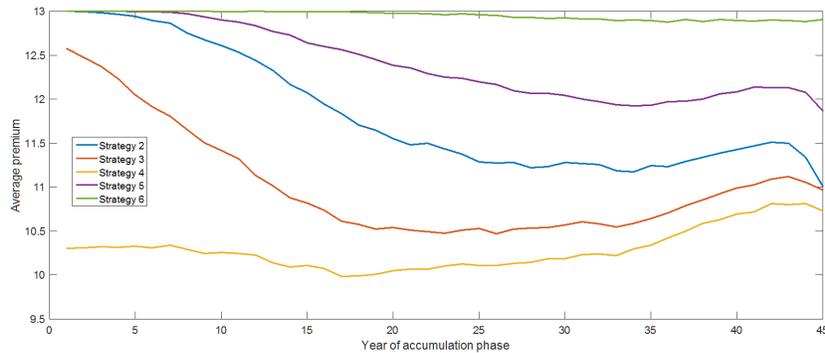
Figure 27: Average premium in percentages payed during each year of the accumulation period for strategies 2 to 6.

For strategies 2 and 3 a similar pattern is observed as for strategy 1, for strategy 5 the result is similar to strategy 7. The more interesting results are observed for strategies 4 and 6. The first chooses to put in very little premium throughout the period, only to increase the average premium close to the retirement age. This indicates why strategy 4 works poorly, it dislikes variance which is increased by putting in more money. Closer to the retirement date this

variance has decreased enabling the strategy to increase its premium. Strategy 6 shows a minor decrease in the average premium when the retirement date approaches. This is due to the nature of the lifecycles which do not allow any strategy to be invested in a high percentage of equities close to retirement, i.e. it forces a strategy to invest in bonds. When investment opportunities in bonds are very bad due to a low level the expected return during the remainder of the period might be lower than the discount rate, thus the maximum premium is not attractive anymore from a risk neutral perspective.
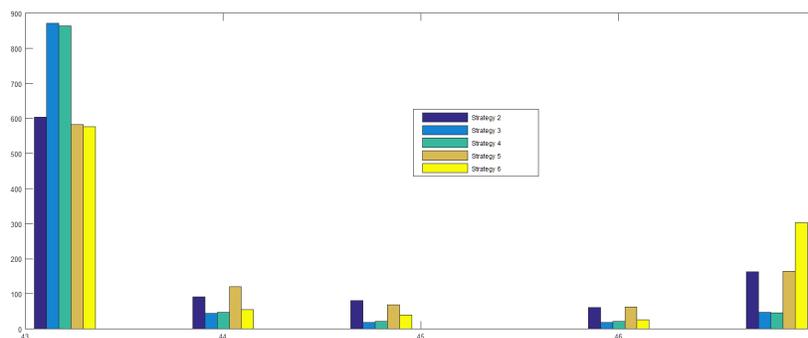

Figure 28: Histogram of realized retirement ages for unselected strategies.

A similar bi-modal tendency is visible in the histogram for strategies 2 to 6 as was the case for strategies 1 and 7. The strategies all mimic the trend found for strategy 7. Most often the participant retires after 43 accumulation years, every following year a declining fraction retires leaving the rest of the probability mass to retire after 47 accumulation years. For strategies 3 and 4 the fraction that retires at the earliest moment is a lot higher which logically results in low fractions elsewhere.

9.3   Fully relaxed parameter space with wage growth

In the early models the wage was normalized and assumed to be corrected for the realized inflation per year. In practice however, people gain experience and grow towards higher functions with a growth in their wage matching their increased skills and experience. Different jobs or sectors might entail different growth paths. For now the wage growth data provided by PGGM which concerns the relatively large and diverse health care sector is considered.

The pension replacement ratio is calculated relative to the final wage received by a participant. Therefore the premia deposited during the early years, in which the wage was roughly half of the final wage, are a lot less compared to the case where wages were normalized. This increased difficulty is reflected in the results shown in Table 36.

Table 36 indeed shows that means have bean dramatically reduced within the same parameter space. The patterns observed before seem unaffected by

Table 36: Table containing results for the fully relaxed parameter space with wage growth. Lifecycle* denotes the mean for the first 30 years in which the lifecycles are constant.

	Mean	Median	prctile 5	prctile 1	Premium	Lifecycle*	APL
Strategy 1	0,97	0,92	0,53	0,39	12,69	3,99	45,34
Strategy 2	0,94	0,87	0,53	0,40	13,24	4,08	44,43
Strategy 3	0,90	0,81	0,42	0,31	15,34	3,96	43,45
Strategy 4	0,86	0,77	0,40	0,30	15,14	3,36	43,47
Strategy 5	0,99	0,95	0,42	0,32	14,70	4,22	44,30
Strategy 6	1,10	0,91	0,38	0,28	16,80	4,65	44,25
Strategy 7	1,02	0,92	0,49	0,39	12,75	4,48	44,82
Unmanaged 1	0,94	0,85	0,45	0,34	13	15	47
Unmanaged 2	0,48	0,43	0,22	0,17	9	15	43
Unmanaged 3	0,91	0,83	0,45	0,33	13	13	47
Unmanaged 4	0,46	0,42	0,22	0,17	9	13	43

the inclusion of wage growth. Percentile results are still the best for managed strategies 1, 2 and 7. With the minimum strategy (1) having a higher average accumulation phase length, the utility curve (7) strategy the highest average premium and the quadratic loss (2) strategy the lowest mean and median result.

9.4 Fully relaxed parameter space with risky wage

The previous model supposes that wage grows via a predefined curve. In reality wage growth is subject to variation. The model is extended to incorporate this uncertainty about the wage development. Log-normal shocks are assumed to avoid negative shocks to the wage. Though such shocks may be observed in reality, they could shrink the pension target which the research chooses to restrict. No positive autocorrelation between the shocks is included to prevent large discrepancies in wages, whereas a negative autocorrelation defeats the purpose of including risky wage growth.

The inclusion of risky wage improves the mean results for all pension strategies, as Table 37 shows. The opposite is true for the fifth and first percentile results. The latter is unsurprising since there is another risk factor, unexpected increases in the wage, which can deteriorate pension replacement ratios. The former may seem counterintuitive at first. On average the final wage is still the same. Only there are now low final wages, for which it is more easy to obtain a high pension replacement ratio, and high final wages, for which more risks are needed driving up the mean return. This phenomenon slightly lifts the mean pension replacement ratio.

After the inclusion of risky wage growth the results for the sub groups are compared. The chances of a care free pension within this parameter space are much smaller for the more realistic risky wage growth variant compared to the normalized flat wage curve. This is due to the final wage and thus target being

Table 37: Table containing results for the fully relaxed parameter space with risky wage growth. Lifecycle* denotes the mean for the first 30 years in which the lifecycles are constant.

	Mean	Median	prctile 5	prctile 1	Premium	Lifecycle*	APL
Strategy 1	1,00	0,93	0,51	0,37	11,96	12,73	45,30
Strategy 2	0,96	0,87	0,51	0,37	12,08	13,30	44,42
Strategy 3	0,92	0,80	0,41	0,28	11,94	15,32	43,43
Strategy 4	0,88	0,78	0,38	0,26	11,31	15,13	43,44
Strategy 5	1,07	0,94	0,40	0,27	12,65	14,93	43,95
Strategy 6	1,14	0,94	0,36	0,25	12,65	16,78	44,27
Strategy 7	1,04	0,93	0,48	0,37	12,47	12,72	44,75
Unmanaged 1	0,97	0,85	0,43	0,32	13	15	47
Unmanaged 2	0,49	0,43	0,22	0,15	9	15	43
Unmanaged 3	0,94	0,83	0,43	0,33	13	13	47
Unmanaged 4	0,47	0,42	0,23	0,15	9	13	43

Table 38: Results conditional on the minimum accumulation phase length being chosen with a pension replacement ratio higher than one.

	Frequency	APL	Premium	Lifecycle*	Mean
Strategy 1	0,28	43,00	10,99	12,36	1,48
Strategy 2	0,28	43,00	11,28	13,24	1,43
Strategy 3	0,32	43,00	11,15	14,45	1,43
Strategy 4	0,28	43,00	10,54	14,32	1,41
Strategy 5	0,25	43,00	12,52	13,82	1,56
Strategy 6	0,17	43,00	12,48	14,37	1,52
Strategy 7	0,18	43,00	11,77	12,60	1,62

higher relative to the starting wage. This also affects the mean of the fraction of care free pensions which inevitably have become smaller. The average premium needed to obtain such a pension has also increased grosso modo a half percent for all strategies, again reflecting the increased difficulty. All these effects seem to be comparable across different strategies, with much of the patterns the same for both wage curves.

As expected after the results for the previous group and from the intuition about the wage curves, Table 39 indeed shows an increased frequency for the pension group scoring lower than the minimum. The accumulation phase length is on average roughly the same for strategies compared across wage curves, the premia are on average a little higher for the risky wage growth case. This is expected since more premium is needed to obtain the target pension and a similar effect was visible in the care free pension group.

The results for the first two groups were comparable across different wage curves, therefore it is no surprise to reach the same conclusion for the last group. This group now contains the majority of probability mass for every

Table 39: Results conditional on the pension replacement ratio lower than the minimum of sixty percent.

	Frequency	APL	Premium	Lifecycle*	Mean
Strategy 1	0,10	47,00	12,88	15,09	0,49
Strategy 2	0,11	46,35	12,80	15,86	0,50
Strategy 3	0,19	43,83	12,52	15,83	0,47
Strategy 4	0,23	43,60	11,82	15,71	0,47
Strategy 5	0,20	43,34	12,58	15,32	0,46
Strategy 6	0,24	43,08	12,51	15,69	0,44
Strategy 7	0,16	45,28	12,81	14,54	0,50

Table 40: Results conditional on not being in the 'care free' or 'below minimum' groups.

	Frequency	APL	Premium	Lifecycle*	Mean
Strategy 1	0,62	46,03	12,24	13,80	0,87
Strategy 2	0,61	44,72	12,31	13,88	0,82
Strategy 3	0,49	43,54	12,22	14,71	0,77
Strategy 4	0,49	43,62	11,52	14,55	0,78
Strategy 5	0,56	44,57	12,73	14,22	1,07
Strategy 6	0,60	45,10	12,75	13,71	1,31
Strategy 7	0,66	45,11	12,58	13,37	1,01

pension strategy and exhibits the same differences as for the flat riskless wage curve.

9.4.1 Additional results for the fully relaxed parameter space with risky wage, job risk and a stability based domain for premia and strategy

Table 41 provides results for the first group of schemes which obtained a care free pension. All strategies put in less premium compared to the overall average. This is a promising result since it shows the ability to cut back in premia when a

Table 41: Results conditional on the minimum accumulation phase length being chosen with a pension replacement ratio higher than one.

	Frequency	APL	Lifecycle*	Premium	Mean
Strategy 1	0,47	43,00	11,60	0,12	1,41
Strategy 2	0,36	43,00	13,17	0,12	1,37
Strategy 3	0,41	43,00	14,40	0,11	1,41
Strategy 5	0,37	43,00	13,50	0,14	1,55
Strategy 6	0,32	43,00	12,70	0,19	2,02
Strategy 7	0,33	43,00	13,53	0,15	1,77

Table 42: Results conditional on the pension replacement ratio lower than the minimum of sixty percent.

	Frequency	APL	Lifecycle*	Premium	Mean
Strategy 1	0,01	47,00	14,94	0,19	0,50
Strategy 2	0,03	46,12	16,02	0,17	0,52
Strategy 3	0,07	43,76	15,95	0,15	0,51
Strategy 4	0,35	43,53	16,06	0,08	0,35
Strategy 5	0,06	43,87	15,77	0,14	0,43
Strategy 6	0,05	43,11	16,20	0,15	0,39
Strategy 7	0,03	45,17	15,64	0,18	0,50

Table 43: Results conditional on not being in the 'care free' or 'below minimum' groups.

	Frequency	APL	Lifecycle*	Premium	Mean
Strategy 1	0,52	45,51	13,76	0,15	0,92
Strategy 2	0,61	44,56	13,80	0,15	0,86
Strategy 3	0,52	43,31	14,53	0,14	0,80
Strategy 4	0,48	43,75	14,08	0,11	0,78
Strategy 5	0,57	45,08	14,40	0,16	1,03
Strategy 6	0,63	45,53	13,67	0,21	1,49
Strategy 7	0,64	45,47	13,98	0,17	1,21

high pension result is expected. Again there is a clear difference between the loss aversion strategies 1, 2 and 3, which put in considerably less premium compared to the other strategies which reward high results. This can also be seen in the means which are higher for the second group. The chances of obtaining the 'care free' pension have also increased which is related to the higher average premia for an important part.

The chance of obtaining a pension replacement ratio below the minimum is also severely reduced. The average premium for this group is much higher than for the 'care free' pensions as well as the percentage which is invested in equities. Poor performances can therefore be improved in a stability based domain for premia, since strategies raise their premium and choose a relatively risky lifecycle.

The final group shows little new insights. The results reconfirm the much higher mean for the second group of strategies compared to the defensive strategies.

9.4.2 Fully relaxed parameter space with risky wage and stability based domain for premia and lifecycle

The previous model had full freedom to choose between the 21 lifecycles every year. Now the restriction is imposed that a maximum shift of two lifecycles

Table 44: Table containing results for the fully relaxed parameter space with risky wage growth and a stability based domain for both premia and lifecycles. Lifecycle* denotes the mean for the first 30 years in which the lifecycles are constant.

	Mean	Median	prctile 5	prctile 1	Premium	Lifecycle*	APL
Strategy 1	1,13	1,03	0,74	0,58	0,14	13,74	44,36
Strategy 2	1,02	0,93	0,66	0,54	0,14	12,93	44,06
Strategy 3	1,01	0,90	0,57	0,41	0,13	14,64	43,23
Strategy 4	0,70	0,70	0,11	0,03	0,10	15,12	43,56
Strategy 5	1,16	1,13	0,56	0,24	0,15	13,94	44,29
Strategy 6	1,54	1,52	0,55	0,27	0,20	16,42	44,60
Strategy 7	1,32	1,25	0,64	0,48	0,16	13,58	44,58

is allowed. The reasoning behind this is that transaction costs and practical limitations provide problems when shifts are too large. When the pool of participants is sufficiently large these shifts can be canceled out for a part, but since participants obtain a similar return in bad years or good years a positive correlation between shifts in lifecycles is expected. Therefore transaction costs and practical limitations might form a real problem.

Undoubtedly the restriction will hurt performance, Table 44 shows to what extent performance is hurted.

All strategies show that they suffer from the restriction. For most indicators only a minimal loss follows from the restriction. This difference can be seen indiscriminately amongst the results. The average lifecycle is also very much comparable to the case without the restriction on the lifecycle. This indicates that large shifts in lifecycles should not form a problem for a managed DC scheme. When individuals are pooled these shifts are reduced even further and the optimal balance between the toughness of the restriction and the associated transaction costs is improved. The tables containing results for the subgroups can be found below.

The 20 year rolling window is more volatile due to the higher weight of an individual observation, such as when the 'beurskrach' of 1929 enters and leaves the window. The geometric average of the equity returns has a minimum of around five percent in this sample, providing us with comfort that the geometric average of 6.5 percent is not too high. The yields for 2015 are much lower than for the 50 year rolling window, since rates have plummeted in the past decade. The two percent long run mean seems much more plausible using the 20 year window and seems a sufficiently pessimistic scenario to research the managed DC schemes in difficult economic circumstances.
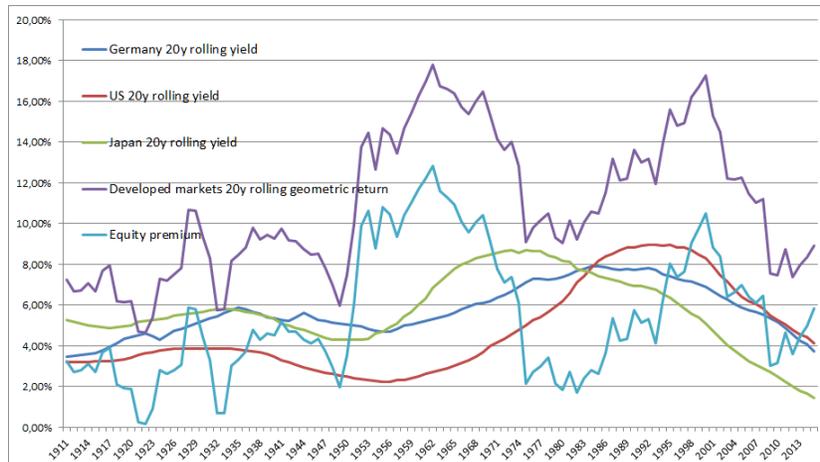
Figure 29: Geometric returns for time series calculated over a moving window of 20 years. Equity premium is calculated by substracting the equally weighted mean of the bonds from the developed markets equities.

10 Sensitivity tests appendix

10.1 Stability

In order for a managed DC scheme to be consumer-friendly, it has to provide a degree of stability similar to the current DB scheme. Due to the collective nature of a DB scheme, a stable portfolio can be held with minor changes in the premium payed by an individual participant. The former ensures that participants suffer only marginally from transaction costs. The latter enables a participant to uphold a stable consumption pattern. Sharp increases in premia will erode trust in the pension scheme, as most participants will wonder why the premia was not higher last year. Sharp increases also can have significant negative effects on the buying power of a participant which is another possible source of discontent.

In order for a managed DC scheme to smoothly replace a managed DB scheme it has to overcome or at least mitigate said problems. First an overview of the magnitude of both problems and the frequency in which they occur is needed. Subsequently a stability based domain for premia and lifecycles is enforced and performance is reevaluated.

10.1.1 Stability of premia

A pension fund would not face much practical issues if premia were to vary a lot. A clever automated administration system should have no difficulty registering changes and keeping track of all payed premia and the built-up pension capital. As stated earlier an individual will suffer from an unstable premium. A large

Table 45: Results for the final model using default parameters and a stability based domain. The yearly change in the premium is restricted to at most one percent. Based on 250 scenarios.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	1,05	1,01	0,67	0,52	12,82	0,14	44,97
Strategy 2	1,08	0,95	0,60	0,44	12,25	0,17	43,50
Strategy 6	1,53	1,58	0,56	0,30	15,83	0,20	45,26
Strategy 7	1,38	1,33	0,64	0,48	13,20	0,17	45,04

collective of individual managed DC schemes can not provide solace to these problems regarding the individual. Therefore the stability based domain is considered. In earlier results using a stability based premium domain, a maximum shift of two percent up or down was allowed. Now a more restrictive value of one percent is investigated to uncover the effect of this restriction.

The results of the standard model including the default economic scenarios in which the premium is allowed to change only one percent on a yearly basis are provided in Table 45. As a starting value fifteen percent is used.

The means and medians are marginally affected by the stronger restriction when comparing results in Table 45 against those in Table 16. The most notable difference is in the lower percentile results. The restriction limits the strategy in the prevention of poor pension replacement results by allowing only a small increase in the premia. The agility used to counter unfortunate returns is diminished but still considerable.

10.1.2 Stability of lifecycle/portfolio

For the practicability of the managed DC scheme it is important that the portfolio of a pension fund shows some stability. E.g. a pension fund that has to reduce the overall portfolio percentage of equities from 70 percent to 30 percent and back to 60 percent the next year would be faced with huge transaction costs among other problems associated with drastically rebalancing a portfolio[25].

In this research each scenario makes use of different underlying economic scenario's. Therefore it is difficult to derive the stability of a consolidated investment portfolio from the results. An approach in which several different 'reference persons'[26] are used each with a different age, reflecting the demographics of the relevant group, is outside the scope of this research. However, from statistical theory follows that including different (age) groups in a portfolio can considerably diminish the variance[27] and hence improve the stability of the

---

[25]Inter alia: flooding the market, the inability to obtain the required amount of assets and the time needed for rebalancing.

[26]Fictional persons used for comparison ends.

[27]Assume X and Y different groups of equal size and equal variance in portfolio weights, but different ages. A portfolio made up entirely of group X will have variance equal to var(P1)=var(X). A portfolio made up for fifty percent out of group X and 50 percent group Y will have variance: var(P2)=1/4*var(X) +1/4*var(Y) + 1/2*ρ_{xy}*std(X)*std(Y) which sim-

asset portfolio.

To get an idea of the stability of an individual's portfolio the average change of all scenarios in the fixed domain setting is shown in Figure 30.
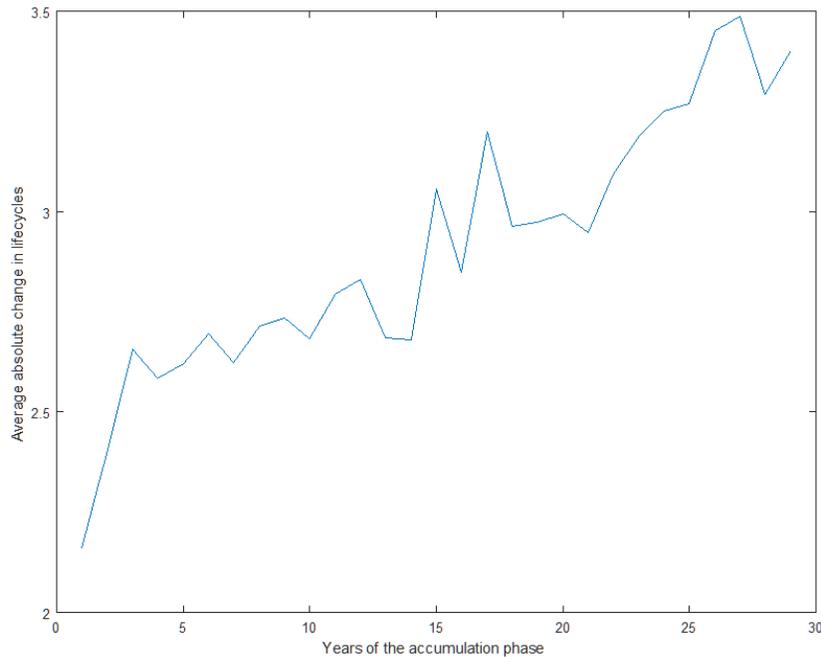

Figure 30: Development of the average absolute change in lifecycles.

Figure 30 shows that the average absolute change in the lifecycle is between 2 and 3.5, or 10 and 17.5 percent equities, on a yearly basis. This is not an alarming value since a pool of portfolios only has to make net changes. Portfolios will tend to make the same changes dependent on changes in the interest rate, thus the benefit from pooling might be low. However transaction costs are relatively limited compared to the benefits of managing ones portfolio and could also be taken into account when deciding whether to switch from lifecycle or not.

Paragraph 6.6.2 showed that restricting the individual portfolio's yearly shift to at most two lifecycles, or ten percent, marginally hurts performance. From Table 30 it is known that in the unrestricted model portfolios are more volatile, occasionally even shifting from the riskiest portfolio to the 'riskless' portfolio. The remainder of this paragraph will look into the effects of restricting the portfolio changes to at most 1, or 5 percent. Results for this model are provided in Table 46.

plifies to $(1/2+1/2*\rho_{xy})$*var(X) using var(X)=var(Y). The correlation coefficient has an upper limit of 1 therefore the following holds: var(P1)$\geq$var(P2). A positive correlation coefficient may be expected in this case, but can still mean a large variance reduction.

Table 46: Results for the final model using default parameters and a stability based domain. The yearly change in the lifecycle is restricted to at most one lifecycle or fivepercent. Based on 250 scenarios.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	1,10	1,03	0,67	0,59	13,55	0,14	44,70
Strategy 2	1,11	0,96	0,65	0,55	12,48	0,17	43,33
Strategy 6	1,61	1,61	0,72	0,35	16,32	0,20	45,27
Strategy 7	1,41	1,32	0,70	0,56	13,63	0,17	45,06

The average lifecyle is higher in Table 46. This has to do with the inability to swiftly change to strong allocation of bonds when bonds are attractive. The strategies therefore remain invested more heavily in equities than desired. The changes in the accumulation phase length and the pension replacement ratio distribution compared to the values in 16 are negligible. Thus the strategies are robust and can handle strong restrictions on changes in lifecycles.

10.1.3 Stability of the accumulation phase length

In a Dutch DB scheme a participant is first informed about the possibilities of retiring prior to the standard age[28] five years before reaching this age. When retirement approaches the participant can choose to exercise the option of retiring early. One of the main benefits of a managed DC scheme is the ability to choose your own preferred retirement age and adjust the scheme to realize this goal. The government is currently debating whether the first pension pillar should become flexible. Such a change would eliminate the gap in the first pension pillar when retiring early. Instead a participant would ideally be able to choose on a monthly basis whether the annuity associated with the currently accumulated capital is sufficient as a monthly allowance for the remainder of the participant's life. The lifecycles should ensure that no large shocks occur close to retirement by investing nominally matched in bonds, which has a similar cash flow pattern as the future annuity.

10.2 Other

10.2.1 Premium cap

An advantage of an individual managed DC scheme is the option of fitting the scheme to ones personal financial situation. If a person goes through a difficult financial period it might be helpful to temporarily cap premia or stop deposits altogether. This might be the case for parents after having a baby or in the period after purchasing a house. To investigate how an anticipated premium cap or stop affects the strategies' performance, the strategies cap premia during year 10 until year 15 of the accumulation period at 5 percent.

---

[28]The age on which one becomes eligible to receive the first pillar pension.

Table 47: Results for the final model using default parameters. With the exception of the starting wage which is 100,000 or 10 times the default starting wage. Based on 250 scenarios.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,94	0,89	0,46	0,37	11,9	11,86	45,54
Strategy 2	0,89	0,80	0,45	0,40	12,4	12,11	44,16
Strategy 6	1,03	0,85	0,33	0,23	15,7	12,40	44,27
Strategy 7	1,00	0,84	0,44	0,37	12,1	12,28	45,01

When less premium is put in, a logical result is a lower pension replacement ratio. A more interesting result is how the premium develops over time. This property is visualized in Figure 31.
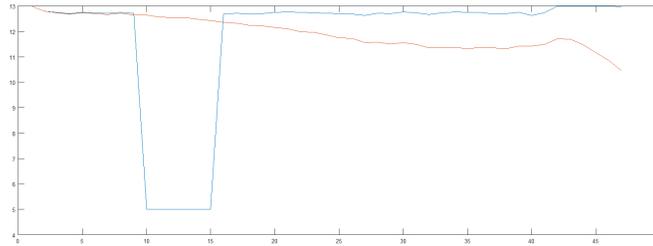

Figure 31: Development of the average premium percentage over time chosen by strategy 1. The blue line denotes the model with the premium cap, whereas the red line denotes the default model.

The blue line is slightly higher during the phase prior to the premium cap, showing the ability of the model to anticipate a future lower premium inflow. The loss associated with the lower premium paid during the cap period is later mitigated by choosing a higher premium on average. These findings are no great surprise, but show the ability of a managed DC model to incorporate a premium cap or even a premium stop, since the model can anticipate restrictions on the premium percentages. In a DB framework with one pension pot such individual preferences or restrictions are much harder to incorporate, let alone anticipate.

10.2.2 Wage level, wage growth and wage risk

To check how much effect the wage level has on the choices made, the wage is inflated with a factor ten to 100.000 euro. The only thing that will change as a result of this setting is the relative effect of retiring early and missing the first pension pillar of 14.000 euro.

From Table 47 one can deduce that inflating the wage level indeed has the anticipated effect. For high wages this gap is relatively small, enabling them to retire earlier than lower wages. This is reflected by the average accumulation

Table 48: Results for the final model using default parameters. With the exception of the wage growth which is 4 percent per year, meaning the final salary is around 6 times as large as the first.

	Mean	Median	Prctile 5	Prctile 1	Lifecycle*	Premium	APL
Strategy 1	0,39	0,36	0,19	0,16	11,9	12,75	47,0
Strategy 2	0,38	0,35	0,18	0,14	14,7	12,68	46,2
Strategy 6	0,28	0,24	0,11	0,08	15,8	12,26	43,1
Strategy 7	0,38	0,35	0,19	0,15	13,6	12,65	46,0

phase length which is lower for all managed strategies by values ranging from 0.23 to 0.65 year. The other results show only very marginal changes, as could be expected.

The wage growth curve used in this research is obtained from an internal PGGM database. The curve, which can be found in Figure 8, shows a relatively flat pattern with little wage growth in real terms in the second phase of the accumulation period. This pattern differs per sector. A 35 years old nurse may expect few possible promotions and limited relevant experience adding to his or her value. A 35 years old lawyer may have different career path and derives a substantial part from his or her personal value from experience, resulting in a steeper wage growth curve. To investigate the effect the steepness of the wage curve has on the pension replacement ratio a yearly wage growth of 4 percent each year is considered. Results for this alteration can be found in Table 48.

The results show that obtaining a pension replacement ratio close to 1 is extremely hard under the default assumptions. Either the premium domain, now ranging from 9 to 13 percent, should be drastically increased or the pension goal should be related to e.g. the wage earned in year 20. The wage growth curve thus has a considerable influence and should be taken into account when determining the optimal pension scheme. For participants with a steep wage growth it seems unattractive to pay a high premium during the early years in which a small salary is earned to ensure that the consumption level during retirement can be around 6 times higher. The assumption that a participant always seeks to minimally maintain his consumption level therefore might be too strict for participants expecting their wage to grow considerably every year.