Erasmus University Rotterdam
Erasmus School of Economics
Bachelor Thesis Business Analytics & Quantitive Marketing

Maintenance optimization for a single wind turbine component considering actual wind variability

Joep Verdegaal (545906)

Supervisor:	Rommert Dekker
Second assessor:	Name of your second assessor
Date final version:	2nd July 2023

Abstract

In this paper, a model for optimizing age replacement policy considering actual wind variability (w-ARP) is introduced. The The policy is aimed at replacing a single-component off-shore wind turbine and extends a period-dependent age-replacement policy (p-ARP). A markov decision process model is formulated to model the component age and wind speeds using wind speed time series data from KNMI. By estimating power a output function for a particular wind turbine we approximate downtime costs for w-ARP and adjust cost functions for the p-ARP, which are compared subsequently. The w-ARP, that allows for higher maintenance age thresholds in the summer periods, results in lower costs and higher savings compared to p-ARP. A simplified implementation of this model demonstrates the potential advantage of including actual wind variability in scheduling maintenance for wind turbines.

Keywords: maintenance, age replacement policy, wind variability, offshore wind turbine.

1 Introduction

By the year 2030, the Dutch government aims to generate 30% of the total energy production from renewable energy sources and by 2050, the energy supply must be almost entirely sustainable (Ministerie van Algemene Zaken, 2023). Consequently, wind farms must supply 16% of all energy in the Netherlands and 75% of current electricity consumption. Although off-shore wind energy is the cheapest large-scale renewable energy source, the rapid expansion of both the number and size of off-shore wind farms presents operational challenges as well as maintenance costs. Nowadays, the operation and maintenance costs make up around $25 - 30\%$ of the total life cycle costs for an off-shore wind farm (Zheng et al., 2020). In order to enhance market competitiveness and attain improved economic returns, it is important to enhance the operation and maintenance management level of wind farms

To keep the wind turbines operating, two types of maintenance can be performed. Preventive maintenance (PM) is carried out to prevent the failure of components, while corrective maintenance (CM) is done to repair a failed component and is typically more expensive. As the failing of a component and maintenance require the turbine to shutdown resulting in production loss, corrective maintenance needs to be performed as soon as possible. Popular PM policies are age-based (ARP), block-based maintenance policies (BRP), and modified block-based maintenance policy (MBRP). Generally, the challenge in maintenance optimisation problems is to find a cost-efficient balance between corrective and preventive maintenance.

Weather conditions play a big role in off-shore wind turbine maintenance and operation scheduling (Seyr & Muskulus, 2019), with wind speeds being the most important environmental factor influencing the scheduling (Zheng et al., 2020) in three main ways. First, Many studies have illustrated that the correlation between wind speed and hazard rate is positive. Second, high wind speeds may restrict maintenance operations because high winds can make maintenance tasks complicated and hazardous (Zhang et al., 2019). Third, the downtime cost is associated with the the wind speed during downtime, meaning higher maintenance costs for high wind speeds. Wind speeds are are characterized by yearly seasonality, with higher wind speeds occurring more frequently in the winter. Therefore, one would expect summer periods are the best moments do do maintenance in. By considering the varying wind speeds in maintenance planning, operators can schedule maintenance activities strategically by avoiding periods of strong winds, operators can minimize the number of service visits, reduce the time spent on each visit resulting in cost savings.

The objective is to determine a preventive replacement policy which minimizes the long-run average costs. To do this, this study adjust the period-dependent age replacement policy (p-ARP) model presented by Schouten et al. (2022), by including expected wind speed as an extra (state) variable. Via discratisation of time, a Markov decision process (MDP) is used to describe the statistical behaviour of the components age. Historical wind speed data is used to discretize wind speeds into wind states and to estimate first order wind speed transition probabilities. This is added to the MDP to also model the behaviour of wind speeds for every period in the maintenance cycle. We use power output data for a certain wind turbine and piece-wise (non-)linear regression method to estimate the power output curve that is then used for downtime costs estimations. (This way, the maintenance costs depend on the wind speeds and not on the time of the year.) Given this model, we present the wind-dependent age replacement policy (w-ARP) and the long-run average maintenance costs can then be optimized using a linear programming (LP) formulation. The resulting policy defines the period, critical age threshold and wind speed for which a PM is to be performed.

To implement the model, this paper considers the maintenance scheduling for a single component Vestas V164-9.5 MW wind turbine in the North sea at IJmuiden, which is comparable to the case in Schouten et al. (2022). This implementation example is adjusted to compare the w-ARP and p-ARP results. This way, we can examine whether the model actually benefits from including actual wind variability. Due to a restriction of heap space memory in the Java Virtual Machine (JVM), the MDP model presented in this paper is restricted in its state space size and therefore some parameter simplifications are introduced. The simplified model serves the purpose of obtaining results and demonstrating the implementation of the proposed model. In addition, some adjustments are made to the costs functions presented by Schouten et al. (2022) to enable cost comparability between p-ARP and w-WARP. For w-ARP, the results show higher critical maintenance age thresholds in the summer periods than in the winter periods, most likely because low wind speeds are more prevalent in summer and postponing maintenance to the next period is more justified. The w-ARP results in lower average annual costs compared to p-ARP with average costs estimates, but higher than p-ARP with optimistic costs estimates. Compared to the constant cases with constant costs, the w-ARP model show annual savings around 11% while the savings for the p-ARP model are small.

The outline of the thesis is as follows. The following section, 2, presents an overview of relevant literature. Next, Section 3 describes the Markov decision process (MDP) model and its parameters. Section 4 describes, mathematical programming formulations of the the maintenance policies. Thereafter, Section 5 explains the historical wind data set and the power output data for the wind turbine. The results are provided and described in Section 6. Next, Section 7 discusses the model implementation and idea's for future research. Lastly, 8 provides an overview of most important findings of this study.

2 Literature review

2.1 Wind speed variability

There exist a variety of studies on off-shore wind turbine maintenance optimization. Seyr & Muskulus (2019) provides an overview of factors that influence planning and cost of maintenance, and maintenance models presented in current literature. The most influential factors in off-shore wind turbine maintenance scheduling are occurrence of failures, availability of maintenance crew, spare parts and vessels, weather and external factors, the chosen maintenance strategy, and economical parameters such as the electricity price and subsidies (Seyr & Muskulus, 2019).

Studies that include wind variability in optimizing maintenance scheduling are Byon et al. (2010) andByon & Ding (2010). Wind variability in maintenance planning is captured by considering the stochastic nature of weather conditions and solve partially observed Markov decision process models. In Schouten et al. (2022), some of these strategies are extended to include time-varying costs, on which our cost functions for the maintenance policies are based. Zhang et al. (2019) examines an opportunistic maintenance strategy for wind turbines considering stochastic weather conditions (and spare parts management) is proposed using a Markov chain model.

Modeling weather conditions with a Markov decision process has advantages. Markov states can be readily updated whenever new wind speed observations are available (Li et al., 2021). A Markov decision model is capable of representing correct wind speed distribution and capturing their persistence (Scheu et al., 2012). In addition, Markov decision model does not require long records of wind speed measurements or assume the independence of successive wind speeds (Sahin & Sen, 2001; Zhang et al., 2019). Moreover, the studies done by Hagen et al. (2013),Sahin & Sen (2001), Shamshad et al. (2005), Nfaoui et al. (2004) and Karatepe & Corscadden (2013) show the ability of a Markov chain model to retain the statistical properties of wind speeds by comparing generated synthetic wind speeds to observed data. As a consequence, a Markov decision process to model the wind speeds per period is the method of choice. which forms the basis for the presented model.

2.2 Maintenance policies

In current literature, the most popular maintenance policies for wind turbines are age- and block replacement policies. The paper from Boland & Proschan (1982) introduces three maintenance policies: the age replacement (ARP), block re- placement (BRP), and modified block replacement policies (MBRP). In the standard ARP policy, a component is replaced preventively once it has reached a certain age. Under a BRP components are replaced at a fixed time interval, regardless of the component age. MBRP is a combination of the two, where components have scheduled maintenance times based on intervals, but maintenance is only performed at these times if the component has reached a certain age. Under mild conditions for a single component system with constant PM and CM costs, the ARP is most optimal with respect to the average costs.

Other maintenance methods that are examined in current literature are opportunity- and condition-based policies. The few studies that include time-varying costs consider opportunity maintenance, where operations are initiated by a stochastic event, consequently ensuring a degree of unpredictability. Condition-based maintenance also lack this plannability of operations. In addition, many studies on the maintenance optimization of wind turbine components assume the availability of sensor data. However, a problem is that not all deterioration can be accurately measured (Schouten et al., 2022). Therefore, models that use a deterioration process of components with known statistical behavior and where failures occur unexpectedly, such as ARP and BRP, are the main focus of Schouten et al. (2022).

In the maintenance literature, it is mostly assumed that the cost rates are known and constant over time. Since maintenance costs depend on varying wind speeds, this assumption would result in non-realistic maintenance policies. The paper from Schouten et al. (2022) that presents period-dependent maintenance policy optimization models (p-ARP, p-BRP and p-MBRP) addresses wind variability by allowing costs to change over periods of a cycle e.g. weeks of a year. The yearly seasonality characteristic of wind speeds is by depending the cost estimates on the period of the year. This study shows that the p-ARP to be the optimal policy with respect to average costs, while the p-BRP has the advantage that maintenance can be planned in advance, and p-MBRP is combination of low costs and plannability. To the best of their knowledge, first time addressing time-dependent cost rates in single- component maintenance optimization. The research done in their paper is the main inspiration for our research. However, as the model is time dependent, it does not account for the actual wind variability. To address this, this paper extends their model by introducing an extra state variable expected wind speed. In the papers of Boland & Proschan (1982) and Schouten et al. (2022), the ARP is proven to be the most optimal with respect to averaged costs in constant cost and time-varying cost case. Therefore, this study extends the p-ARP model to account for actual wind variability. For further research, it would be interesting to investigate incorporating wind variability in block based BPR and MBRP.

Overall, modeling a state variable indicating the actual wind speed at a wind turbine site and incorporating it into the MDP framework can provide a more accurate representation of the system's performance and enable more effective maintenance scheduling to minimize maintenance average costs. To the best of knowledge, it is first study addressing wind speed depended cost rates in single-component maintenance optimization for an off-shore wind turbine.

3 Methodology

This section describes the Markov decision Process (MDP) model and its parameters that are used to model the age of the component and the expected weekly average wind speed. This extends the MDP presented in Schouten et al. (2022) that is used for the p-ARP and the formulation of this method is provided in Section A.1. Due to a heap space memory (error) restriction in the Java Virtual Machine (JVM) of the software of choice, Eclipse 2019-12 (4.14.0), there is relative low limit to the number of states in the MDP that could be examined. In this paper, this restriction is referred to as the 'memory restriction'. As this study investigates periods of weeks ($N = 52$), this restriction allows for $W = 3$ wind states at most. The number wind states W is preferred to be high as possible because wind speed discretisation into a small number of states leads to inaccurate power output - and donwtime cost estimations. In addition, this restriction does not allow for examining multiple years in a maintenance cycle ($m > 1$). Hence, this section will describe the parameters for a restricted and an unrestricted model.

Table 1: $NomenclatureExtended$.

Parameter	Meaning
N	Number of periods within a year
M	Large number representing maximum age of component
m	Number of years in a PM cycle
W	number of wind speed states
v	wind speed (m/s), measured at 10m height (relative to sea level)
v_{rh}	wind speed (m/s) at height of the rotor (relative to sea level)
v	Maximum wind speed above which maintenance prohibited
$\mathbb{N}^+$	Set of positive integers, $\{1, 2, ..\}$
$\bar{\mathbb{N}}$	Set of extended natural numbers, $\{0, 1, 2, ..., \infty\}$
I_1	Set of periods within a year
I_2	Set of component ages
I_3	Set of wind speed states
I	State space of the Markov decision process, $I_1 \times I_2$
I^b	Set of states representing a failed component, $I_1 \times \{0\}$
I^{bw}	Set of states representing a failed component and high wind speeds, $I_1 \times \{0\} \times \{W\}$
X	Lifetime of component
α	Scale parameter of Weibull distribution
β	Shape parameter of Weibull distribution
$A(i_1, i_2)$	The set of possible actions in MDP in state $(i_1, i_2) \in I$
$\pi_{(i_1,i_2)(j_1,j_2)}(a)$	Transition probability from state $(i_1, i_2) \in I$ to state $(j_1, j_2) \in I$ under action $a \in A(i_1, i_2)$
p_x	Probability that component fails in period directly before reaching age x
$c_{(i_1,i_2)}(a)$	Cost of taking action $a \in A(i_1, i_2)$ in state $(i_1, i_2) \in I$
$c_f(i_1)$	CM cost for expected winds speeds $i_3 \in I_3$
$c_p(i_1)$	PM cost for expected winds speeds $i_3 \in I_3$
$c_d(i_3)$	downtime cost for expected winds speeds $i_3 \in I_3$

3.1 Maintenance model description

This paper considers a single-component in a continuously operating off-shore wind farm. To prevent failure of the component, preventive maintenance (PM) replaces the component retentively, while corrective maintenance (CM) in case of a component failure. The system is maintained by replacing the component by an as-good-as-new component. For every period, we plan maintenance based on the state of the component and the expected weekly average wind speeds, where we assume wind speeds are perfectly forecasted. When a component fails, it is assumed that the wind turbine stops operating and income is missed (downtime costs). Therefore, Corrective maintenance (CM) has to be performed directly. In case of failure and wind speeds for a period exceed a high wind speed threshold v_w , maintenance is not allowed and therefore postponed. The expected downtime costs depend on the expected average wind speed, corrected for the height of the rotor. These lost production costs can be computed using the power output curve and the duration of the maintenance which is assumed to be known for PM and CM. All other costs related to maintenance operations are assumed to be constant over time. The transition probabilities are determined by the failure probability of the component, which depends on its age, and the wind speed transitions probability between consecutive periods. For simplicity, this papers assumes that the wind speeds do not affect the failure probability of the component. The goal is to determine a preventive replacement policy which minimizes the long-run average costs. The policy should specify the age, the period (of the cycle) and the winds speed in which a PM is to be performed. This model considers a yearly maintenance cycle but the model could also be used for any other cycle lengths and could also be applied to other wind turbines in other systems.

3.2 Markov decision process (MDP)

The life of the component is modeled by a discrete-time Markov decision process (MDP) with a partially ordered state space $I = I_1 \times I_2 \times I_3$, consisting of periods of a year, the component age and the expected weekly average wind speeds. Time is discretized into N periods presented by the set $I_1 \in \{1, 2, ..., mN\} \subseteq \mathbb{N}^+$, where N is the number of periods in a year ($N = 52$ for weeks) and m is the number of years in a maintenance cycle. The component age is presented by the set $I_2 \in \{0, 1, 2, ...\} \subseteq \bar{\mathbb{N}}$ and the lifetime of the component is denoted by X and $\mathbb{P}(X = k) > 0$ for all $k \in \bar{\mathbb{N}} \setminus \{\infty\}$, where $\bar{\mathbb{N}} = \{0, 1, 2, \ldots, \infty\}$ is the set of extended natural numbers. The expected weekly average wind speeds for week i_1, $I_3 = \{1, 2, ..., W\}$, is the set of discretized expected weekly average wind speeds, where W is the total number of wind speed states and also represents the state with the highest wind speeds.

3.3 Wind speed discretization

The wind speed at the height of the rotor is larger than the wind speed at the measured height of 10 meters KNMI (n.d.). Consequently, the expected power output should be estimated using wind speeds at the height of the rotor. To correct the the measured wind speed v to the wind speed at the height of the rotor v_{rh}, we use $v_{rh} = C_{rh}v$, where C_{rh} is the height-correction-factor. A more detailed explanation of the wind correction method is given Schouten et al. (2022). To estimate wind speed transition probabilities and downtime costs, first, weekly average wind speeds are corrected for the height of the rotor (v_{rh}) and then discretized into states.

Discretization of wind speeds is applied to simplify the representation of the continuous range of wind speeds into a finite number of discrete states, represented by $I_3(i_1) = \{1, 2, ..., W\}$. For every period i_1, we allow the range of each wind speed state to be different, as 'high' wind speeds is relative to the period of the year. Using historical wind speed data, the wind speed distribution with pdf $f_{i_1}(v)$ for every period i_1 of the year can be derived. Next, the discretization criteria (i.e. the ranges of each wind speed state) can be determined using the percentiles of $f_{i_1}(v)$. Through the utilization of percentiles, you ensure that the wind speed ranges are determined based on the distribution of the wind speed data, which ensures that the model accounts for the annual seasonal pattern observed in wind speeds. As a result, the first state of period i_1 contains the winds speeds 0 m/s up to the wind speed value of the first $100/(W)^{th}$ percentile (i.e. lowest wind speeds), the second state contains wind speeds between the value of the $\frac{100}{W}th$ and the $2 * 100/(W)^{th}$ percentile, and so on. The Wth state contains the high wind speeds for which maintenance is not allowed (v_W). The wind speed state in period i_1 is

$$i_3 = \begin{cases} 1 & \text{for } p_0^{i_1} \leq v_{rh} < p_{100\backslash(W)}^{i_1} \\ 2 & \text{for } P_{100\backslash(W)}^{i_1} \leq v_{rh} < p_{200\backslash(W)}^{i_1} \\ ... & \\ W-1 & \text{for } p_{100(W-2)\backslash(W)}^{i_1} \leq v_{rh} < v_W \\ W & \text{for } v_W \leq v_{rh} < p_{100}^{i_1} \end{cases} \tag{1}$$

, where $p_{100(W-1)\backslash(W)}^{i_1}$ is the $(W-1)^{th}$ percentile of wind speed pdf $f_{i_1}(v)$.

To keep the state space small, the wind speeds are discretized into three states. In addition, the wind speed state ranges for all periods are set equal, which is done for simple use and because of time restrictions. Maintenance actions are only carried out when the wind speed is less than 10 m/s (Zhang et al., 2019). Subsequently, the other two wind states ranges are set to have equal ranges. This results in the following wind speed discretization

$$i_3 = \begin{cases} 1 & \text{for } 0 \leq v_{hr} < 5 \\ 2 & \text{for } 5 \leq v_{hr} < 10 \\ 3 & \text{for } 10 \leq v_{hr} \end{cases} \tag{2}$$

3.4 Action space

In each state $i = (i_1, i_2, i_3)$, except for the states with a failed component and high wind speeds, there are two actions possible: either replace the component ($a = 1$) by performing a PM or to do nothing ($a = 0$). For the states with a failed component, denoted by age $i_2 = 0$, only CM can be performed as is must be performed directly. However, for the states with high wind speeds $i_3 = W$ no maintenance can be performed. The state-dependent action space can be expressed as follows

$$A(i_1, i_2, i_3) = \begin{cases} \{1\} & \text{if } i_2 \in \{0, M\}, i_3 \neq W \\ \{0, 1\} & \text{if } i_2 \notin \{0, M\}, i_3 \neq W \\ \{0\} & \text{if } i_3 = W, \forall i_2 \end{cases} \tag{3}$$

, where $i_1 \in I_1$, $i_2 \in I_2$, M represents the maximum component age at which a PM is performed, and W represents state containing the wind speeds for which maintenance is not carried out. In case the maximum age $M = \infty$, PM is never performed. In case of failure and wind speeds exceed threshold v_w, no maintenance is performed. For all other wind speeds, the action depends on the state of the component.

3.5 Transition probabilities

The transitions of the Markov chain are dependent on whether we plan a PM, a failure occurs and high wind speeds are expected to be present. For simplicity, we assume that the wind speeds do not affect the failure probability of the component. As a result, the transition probabilities for the components age and wind speed state are independent.

The probability of component failure depends on the age of the component, but is assumed to be independent of the time of the year. The system jumps to a state with age 0 and a higher time period in case of failure, whereas there is a transition to a state with higher age and time period if no failure occurs. In case of PM, there is an instantaneous jump to age 0, after which the component can reach age 1 at the end of the period, or have a failure and end with age 0. The transition probability $\pi_{(i_1,i_2)(j_1,j_2)}(a)$ from state (i_1, i_2) to state (j_1, j_2) under action $a \in A(i_1, i_2)$ is given in Section A.1.2. The failure probabilities from Schouten et al. (2022) under action $a = 0$ are corrected, since they depend on p_{i_2+1} rather than on p_{i2}, as the probability of failure between i_2 and $i_2 + 1$ is required.

If we assume in our model that PM to last one period length and PM is performed in period i_1, the age of component reaches 0 at beginning of the next period $i_1 + 1$. The age can then reach 1 at the end of the period $i_1 + 1$, or have failure at end with age 0. Therefore the transition probabilities would not be applicable in our model. Nevertheless, for reproducibility, conditions for the transition probabilities from Schouten et al. (2022) are not adjusted.

The transition probabilities for wind speeds are assumed to be period dependent because of the yearly seasonality nature of wind speeds. Under the Markov chain property, it is assumed that the current state of the wind contains all the relevant information about the wind situation and its possible future development Hagen et al. (2013). The conditional transition probabilities from state (i_1, i_3) to (j_1, j_3) are independent on time and can be written in terms of matrix elements denoted in (4a). Using daily historical time series data on wind speeds, we can estimate transition probabilities for every period i_1 with wind state $i_3 \in I_3$ to $j_1 = i_1 + 1$ with wind speeds $j_3 \in I_3$. Empirical frequencies are used to estimate the wind speed transition probabilities, which gives the maximum likelihood estimators $\hat{p}_{i_3,j_3}(i_1)$ given in (4b) (Hagen et al., 2013; Shamshad et al., 2005).

$$p_{i_3,j_3}(i_1) = \mathbb{P}(w_{i_1+1} = i3 \| w_{i_1} = j_3) \tag{4a}$$

$$\hat{p}_{i_3,j_3}(i_1) = \frac{m_{i_3,j_3}(i_1)}{\sum_{j_3=0}^{W} m_{i_3,j_3}(i_1)} \tag{4b}$$

, where w_{i_1} is the wind speed in period i_1, $m_{i_3,j_3}(i_1)$ is the amount of transitions from state i_3 in period i_1 to state j_3 in week $j_1 = i_1 + 1$, and W the number of wind states. This results in a total of N transition probability matrices, one matrix for every period i_1 to period $j_1 = i_1 + 1$. The model assumes piecewise stationarity by only considering data from one period and its consecutive period when estimating the transition probabilities (Hagen et al., 2013). To ensure

It can be checked that $\sum_j^W \hat{p}_{i_3,j_3}(i_1) = 1$, for $i_1 \in I_1, i_3, j_3 \in I_3$, ensuring discrete Markov chain property holds (Shamshad et al., 2005; Hagen et al., 2013).

Although, studies have shown that the correlation between wind speed and hazard rate of the wind turbine is positive (Zheng et al., 2020), for simplicity, it is assumed that the failure probability of the component is unaffected by winds speeds. Hence, the transition probabilities $\pi_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(a)$ from state $i = (i_1, i_2, i_3)$ to state $j = (i_1, i_2, i_3)$ under action $a \in A(i_1, i_2, i_3)$ are defined as follows:

$$\pi_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(a) = \pi_{(i_1,i_2)(j_1,j_2,)}(a) \times \hat{p}_{i_3,j_3}(i_1)$$

, for $i_1, j_1 \in I_1, i_2, j_2 \in I_2, i_3, j_3 \in I_3, a \in A(i_1, i_2, i_3)$. The transition probabilities are given in Equations 5,6 and 7, where p_{i_3,j_3} is estimated by $\hat{p}_{i_3,j_3}$.

$$\pi_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(1) = \begin{cases} (1-p_1) \times p_{i_3,1}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = 1, i_3 \neq W, j_3 = 1 \\ ... & \\ (1-p_1) \times_{i_3,W}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = 1, i_3 \neq W, j_3 = W \\ p_1 \times p_{i_3,1}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = 0, i_3 \neq W, j_3 = 1 \\ ... & \\ p_1 \times p_{i_3,W}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = 0, i_3 \neq W, j_3 = W \\ 0 & \text{else,} \end{cases} \tag{5}$$

$$\pi_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(0) = \begin{cases} (1-p_{i_2+1}) \times p_{i_3,1}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = i_2 + 1, i_2 \notin \{0, M\}, j_3 = 1 \\ ... & \\ (1-p_{i_2+1}) \times p_{i_3,W}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = i_2 + 1, i_2 \notin \{0, M\}, j_3 = W \\ p_{i_2+1} \times p_{i_3,1}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = 0, i_2 \notin \{0, M\}, j_3 = 1 \\ ... & \\ p_{i_2+1} \times p_{i_3,W}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = 0, i_2 \notin \{0, M\}, j_3 = W \\ 0 & \text{else,} \end{cases} \tag{6}$$

In case $i_2 \in \{0, M\}$ and $i_3 = W$, no maintenance is done $(a = 0)$ because we assume safety of maintenance crew is of greater importance than incurring downtime costs. For states $i = (i_1, i_2, i_3)$ with $i_2 = 0, i_3 = 0$ and $j = (j_1, j_2, j_3)$ with $j_2 = 0$, we have $\pi_{(i_1,0)(j_1,0)}(0) = 1$, because not maintaining a failed component in period i_1 will lead to a failed component in period $i_1 + 1$, and $\pi_{(i_1,0)(j_1,j_2)}(0) = 0$ for $j_2 \in I_2 \setminus \{0\}$. This is the same for states with $i_2 = M, i_3 = 0$, where $\pi_{(i_1,M)(j_1,M+1)}(0) = 1$. The transition from state $i = (i_1, i_2, i_3)$ to state $j = (j_1, j_2, j_3)$ then only depends on the wind transition probabilities $p_{i_3,j_3}(i_1)$, $i_3, j_3 \in I_3$, where p_{i_3,j_3} is estimated by $\hat{p}_{i_3,j_3}$. These probabilities are:

$$\pi_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(0) = \begin{cases} p_{W,1}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, i_2 = 0, j_2 = 0, j_3 = 1 \\ ... & \\ p_{W,W}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, i_2 = 0, j_2 = 0, j_3 = W \\ p_{W,1}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, i_2 = M, j_2 = M+1, j_3 = 1 \\ ... & \\ p_{W,W}(i_1) & \text{for } j_1 = i_1 + 1 \pmod N, i_2 = M, j_2 = M+1, j_3 = 1 j_3 = W \\ 0 & \text{else,} \end{cases} \tag{7}$$

It can checked that $\sum_j^{|I|} \pi_{ij}(a) = 1$, for $i = (i_1, i_2, i_3), j = (j_1, j_2, j_3) \in I$ and $a \in A(i)$, where $|I|$ is the cardinality of state space set i.e. the total number of states. Lastly, it can be verified that the Markov decision chain defined by 5,6 and 7 is unichain.

3.6 Cost parameters

Maintenance costs typically consist of the following components: man- power, material, and lost production costs due to downtime. The costs parameters used for p-ARP as presented by Schouten et al. (2022) are proveded in Section A.1.3. In our model, the costs depend on the wind winds speed i_3 and on the period i_1, since the discretization of wind speeds is period dependent. The costs for PM and CM are

$$c_p(i_1, i_3) = c_p^c + c_d(i_1, i_3) \tag{8a}$$

$$c_f(i_1, i_3) = c_f^c + c_d(i_1, i_3) \tag{8b}$$

, where c_p^c and c_f^c are all the other costs related to PM and CM (i.e. manpower and material costs), respectively, and $c_d(i_3)$ are the lost production costs due to downtime. The costs of taking action a in state $i = (i_1, i_2, i_3)$ is

$$c_{(i_1,i_2,i_3)}(a) = \begin{cases} 0 & \text{if } a = 0, i_2 \neq 0 \\ c_d(i_1, i_3) & \text{if } a = 0, i_2 = 0 \\ c_p(i_1, i_3) & \text{if } a = 1, i_2 \neq 0 \\ c_f(i_1, i_3) & \text{if } a = 1, i_2 = 0 \end{cases} \tag{9}$$

, where action a can only be 1 if $i_3 \neq W$. When a a component fails and the wind speeds are too high to maintain immediately (i.e. $i_2 = 0$ and $i_3 = W$), the downtime costs are the only costs. $c_d(i_3)$ is related to expected the power output for (expected) wind speeds i_3 in period i_1. This can be computed using power output function of wind turbine $P(v)$ and the downtime, which we assume is known. The power output function for a certain wind turbine can be estimated using piece-wise (non-)linear regressions on power output and wind speed data (for this particular turbine). By fitting functions for different wind speed intervals, we can estimate $P(v)$ with $\hat{P}(v)$. The relation between the power output of a wind turbine and the wind speed is non-linear and Figure 1 shows such a relation. Therefore, we estimate the average power output for every state i_3 in period i_1 using the wind speed distribution of this period. Since wind speeds are discretized, for each wind state i_3 we compute the expected power output by taking the average of power outputs for all wind speeds in the wind speed range of i_3. This can be computed by taking the integral of $\hat{P}(v)$ over the interval and dividing by the length of the interval. That is, Expected/average power output for wind speed state i_3 in period i_1 with wind speed pdf $f_{i_1}(v)$

$$\bar{P}(i_1, i_3) = \frac{1}{b_{i_1,i_3} - a_{i_1,i_3}} \int_{a_{i_1,i_3}}^{b_{i_1,i_3}} \hat{P}(v)\, dv \tag{10}$$

where a_{i_1,i_3} and b_{i_1,i_3} are the wind speed ranges for state i_3 in i_1 (see Equation 1). The higher the number of wind states we choose, the more accurate power output estimates we obtain for every (expected weekly average) wind speed v. Subsequently, the daily downtime costs for period i_1, $c_d(i_1, i_3)$ can be estimated by the following equation, where we assume a constant average hourly electricity price estimate p_e (euro/kWh):

$$c_d(i_1, i_3) = 24p_e \times \bar{P}(i_1, i_3) \tag{11}$$

In this paper, comparisons are made with the case where PM and CM costs are constant throughout the year. For this case, we assume the expected power output for period i_1 is the historical average power output, resulting in constant yearly (downtime) costs. We estimate the historical (weekly if $N = 52$) average power output using the same wind speed data used for estimating the transition probabilities. With the estimated power output function $\hat{P}(v)$, the power output can be averaged over all periods and all years. For the p-ARP, the constant PM and CM cost rates are simply the the yearly average PM and CM costs (see Section A.1.3). We refer to this case as the constant cost case.

The wind speeds for a simplified model are discretized into $W = 3$ states, which are not time dependent (see Section 6.2). Therefore, the simplified (downtime) cost parameters will only depend on wind speed. Each wind speed state power output can be approximated using $\hat{P}(i_3)$. In this paper, the wind speed upper bound for the state $i_3 = 3$ range is set as the maximum recorded wind speed for the historical wind speed data set, also used for the constant cost case average power estimation.

4 Maintenance policies

In the paper by Schouten et al. (2022), three period-dependend replacement policies are examined. Namely, the age-based, block-based and modified block-based maintenance policies (ARP, BRP, and MBRP, respectively). This paper limits its scope to the ARP policy, which is proven to be the most optimal in with respect to costs under mild conditions (2). The ARP is adapted by Schouten et al. (2022) to the time-varying cost case by defining a separate critical maintenance age for each time period, leading to the p-ARP. Subsequently, the p-ARP is adjusted to account for actual wind speed variability, resulting in the w-ARP.

4.1 Period-dependent age replacement policies (p-ARP)

A p-ARP policy in which in each period $i_1 \in I_1$ of the year, there is period-dependent critial maintenance age $t(i_1) \in \bar{\mathbb{N}} \setminus \{0\}$ at or above which PM is performed (Schouten et al. (2022)). The following LP leads to the optimal policy that minimize the long-run average cost for p-ARP

$$\text{(p-ARP)}\ \min \sum_{i=(i_1,i_2)\in I\setminus I^b} c_p(i_1)x_{i,1} + \sum_{i=(i_1,i_2,)\in I^b} c_f(i_1)x_{i,1} \tag{12a}$$

$$\text{s.t.} \sum_{a\in A(i)} - \sum_{j\in I}\sum_{a\in A(j)} \pi_{ji}(a)x_{j,a} = 0, \forall i = (i_1,i_2) \in I \tag{12b}$$

$$\sum_{i_2\in I_2}\sum_{a\in A(i_1,i_2)} x_{i_1,i_2}(a) = \frac{1}{mN}, \forall i_1 \in I_1 \tag{12c}$$

$$x_{i0} = 0, \forall i = (i_1,i_2) \in I : i_2 \in \{0, M\} \tag{12d}$$

$$x_{i,a} \geq 0, \forall i = (i_1,i_2) \in I, a \in A(i) \tag{12e}$$

, where $I^b = I_1 \times \{0\}$ is the set of states representing a failed component. $x_{i,a}$ can be interpreted as the long-run probability of being in state $i = (i_1, i_2, i_3) \in I$ at the beginning of the period and action $a \in A(i_1, i_2, i_3)$ is chosen. Constraint (12d) as presented by Schouten (2019) is added to the model to ensure that maintenance is done if the age of the component is either 0 or M.

4.2 Wind-dependent age replacement policy (w-ARP)

The w-ARP opperates the same as p-ARP, except that the policy also depends on the expected weekly average wind speed, i.e. the expected maintenance costs for period i_1 depend on the expected wind speeds i_3 for period i_1 and maintenance is not done if winds speeds are too high ($i_3 = W$). The following LP leads to the optimal policy that minimizes the long-run average cost for w-ARP:

$$\text{(w-ARP)}\ \min \sum_{i=(i_1,i_2,i_3)\in I\setminus I^b} c_p(i_1,i_3)x_{i,1} + \sum_{i=(i_1,i_2,i_3)\in I^b} c_f(i_1,i_3)x_{i,1} + \sum_{i=(i_1,i_2,i_3)\in I^{bw}} c_d(i_1,i_3)x_{i,0} \tag{13a}$$

$$\text{s.t.} \sum_{a\in A(i)} - \sum_{j\in I}\sum_{a\in A(j)} \pi_{ji}(a)x_{j,a} = 0, \forall i = (i_1,i_2,i_3) \in I \tag{13b}$$

$$\sum_{i_2\in I_2}\sum_{i_3\in I_3}\sum_{a\in A(i_1,i_2,i_3)} x_{i_1,i_2,i_3}(a) = \frac{1}{mN}, \forall i_1 \in I_1 \tag{13c}$$

$$x_{i0} = 0, \forall i = (i_1,i_2,i_3) \in I : i_2 \in \{0, M\}, i_3 \neq W \tag{13d}$$

$$x_{i1} = 0, \forall i = (i_1,i_2,i_3) \in I : i_3 = W \tag{13e}$$

$$x_{i,a} \geq 0, \forall i = (i_1,i_2,i_3) \in I, a \in A(i) \tag{13f}$$

The objective function (13a) represents the long-run average cost of the policy. (13b) ensures the inflows of the states are equal to the outflows. The long-run probabilities of being in each state must be $\frac{1}{mN}$, since N periods over a cycle of m years are considered, which is defined by restriction (13c). Constraint (13d) ensures that we do maintenance when age is 0 or critical age M and the wind speeds are not too high ($i_3 = W$). The additional constraint (13e) ensures that we do no maintenance when wind speeds are too high.

5 Data

5.1 Wind speed data

For modeling wind speeds using MDP, this paper uses historical weather data from KNMI (2023) containing daily average wind speeds in the coastal town IJmuiden in the Netherlands . This time series data is updated daily and we select the years 1971 to 2023 as the aim is to incorporate data for complete annual periods. The total mean wind speed is 6.63 m/s and the highest recorded 22.6 m/s. The wind speed are measured at 10m, meaning the wind speeds need to be corrected to wind turbines with rotors higher than 10m for cost estimations. In total there are 129 missing observations, which are replaced by the total sample average of 6.63 m/s. A more sophisticated method would be replacing the missing observation by the average wind speed of the month in which the missing observation is measured. This gives a more accurate sample average. However, due to time restrictions and the demand for coding more data preparation, this latter method is left for extended research. In addition, as only 129 of the total 18993 observations is unobserved, this method might not have a noticeable effect on average cost approximations.

Estimating transition probabilities based on non-stationary data leads to time varying estimates, which introduces additional complexity to the modeling and analysis of the Markov chain. To examine this, using R Studio 4.2.2., the Augmented Dickey-Fuller (ADF) test is performed to test whether the wind speed time series is stationary. The test statistic of -20.881 with p-value of 0.01 indicates that the wind speeds data is stationary. Therefore, the estimated transition probabilities are considered time-independent.

5.2 Wind turbine power output data

Data from Commissie-M.E.R. (2016) is used for estimating the power output function for a Vestas V164-9.5 MW turbine. This data contains the the power output per wind speed for a Vestas V164-8.0 MW turbine. The power output for the 9.5 MW turbine is approximated by linearly scaling the data for the 8.0 MW turbine, using the same scaling method as explained by Schouten (2019). Here, the power output data (kW) is scaled from [4, 13] to [3.5, 14] and the wind speed values (m/s) from $[0, 8]$ to $[0, 9.5]$. The data set starts for wind speed 3.5 m/s with power output 194 kW, and since we do not know the specific cut-in power output for 9.5 MW turbine, scaling from $v = 0$ m/s wont give correct scaled data. Therefore, it is assumed that the cut-in power for 9.5 MW turbine is the same as the 8MW turbine, and we scale the power value's from $[0.194, 8]$ to $[0.194, 9.5]$. Data for the 8 MW turbine and the scaled data for the 9.5 MW turbine are shown in Table 3 in Section A.2. The linear scaling functions for the power output and wind speeds are also provided in this Section. The power output function for

9.5 MW is plotted in Figure 1, which shows the non-linear relationships between power output and wind speed.

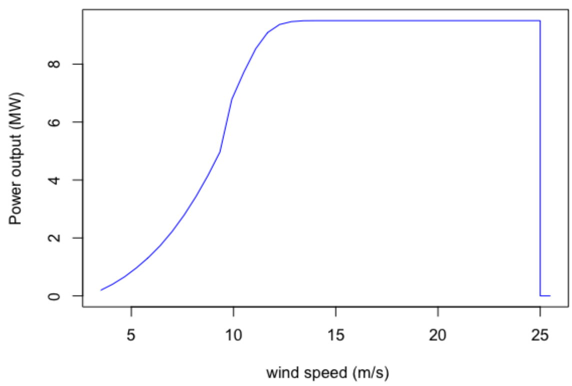


Figure 1: Power output for a Vestas V164-9.5 MW wind turbine

6 Results

To implement the model, this paper considers the maintenance scheduling for a single-component Vestas V164-9.5 MW wind turbine in the North sea at IJmuiden. Due to heap space restriction (see Section 3), the model considers a maintenance cycle of one year $m = 1$, discretized into weeks ($N = 52$). In addition, the number of wind states W and the maximum critical age M of the component are restricted to 3 and 53 respectively. Only the gearbox is maintained throughout life of the wind turbine and maintenance is performed by replacing the component by an as-good-as-new component. The scheduling is based on the age of component and the weekly (expected) average wind speed, where we assume weekly weather forecasts to be accurate. Arwade & Gioffrè (2014) show that the assumption of stationarity of the wind speed process over a time period of approximately a week is reasonably well justified. This implies that the mean and variance of wind speeds in a period of a week do not change significantly, and the expected average wind speed for a week is a good indication for the daily wind speed of that particular week. Therefore, it is assumed that if wind speed i_3 allows for maintenance in week i_1, say PM last one week, there is not a day in that week that will not allow for maintenance. This study refers to an offshore wind turbine, but the model can also be applied to other installations with predictable time-varying production

This particular implementation influences the parameters of the model in the following way. We assume that deterioration of the component is not visible, but assume that the lifetime distribution follows a Weibull distribution. For simplicity, it is assumed that wind speeds do not affect the hazard rate of the wind turbine components. For discretization, wind speeds are corrected for the height of the rotor which is assumed to be 138 meters. The downtime costs are dependent on wind speed, assuming the rest of the other costs (manpower, material) are constant over time. For comparison reasons, the downtime costs function for the p-ARP is slightly adjusted to obtain results. In the following subections (6.1-6.3), this implementation for the parameters is explained in more detail. Under these parameters, the model is optimized using CPLEX 22.1.0 in Java and the implementation results are provided in Section 6.4.

6.1 Lifetime distribution

It is assumed that the component's lifetime is distributed according to a Weibull distribution, with cdf $F(x) = 1 - exp((-\frac{x}{\alpha})^{\beta})$ and $\mathbb{P}(X = x) = F(x) - F(x-1)$ for $x \in \bar{\mathbb{N}}$, where scale parameter $\alpha > 0$, denotes the characteristic life of the component, and shape parameter $\beta > 1$ indicates an increasing component failure pattern. Hence, the probability mass is shifted to the right and the distribution is strictly positive (Schouten et al., 2022). Both x and α are measured in Weeks. We choose $\beta = 3$ and $\alpha = mN = 52$, implying an average time to failure of 46.43 weeks.

6.2 Wind speed discretization

For the Vestas V164-9.5 MW wind turbine turbine, we assume a rotor height of 138 meters (Schouten, 2019). The wind speed at this height ($v_{rh} = v_{138}$) is larger than the wind speed at the measured height of 10 m (KNMI, n.d.). The relation between these wind speeds is $v_{138} = 1.181v$, which means that the average wind speed in IJmuiden 1971-2023 at the rotor of the wind turbine is 7.83 m/s instead of 6.63 m/s, while the percentage wind speed fluctuations remain the same.

6.3 Maintenance costs

Maintenance costs consist of the following components: man-power, material, and lost production costs due to downtime for PM and CM. Wind variability affects the downtime costs and assuming the rest of the costs are constant over time. The downtime costs for p-ARP and w-ARP are computed using a different approach. Both relate the downtime costs to the expected downtime of the components, the power output curve, and historical wind speed data. However, the downtime costs for w-ARP depend on the expected power output for wind state i_3, where as the downtime costs for p-ARP depend on the expected power output for period i_1. For each maintenance action, we assume that we know the downtime of the component. First, in Subsection 6.3.1the power output curve for the Vestas V164-9.5 MW turbine is estimated and used to derive the cost functions for the w-ARP in Subsection 6.3.2. Next, the cost functions for the p-ARP are formulated in Subsection 6.3.3, by adjusting the ones presented in Schouten et al. (2022).

6.3.1 Estimating power output curve

The energy output of a wind turbine increases approximately with a third power of the wind speed up to a level of bout 12m/s, after which the energy output generated remains constant (Figure 1). The typical power output function of a wind turbine is described by the follwing function (Jin & Tian, 2010; Tian & Wang, 2020)

$$P(v) = \begin{cases} 0 & , v < v_c, v \geq v_s \\ 0.5\mu_{max}\rho A x^3 & , v_c \leq v \leq v_r \\ P_r & , v_r \leq v \leq v_s \end{cases} \quad (14)$$

,where v_c is the cut-in wind speed, v_r the rated wind speed and v_s the cut-out wind speed of the turbine in m/s. For the Vestas V164-9.5MW turbine power output data, plotted as the blue line in Figure 2 on the interval $v = [0, 15]$, it is known that $v_c = 3.5$, $v_r = 14$, $v_s = 25$, and $P_r = 9.5$ (MW).

To fit a function that describes non-linear part of the the scaled 9.5MW data, piecewise non-linear regression is used in R-studio (4.2.2). The more and smaller the intervals, thus more regressions are performed, the better we can capture the curvature of the underlying function of the data. The downside of considering a large number of intervals, is the increase of computational requirements and the possibility of overfitting. As the data is of one type of turbine on a particular location, an overfitted estimated function would not be a good representative for other systems. Therefore, the number of intervals is selected as low as possible.

Figures 1 and 2 show that the underlying function describing this data is convex on the approximated interval $[3.5, 10]$, concave on $[10, 13]$ and linear for wind speeds above $\pm$13m/s. Therefore, the power output curve $P(v)$ is estimated by a hybrid-function $\hat{P}(v)$ consisting of three different functions. As the function for wind speeds on the last interval is known, $P_r = 9.5(MW)$, only the functions for the first and second interval need to be estimated using non-linear regressions on the 9.5MW power output data. The interval bounds are selected by examining different interval (lengths) and examining the plot compared to 9.5MW power output data. For the first function on interval $[3.50, 10.50]$, we assume the cubic relationship $y = ax^3$ and obtain estimated coefficient $\hat{a} = 6.54817$ (p-value $< 2 \times 10^{-16}$). The function for the second interval $[10.50, 12.83]$ is assumed to have the quadratic relation ship $y = b - a(x - c)^2$. As the maximum power the wind turbine can generate is 9500 kW, parameter b is set equal to 9500. The estimated coefficients are $\hat{a} = 341.58696$ (p-value $= 5.11 \times 10^{-4}$) and $\hat{c} = 12.77971$ (p-value $=2.34 \times 10^{-7}$). This leads to the following estimated power output function for the 9.5MW turbine:

$$\hat{P}(v) = \begin{cases} 0 & , v < 3.50, v \geq 25.00 \\ 6.54817v^3 & , 3.5 \leq v < 10.50 \\ 9500 - 341.59(v - 12.78)^2 & , 10.50 \leq v < 12.83 \\ 9500 & , 12.83 \leq v < 25.00 \end{cases} \quad (15)$$

,where $\hat{P}(v)$ is in kW and v in m/s. This function is plotted as the red line in Figure 2, which shows in combination with the significant p-values that $\hat{P}(v)$ is a good fit the 9.5MW wind turbine power output data. For the simplified wind states, i.e. the three wind states, the (expected) power output is calculated by taking the integral of $\hat{P}(v)$ over the interval of the wind speed state range, divided by the wind speed interval lengths of the state range (equation

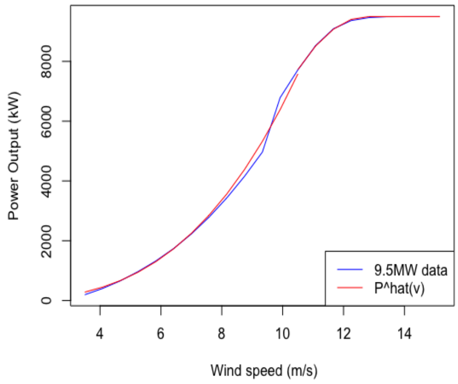


Figure 2: Power output function fits

2). To compute power output for the 3rd wind state, we take the wind speed upperbound for the 3rd state as the maximum wind speed recorded at IJmuiden 1971-2023 KNMI (2023), which is approximately 23 m/s. This leads to the following power outputs in kW:

$$\bar{P}(i_3) = \begin{cases} 155 & \text{if } i_3 = 1 \\ 3069 & \text{if } i_3 = 2 \\ 9296 & \text{if } i_3 = 3 \end{cases} \tag{16}$$

6.3.2 Costs function w-ARP

Maintenance costs for both PM and CM consist of the following components: man-power, material, and lost production costs due to downtime. Missed income due to downtime are related to the expected power output for wind state i_3, which represent the weekly average wind speed at sea level, corrected for the height of the rotor, and the downtime of the component. For the downtime costs we use a constant electricity price of 0.06 euro/kWh.(Schouten et al., 2022), and we assume for each maintenance action that we know the downtime of the component. The other costs (i.e. set-up costs and costs of the maintenance action itself) are considered constant over time. Papatzimos et al. (2018) state that some 10 days are needed for the PM of a gearbox for a 2.3 MW turbine and estimate the maintenance costs as 148.20 thousand euro. Schouten et al. (2022) therefore assumes the downtime of 40 days and maintenance cost of 592.80 for CP. To fit the time discretization in weeks, this study assumes PM and CM to take up 7 and 28 days, respectively. Adding these constant costs to the downtime costs, we arrive at the following costs values for w-ARP in thousand euro. A more detailed derivation of the following functions is shown in Appendix Section A.3.1.

$$c_p(i_3) = 103.740 + 0.01008\bar{P}(i_3) \tag{17a}$$

$$c_f(i_3) = 414.960 + 0.04032\bar{P}(i_3) \tag{17b}$$

For the constant cost case, we use the historical daily average power output for the wind turbine for the years 1971 to 2023 from KNMI (2023). For every day, the power output is estimated using $\hat{P}(v_{138})$. Next, for every week the power output is averaged over all 52 years, resulting in a historical weekly average power output of 3.853 MW, which is will be the constant expected power output for every period.

6.3.3 Adjusted costs function p-ARP

For the p-ARP, Schouten et al. (2022) estimate period dependent downtime costs by estimating power the historical power output for every week of the year. They use the historical wind speed data from Commissie-M.E.R. (2016), but for the years 1971 to 2017, and the V164-8.0 MW turbine power output data that is scaled up to the 9.5 MW turbine. Two output approximation are obtained, the first is computed by taking the average of daily average power outputs over all years within each week (i.e. historical weekly mean), and the second by taking the lowest daily wind speed per week, averaged over the 47 years (i.e. historical weekly minimum). To both power output estimations, a cosine approximated is fitted. Figure 3a show the historical weekly mean (upper data) and minimum (lower data) power output approximations for a Vestas V164-9.5 MW turbine with the corresponding cosine fits. These cosine fits are used to construct the time-dependent downtime costs for p-ARP and can be compared to the downtime costs for w-ARP.

Using the estimated power output curve $\hat{P}(v)$ obtained in Section 6.3.1 and the same estimation method presented in Schouten et al. (2022), we can estimate the average historical power output for each week of the year. Although he approximated power output function shows a good fit to the estimated power output data for a 9.5 MW turbine, the historical power output estimates using this function do not align with the estimates from Schouten et al. (2022), which is shown in Figure 3b. This is in contrast to my expectation, since $\hat{P}(v)$ shows a good fit to the scaled power output data for the 9.5MW turbine and the same historical wind speed data set is used. This significant difference in estimations would not lead to comparable downtime costs results for w-ARP and p-ARP. Given the time restriction, I was not able to locate the cause of this estimation difference and is left for extended research. In order to obtain comparison results, the cosine fits from Schouten et al. (2022) are transformed downward with 1000 kw, to approximately match the historical average power output estimated by $\hat{P}(v)$ (see Figure 3c). The adjusted downtime costs in thousand euro using mean wind speed estimates is given in equation (18a) and using lowest wind speeds is given equation (18b).

$$C_d^{mean}(t) = 5.396 + 1.295cos(\frac{2\pi t}{52} + 0.034) \tag{18a}$$

$$C_d^{low}(t) = 1.917 + 0.850cos(\frac{2\pi t}{52} + 0.020) \tag{18b}$$

These downtime costs can be added to the adjusted constant maintenance costs for PM and CM (see Section 6.3.2). The obtained PM and CM cost in thousand euro are given by equation 19 for mean wind speeds, where the same constant electricity price is used as for the w-ARP costs functions. The cost values for p-ARP can then be compared to the values for the w-ARP.

$$C_p^{mean}(t) = 141.512 + 9.065cos(\frac{2\pi t}{52} + 0.034)) \tag{19a}$$

$$C_f^{mean}(t) = 566.048 + 36.26cos(\frac{2\pi t}{52} + 0.034)) \tag{19b}$$

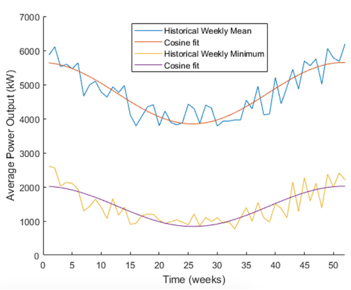
The PM and CM costs using low wind speeds power output estimates in thousand euro are shown in section A.3.1 in the Appendix, as well as the constant cost functions.

6.4 Implementation results

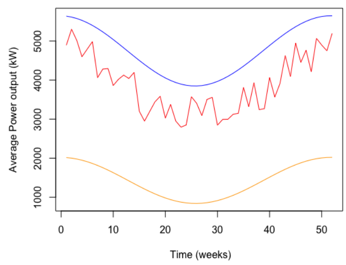
The single-component p-ARP and w-ARP models are optimized for $N = 52, M = 53, W = 3, m = 1, \alpha = 52$ and $\beta = 2$. For every period and wind speed, the critical maintenance age is the first the lowest age for which PM is performed, as it is also optimal to do PM for ages higher than this critical age threshold Schouten et al. (2022). The critical maintenance ages (in weeks) for p-ARP and the w-ARP are shown in Figure 4, where maximum age for the component $M = 53$.

Figure 4a shows that for both average costs estimates using (18a) and low costs estimates using (18b), the critical ages for p-ARP are identical for most periods. The critical ages threshold in the winter periods are set at M more frequently compared to summer. The reason could be that, due to lower wind speeds occurring more frequently in the summer periods, maintenance costs are lower compared to winter months and therefore there are higher critical age threshold is in the winter compared to summer.

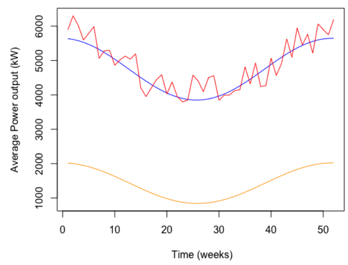
Figure 4b shows the critical maintenance ages for wind states $i_3 = 1$ and $i_3 = 2$ (for wind state $i_3 = W = 3$ we do no maintenance). Critical aintenance ages for lower wind speeds are lower than for higher wind speeds, which is intuitive because the downtime costs are lower. leading to a lower age threshold for PM. Moreover, critical maintenance ages for all wind speeds



(a) Power output approximations graph from Schouten et al. (2022).



(b) Un-adjusted cosine fits. Blue: cosine fit historical weekly mean (28a). Orange: cosine fit historical weekly minimum (28b).



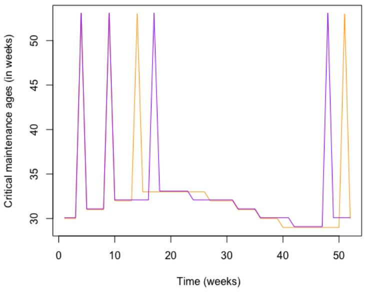
(c) Adjusted cosine fits. Blue: cosine fit historical weekly mean (18a) Orange: cosine fit historical weekly minimum (18b)

Figure 3: Approximated power output of Vestas V164-9.5 MW in IJmuiden

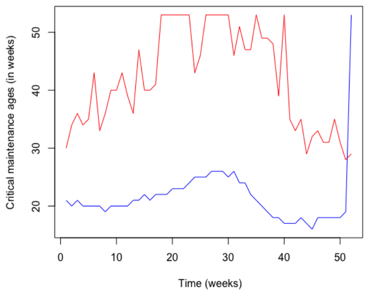
are higher in the summer than in the winter. This seems contradictory to the patterns of the p-ARP and the results from Schouten et al. (2022), where the maintenance ages tend to be lower than in the winter months. However, in w-ARP model the maintenance costs depend on the wind speeds and not on the period of the year (p-ARP), therefore the critical maintenance age for PM is determined differently. In the p-ARP model the period costs are relative to the other periods in the year, whereas in the w-ARP model the period costs are relative to the wind speeds prevailing in that particular period. The reasoning behind the resulting age thresholds for w-ARP could be the following. Intuitively, it makes sense to have a lower critical maintenance age in periods where preventive maintenance is relatively cheap. Maintenance costs are on average more expensive in winter periods because higher wind speeds are more prevalent compared to summer leading to higher power outputs (See Figure 3). Consequently, the occurrence of low wind speeds in the winter leads to relatively low maintenance costs for that time of the year compared to summer. Because of this, the critical age threshold for PM in the summer is higher than in the winter weeks for every wind speed. Compared to winter periods, the probability of the next summer period having low wind speeds is higher, therefore postponing maintenance is more justified in the summer, resulting in a higher maintenance age thresholds for PM. This behaviour of the critical maintenance age does not imply that we prefer winter periods to summer periods to do PM, rather it shows that we allow for higher component ages in the summer than in the winter.

In Figure 4c the critical maintenance ages for the constant cost cases are plotted. The orange line represents the constant critical maintenance age of 31 weeks for p-ARP constant cost case, for both the 'mean' and 'low' cost estimations using Equations (18a) and (18b). The blue and red line in Figure 4c correspond to w-ARP ages for wind state $i_3 = 1$ and $i_3 = 2$, respectively. The critical maintenance age for wind speed state $i_3 = 2$ remains around 30 weeks, while the age for state $i_3 = 1$ fluctuates considerably between $\pm$ 30 and $M = 53$ weeks. This implies that for some periods, maintenance will be performed for winds speeds $i_3 = 2$ rather than for wind speeds $i_1 = 2$, which is not logical since the costs for PM and CM are constant. I would expect every wind speed to have the same constant PM age over the year. The results might be caused by winter months having a low probability of low wind speeds compared to high wind speeds (with equal downtime costs), therefore the model relates this to a low probability of performing maintenance with wind speeds $i_3 = 1$ in the winter and sets a high critical age threshold. However, this would not explain why the maintenance ages for wind speed $i_3 = 2$ are almost constant throughout the year. Therefore, identifying the true underlying cause of this result and perfecting the model requires extended research.

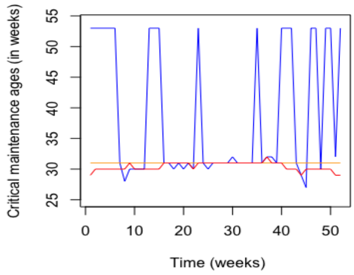
Table 2 shows the annual (long-run) maintenance costs in thousand euros. We see that costs w-ARP are lower than for the p-ARP using the mean cost function (18a) and higher than p-ARP using the low costs function (18b). For the latter, the costs approximates are only based on historical average lowest daily wind speed per week, leading to very optimistic cost estimates. These costs only incur if maintenance is planned on the day of the selected week with the lowest wind speeds, which would not be realistic estimates when is preventive maintenance is assumed



(a) p-ARP
Purple: average costs using (18a).
Orange: low costs using (18b).



(b) w-ARP
Blue: ages for wind speeds in $i_3 = 1$. Red: ages for wind speeds $i_3 = 2$.



(c) Constant costs cases. Blue: w-ARP wind state 1. Red: w-ARP wind state 2. Orange: p-ARP constant costs, using mean (18a) and low wind speeds (18b).

Figure 4: Critical maintenance ages for p-ARP and w-ARP

Table 2: Long-run annual maintenance costs (in thousand euro), $N = 52, m = 1$

	costs	constant cost case	savings
p-ARP using average costs (18a)	501.181	501.564	0.08%
p-ARP using low costs (18b)	382.887	383.287	0.10%
w-ARP	453.287	510.568	11.22%

to last a week. When comparing the maintenance costs to the constant costs cases for the case $N = 52, m = 1$, we see that the w-ARP has 11.22% savings, while for p-ARP there are barely saving. The savings for p-ARP compared to the results from Schouten et al. (2022), that examine the case for $N = 52$ and $m = 4$, are considerably lower. One reason for this could be the the adjustments of the p-ARP cost functions or the lower $m = 1$. The transformation of the downtime cost functions for the p-ARP heavily influences the p-ARP costs results, which should be carefully observed when comparing p-ARP and w-ARP average costs estimates.

7 Discussion

The heap space restrictions limits the scope of this research to a relatively small state space. Increasing the state space size would allow for a more accurate representation of the off-shore wind turbine maintenance system. Increasing the number of wind speed states leads to more accurate maintenance costs estimates. In our implementation example where the number of wind states $W = 3$ is small, the average power output per wind state is computed over a larger wind speed interval. Therefore, the power output for state i_3 does not give a accurate approximation for every wind speed v in state i_3, resulting in less realistic cost estimations. In addition, since wind speeds can vary significantly on hourly basis, it would be interesting to consider periods lengths less than a week, say days, and examine how this would affect the maintenance scheduling. Moreover, the results form Schouten et al. (2022) show that the annual average maintenance costs decrease for an increase in the number years in a maintenance cycle (m). With sufficient heap space availability, further research could investigate larger state space sizes and can give more insights in maintenance policy optimization and scheduling. Furthermore, future research can investigate other maintenance policies such as (p-)BRP and (p-)MBRP considering actual wind variability. The addition of a maximum wind speed constraint for maintenance could lead to interesting block-based modeling as it could affect the fixed block times the BRP and MBRP rely on.

In this research, some simplifications and adjustements are made with the purpose of obtaining results and/or because of time constraint. These include, setting the wind speeds discretization ranges for all periods equal i.o. using the period wind speed distributions. Next, Missing wind speed values setting as total sample mean i.o. the period of observation mean. In addition, the downtime costs functions for p-ARP model presented by Schouten et al. (2022) are adjusted because the power output estimates did not align accurately. Moreover, the constant cost critical ages for w-ARP appear not to align with the expected results that are also presented in the paper from Scheu et al. (2012). Elaborating on these aspects and investigating their implementation in the model should result in a more accurate representation of the single-component off-shore wind turbine maintenance system and requires extended research. All in all, because of some these model simplifications and adjustments, one should not base their choice of using and implementation of p-ARP or w-ARP solely on the results of this research. Nevertheless, this research shows the potential of including wind variability in maintenance optimization for a single component off-shore wind turbine.

8 Conclusion

In this paper, the single-component p-ARP model for time varying costs presented by Schouten et al. (2022) is extended to include actual wind variability. A Markov decision process is used to model the age of a component and wind speeds, for which historical daily wind speed data is used from KNMI (2023). Regression techniques and power output data are used to estimate power output curve a wind turbine, on which we base maintenance cost approximations. The maintenance of a single gearbox in a Vestas V164 9.5 MW off-shore wind turbine setting is investigated and some simplifications are introduced to illustrate the implementation of the model. A Linear Problem for an age replacement policy considering wind variability is formulated (w-ARP) and optimized. The w-ARP is compared to the period dependent age replacement policy and a case with constant costs, with respect to the long-run annual average maintenance costs and critical maintenance age for PM. Around 11% savings for w-ARP are obtained, while for p-ARP there are barely saving. The w-ARP is also the most optimal with respect to the annual average costs. In addition, the p-ARP sets higher critical age thresholds for PM in the winter periods because maintenance is more expensive than in summer periods. On the other hand, the w-ARP model allows for higher threshold in summer periods compared to winter periods, which is most likely due to the yearly seasonality in wind speed patterns.

In this research, an age replacement policy considering time-varying costs is extended to account for actual wind variability. Some model simplifications and adjustments are made with the purpose of comparing policies and demonstrating the implementation of the the model, which should be carefully observed when interpreting the results in this paper. Nevertheless, this research shows the potential advantage of including wind variability in optimizing maintenance scheduling for a single component off-shore wind turbine.

References

Arwade, S. R. & Gioffrè, M. (2014). Validity of stationary probabilistic models for wind speed records of varying duration. *Renewable energy*, *69*, 74–81.

Boland, P. J. & Proschan, F. (1982). Periodic replacement with increasing minimal repair costs at failure. *Operations Research*, *30*(6), 1183–1189.

Byon, E. & Ding, Y. (2010). Season-dependent condition-based maintenance for a wind turbine using a partially observed markov decision process. *IEEE Transactions on Power Systems*, *25*(4), 1823–1834.

Byon, E., Ntaimo, L. & Ding, Y. (2010). Optimal maintenance strategies for wind turbine systems under stochastic weather conditions. *IEEE transactions on reliability*, *59*(2), 393–404.

Commissie-M.E.R. (2016). Retrieved from `https://www.commissiemer.nl/projectdocumenten/00002269.pd`

Hagen, B., Simonsen, I., Hofmann, M. & Muskulus, M. (2013). A multivariate markov weather model for o&m simulation of offshore wind parks. *Energy Procedia*, *35*, 137–147.

Jin, T. & Tian, Z. (2010). Uncertainty analysis for wind energy production with dynamic power curves. In *2010 ieee 11th international conference on probabilistic methods applied to power systems* (pp. 745–750).

Karatepe, S. & Corscadden, K. W. (2013). Wind speed estimation: Incorporating seasonal data using markov chain models. *International Scholarly Research Notices*, *2013*.

KNMI. (n.d.). *Windmetingen.* Retrieved from `https://www.knmi.nl/kennis-en-datacentrum/uitleg/windm`

KNMI. (2023). *Daggegevens van het weer in nederland.* Retrieved from `https://www.knmi.nl/nederland-nu/klimatologie/daggegevens`

Li, W., Jia, X., Li, X., Wang, Y. & Lee, J. (2021). A markov model for short term wind speed prediction by integrating the wind acceleration information. *Renewable Energy*, *164*, 242–253.

Ministerie van Algemene Zaken, R. (2023, Mar). *Rijksoverheid stimuleert duurzame energie.* Ministerie van Algemene Zaken. Retrieved from `https://www.rijksoverheid.nl/onderwerpen/duurzame-energie/meer-duurzame-energie-in-de-toek`

Nfaoui, H., Essiarab, H. & Sayigh, A. (2004). A stochastic markov chain model for simulating wind speed time series at tangiers, morocco. *Renewable Energy*, *29*(8), 1407–1418.

Papatzimos, A. K., Dawood, T. & Thies, P. R. (2018). Data insights from an offshore wind turbine gearbox replacement. In *Journal of physics: Conference series* (Vol. 1104, p. 012003).

Sahin, A. D. & Sen, Z. (2001). First-order markov chain approach to wind speed modelling. *Journal of Wind Engineering and Industrial Aerodynamics*, *89*(3-4), 263–269.

Scheu, M., Matha, D., Hofmann, M. & Muskulus, M. (2012). Maintenance strategies for large offshore wind farms. *Energy Procedia*, *24*, 281–288.

Schouten, T. N. (2019). Optimal maintenance policies for wind turbines under time-varying costs". *N. p.*

Schouten, T. N., Dekker, R., Hekimoğlu, M. & Eruguz, A. S. (2022). Maintenance optimization for a single wind turbine component under time-varying costs. *European Journal of Operational Research*, *300*(3), 979–991.

Seyr, H. & Muskulus, M. (2019). Decision support models for operations and maintenance for offshore wind farms: a review. *Applied Sciences*, *9*(2), 278.

Shamshad, A., Bawadi, M., Hussin, W. W., Majid, T. A. & Sanusi, S. (2005). First and second order markov chain models for synthetic generation of wind speed time series. *Energy*, *30*(5), 693–708.

Tian, Z. & Wang, H. (2020, Sep). *Wind power system reliability and maintenance optimization considering turbine and wind uncertainty.* Emerald Publishing Limited. Retrieved from `https://www.emerald.com/insight/content/doi/10.1108/JQME-03-2020-0017/full/pdf?title=wind-`

Zhang, C., Gao, W., Yang, T. & Guo, S. (2019). Opportunistic maintenance strategy for wind turbines considering weather conditions and spare parts inventory management. *Renewable Energy*, *133*, 703–711.

Zheng, R., Zhou, Y. & Zhang, Y. (2020). Optimal preventive maintenance for wind turbines considering the effects of wind speed. *Wind Energy*, *23*(11), 1987–2003.

Programming code

In this research, the following programs and software are used. The Linear problems are optimized using CPLEX 22.1.0 Java. The Java IDE that is used is Eclipse Version 2019-12 (4.14.0). This program is also used to prepare the used data in this research. R studio 4.2.2. is used for data preparation, estimating the power output curve and plotting the necessary figures.

The Java class 'windData2022' is mainly used to process the historical wind speed data from KNMI (2023) and to discretize the wind speeds. The wind speed transition probabilities for every period are calculated using class 'WindTransProbCalculator'. In addition, the class'PowerOutputData' is used to examine historical average weekly power output using the historical wind speed data set for the years 1971 to 2017 (as used in Schouten et al. (2022) and the estimated power output curve. To optimize the p-ARP and the w-ARP models, the Java class files with title p_ARP and w_ARP respectively, can be used. The classes with code to model the p-BRP and p-MBRP with titles p_BRP and p_MBRP are also included. The classes are structure identically. Namely, at the beginning of the file, parameters can be inputted to set the desired size of state space of the model. Based on these inputs and parameters, the decision variables, objective function and constraints are formulated for the maintenance policy. The linear programming (LP) code for p_ARP and w_ARP models the formulations in Sections 4.1 and 4.2, respectively. The models are optimized using CPLEX 22.1.0 in Eclipse, and the values of the objective value and variables are examined.

The R Script 'StationaryTest' is used to test for stationarity in the KNMI historical wind speed data set. data set is imported and lastly, the adf test is performed. The R Script 'OutputTurbineFit' is used to estimate the power output function for the 9.5 MW turbine using piecewise non-linear regression on different data intervals. The R Script 'YearlyOutput2017Final' is used to plot the historical weekly average power output, estimated with the power output function approximated using R Script 'OutputTurbineFit'. In addition, it plots the (adjusted) cosines power output functions presented by Schouten et al. (2022). Lastly, the R Script 'critical ages output' is used to plot the critical maintenance ages, which are the results of the different cost cases obtained from the Java classes p_ARP and w_ARP.

A Appendix

A.1 Methodology

A.1.1 Action space

In each state $i = (i_1, i_2)$, except for the states with a failed component, there are two actions possible: either replace the component (a = 1) by performing a PM or to do nothing (a = 0). For the states with a failed component, denoted by age i2 = 0, only CM can be performed as is must be performed directly. The state-dependent action space can be expressed as follows:

$$A(i_1, i_2) = \begin{cases} \{1\} & \text{if } i_2 \in \{0, M\} \\ \{0, 1\} & \text{if } i_2 \notin \{0, M\} \end{cases} \tag{20}$$

where $i_1 \in I_1, i_2 \in I_2$, M represents the maximum component age at which a PM is performed. In case the maximum age $M = \infty$, PM is never performed.

A.1.2 Transition probabilities

Let $\pi_{(i_1,i_2)(j_1,j_2)}(a)$ be the transition probability from state (i_1, i_2) to state (j_1, j_2) under action $a \in A(i_1, i_2)$, we have:

$$\pi_{(i_1,i_2)(j_1,j_2)}(0) = \begin{cases} 1 - p_{i_2+1} & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = i_2 + 1, i_2 \notin \{0, M\} \\ p_{i_2+1} & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = 0, i_2 \notin \{0, M\} \\ 0 & \text{else,} \end{cases} \tag{21}$$

$$\pi_{(i_1,i_2)(j_1,j_2)}(1) = \begin{cases} 1 - p_1 & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = 1 \\ p_1 & \text{for } j_1 = i_1 + 1 \pmod N, j_2 = 0 \\ 0 & \text{else,} \end{cases} \tag{22}$$

,where $p_x = \mathbb{P}(X = x \| X \geq x)$ indicates the failure probability at age i_2 and mod is the modulo operator. In addition, the failure probabilities from Schouten et al. (2022) under action 0 are corrected, since they depend on p_{i_2+1} rather than on p_{i2}, as the probability of failure between i_2 and $i_2 + 1$ is required.

A.1.3 Costs

In Schouten et al. (2022), the costs of maintenance are dependent only on the period of the year $i_1 \in I_1$. They are denoted by $c_p(i_1)$ and $c_f(i_1)$ for PM and CM respectively. The cost of taking an action $a \in A(i_1, i_2)$ in state $i = (i_1, i_2) \in I$, denoted by $c(i_1, i_2)(a)$, can thus be calculated as:

$$c_{(i_1,i_2)}(a) = \begin{cases} 0 & \text{if } a = 0 \\ c_p(i_1) & \text{if } a = 1, i_2 \neq 0 \\ c_f(i_1) & \text{if } a = 1, i_2 = 0 \end{cases} \tag{23}$$

The cost equations take the following form

$$c_p(i_1) = \bar{c_p} + \Delta_p \cos(\frac{2\pi i_1}{N} + \phi) \tag{24a}$$

$$c_f(i_1) = \bar{c_f} + \Delta_f \cos(\frac{2\pi i_1}{N} + \phi) \tag{24b}$$

For the (p-ARP) constant costs case, the costs value's are estimated with the cyearly average PM and CM cost rates, denoted by

$$\bar{c}_p := \frac{1}{N} \sum_{i_1=1}^{N} c_p(i_1) \ , \ \bar{c}_f := \frac{1}{N} \sum_{i_1=1}^{N} c_f(i_1) \tag{25}$$

A.2 Data

Table 3: Vestas V164 8MW - to 9.5MW wind turbine scaled data

8 MW		9.5 MW	
Wind speed (m/s)	Power (kW)	Wind speed (m/s)	Power (kW)
4.0	194	3.500	193.560
4.5	369	4.083	402.628
5.0	583	4.667	657.750
5.5	843	5.250	967.712
6.0	1141	5.833	1322.976
6.5	1489	6.417	1737.847
7.0	1899	7.000	2226.633
7.5	2372	7.583	2790.525
8.0	2912	8.167	3434.291
8.5	3520	8.750	4159.124
9.0	4197	9.333	4966.217
9.5	5724	9.917	6786.645
10.0	6504	10.500	7716.530
10.5	7186	11.083	8529.583
11.0	7657	11.667	9091.090
11.5	7888	12.250	9366.479
12.0	7972	12.833	9466.621
12.5	7997	13.417	9496.425
13.0	8000	14.000	9500.000
13.5	8000	14.583	9500.000
...	...	...	...
24,5	8000	24.500	9500.000

Data linear scaling functions:

$$x_{scaled} = 1.16667x - 1.16668 \tag{26a}$$

$$y_{scaled} = 1.19216y - 37.279 \tag{26b}$$

A.3 Results

A.3.1 Cost equations

w-ARP) PM and CM costs in thousand euro:

$$c_p(i_3) = 7 * (14820 + \hat{P}(i_3) * 0.06 * 24)/1000 \tag{27a}$$

$$c_p(i_3) = 103.740 + 0.01008\hat{P}(i_3) \tag{27b}$$

$$c_f(i_3) = 28 * (14820 + \hat{P}(i_3) * 0.06 * 24)/1000 \tag{27c}$$

$$c_f(i_3) = 414.960 + 0.04032\hat{P}(i_3) \tag{27d}$$

p-ARP) Daily downtime costs in thousand euro, unadjusted:

$$C_d^{mean}(t) = 6.836 + 1.295cos(\frac{2\pi t}{52} + 0.034) \tag{28a}$$

$$C_d^{low}(t) = 2.061 + 0.850cos(\frac{2\pi t}{52} + 0.020) \tag{28b}$$

p-ARP) Daily downtime costs in thousand euro, with adjustment to match 9.5 MW power output data:

$$C_d^{mean}(t) = 6.836 + 1.295cos(\frac{2\pi t}{52} + 0.034) - 1.44 \tag{29a}$$

$$C_d^{low}(t) = 2.061 + 0.850cos(\frac{2\pi t}{52} + 0.020) - 1.44 \tag{29b}$$

p-ARP) PM and CM costs using mean wind speeds power output estimates in thousand euro:

$$C_p^{mean}(t) = 7 * (20.216 + 1.295cos(\frac{2\pi t}{52} + 0.034)) \tag{30a}$$

$$= 141.512 + 9.065cos(\frac{2\pi t}{52} + 0.034)) \tag{30b}$$

$$C_f^{mean}(t) = 28 * (20.656 + 1.295cos(\frac{2\pi t}{52} + 0.034)) \tag{30c}$$

$$= 566.048 + 36.26cos(\frac{2\pi t}{52} + 0.034)) \tag{30d}$$

p-ARP) PM and CM costs using lowest wind speeds power output estimates in thousand euro:

$$C_p^{low}(t) = 7 * (15.441 + 0.850cos(\frac{2\pi t}{52} + 0.020)) \tag{31a}$$

$$= 108.087 + 5.95cos(\frac{2\pi t}{52} + 0.020)) \tag{31b}$$

$$C_f^{low}(t) = 28 * (15.441 + 0.850cos(\frac{2\pi t}{52} + 0.020)) \tag{31c}$$

$$= 432.348 + 23.8cos(\frac{2\pi t}{52} + 0.020)) \tag{31d}$$

p-ARP) constant costs functions in thousand euro:

$$\bar{C}_p^{mean}(t) = 141.512 \tag{32a}$$
$$\bar{C}_f^{mean}(t) = 566.048 \tag{32b}$$

$$\bar{C}_p^{low}(t) = 108.087 \tag{33a}$$
$$\bar{C}_f^{low}(t) = 432.348 \tag{33b}$$