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Resilience of Private Equity Subsidiaries vs. Public Firms Post-COVID-19 Crash

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Preface and Acknowledgements

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The views stated in this thesis are those of the author and not necessarily those of the supervisor, second assessor, Erasmus School of Economics or Erasmus University Rotterdam.

Abstract

Are private equity firms locusts or butterflies? This thesis investigates the impact of the COVID-19 crisis on innovation. It specifically focuses on the different impacts the crisis had on subsidiaries of private equity firms and compares them to the impacts the crisis had on public firms. The results of this study indicate that while the pandemic had a negative impact on innovation overall, private equity backed firms saw a smaller decline in patent filings and thus, demonstrated greater resilience compared to their public counterparts. This outcome shows that private equity firms play a stabilising role during times of crisis. By enabling continued innovation, the private equity firms not only help their subsidiaries navigate the unstable financial landscape but also contribute to the post crisis economic recovery. The results of this thesis demonstrate the important role that private equity firms play in our economy, especially during times of crisis.

Keywords: Private Equity, COVID-19, Resilience, Innovation, Crises
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1 Introduction

There is much debate about whether private equity firms are beneficial to the economy or are value destroying. This thesis tries to contribute to the literature by examining if private equity firms have a positive or negative impact on the economy when it is at its weakest; during times of crisis. When formulating a hypothesis to this question there are two main logical approaches. On one hand, private equity firms could deplete and liquidate their subsidiaries' assets during tough times to help themselves stay afloat, acting like locusts ravaging a wheat field. On the other hand, because the subsidiaries are their main investments, they could do everything in their power to aid them during these tough times and help them navigate the financial storm, acting like a butterfly nurturing a flower. This research hypothesises the latter. The idea being that private equity firms are used to having long investment horizons and thus, a short-term setback should not be enough for private equity firms to abandon their long-term vision. Therefore they will do everything they can to preserve their investments during times of crisis.

The COVID-19 pandemic has had a large impact on the financial world, leading to disruptions across industries. One of the areas most affected by the crisis is innovation, an important driver of long-term economic growth. This research will use innovation as an indicator for firm resilience. The underlying idea is that if a firm can maintain their usual levels of innovation during and after a crisis, it demonstrates that the firm is resilient and well-equipped to navigate difficult times. Innovation, in turn, will be measured by looking at the amounts of patents filed yearly. This study compares the resilience of subsidiaries of private equity firms and public firms and examines whether being owned by a private equity firm enhances resilience during times of crisis.

The literature review provides insight into how private equity firms operate and gives evidence that economic crises lead to cuts in innovation as firms prioritise short-term survival over long-term investments. The role that private equity plays in this is unexplored. The private equity firms may offer a buffer against economic downturns with their access to capital and strategical expertise. This support may enable the subsidiaries to continue their innovative output during tough financial times. This study seeks to fill in the missing gap by examining if private equity backed subsidiaries are more resilient during times of crisis when compared to public firms.

For the analysis, the framework proposed by Conz and Magnani (2020) will be adopted. This framework suggests that firm resilience should be analysed through temporal dimensions, capabilities contributing to resilience and the outcomes of being resilient.

The research is conducted using an OLS regression on matched firm data. The main results conclude that COVID-19 had a significant negative impact on the amounts of patents filed yearly for all firms. Furthermore, subsidiaries of private equity firms exhibited a lesser decline in patent filings compared to public firms. This is in line with the hypothesis that subsidiaries of PE firms are more resilient during a crisis than public firms. This result is then tested using a weighted regression, that puts a higher weight on firms that have higher levels of patenting activities, and the results remain consistent.

Further testing aims to provide a clearer picture of what drives this increased resilience. This is done by comparing the most resilient subsidiaries to the less resilient subsidiaries, and comparing the most resilient subsidiaries to the most resilient public firms. This way, a comprehensive comparison can be made to see if other factors, like a firm's turnover or their employee

count, influence their resilience during times of crisis. The results of these tests indicate that the firm specific factors do not influence resilience between these groups and that the main driver of this remains the support of a private equity firm.

The remainder of this thesis follows the following structure: the first section discusses the existing literature on private equity firms, firm resilience, economic crises and hypothesis development. This is followed by discussing and presenting the data used for this study, including the data collection, data description and how it was prepared. The methodology section that comes next, explains in great detail how the research is conducted and what methods are used. Thereafter, the results section presents the findings of the analysis. The results are further interpreted in the discussion section. Finally, the conclusion will include a summary of the main findings and their implications for the existing literature and practical use.

This study makes a substantial contribution to the existing literature on private equity firms and the impact of a financial crisis. First, it fills a critical gap in the literature by finding that during financial crises, private equity firms play a supporting role to their subsidiaries. By aiding them during challenging times, their subsidiaries can maintain increased levels of innovation when comparing them to their public counterparts. This in turn, not only benefits their subsidiaries but provides a platform for a quicker economic recovery after the crisis. Second, this study is an addition to the literature that argues that private equity firms mainly focus on long-term investment horizons. By suppo rting their subsidiaries during short-term shocks, they show their commitment to long-term returns on investment. Third, this study provides more insights into financial crises and their effect on firms. It reinforces the idea that most firms focus on keeping the main short-term business operations running by decreasing operations such as R&D that only yield benefits in the long term. Lastly, this study contributes to the existing literature on innovation. The findings emphasise the importance that ownership structure has on influencing a firm's patenting trends during times of crisis. This opens up avenues for more research on innovation, resilience and corporate governance. This study does not only expand on the existing literature on private equity but also provides a deeper understanding of how financial crises impact innovation and the resilience of firms.

2 Theoretical Background

To comprehensively analyse the resilience of subsidiaries versus public firms after the COVID-19 crash, this chapter will present the established literature on private equity (PE) firms and their subsidiaries, firm resilience measured in patents filed and a background on economic shocks. The hypotheses of this paper will also be developed.

2.1 Private Equity and Subsidiaries

2.1.1 Private Equity Explained

Private Equity funds specialise in investment opportunities that are usually not listed on public stock exchanges. Gompers et al. (2016) explain that PE investors are mainly leveraged buyouts (LBO) and growth equity investors. PE firms use these strategies to acquire companies they see upside in. They go on to explain that PE investors anticipate adding value to portfolio companies, with a greater focus on increasing growth than reducing costs. Venture capital (VC) is also a form of private investment. However, this paper will only take buyout private equity firms into account. As Wright and Robbie (1998) explain; venture capital involves the financing of new or radically changing firms. PE firms that use the LBOs focus mainly on larger, well established firms. As this paper aims to compare subsidiaries of PE firms to public firms the latter is more appropriate. Henceforth for simplicity when a "PE firm" is mentioned it will be referring to buyout PE firm.

This relationship however, is not only beneficial for the PE firms. Renneboog et al. (2007) show that pre-transaction shareholders on average receive a premium of 40% and that there is a 30% increase in the share price after a public to private transaction announcement. This can be seen as an indication that in general the market views PE takeovers as value-creating. They show that the main avenues of value creation are undervaluation of the pre-transaction target firm, increased interest tax shields and incentive realignment. Dong et al. (2020) strengthen this positive sentiment by finding that after subsidiaries were relisted on public stock exchanges they bolstered superior profitability compared to benchmark firms.

2.1.2 Limited and General Partners

Private equity funds are limited partnerships where General Partners (GPs) raise capital from institutional investors known as Limited Partners (LPs). This relationship yields separate responsibilities. The GPs are responsible for sourcing, managing and overseeing investments while the LPs provide the capital for a fund and do not participate in its management. This relation is an agency relationship governed by a management contract that is signed at the inception of the fund as per Robinson and Sensoy (2013). They go on to explain the critical importance of these contracts for the LPs: investing in PE involves long-term financial commitments (usually 10 to 13 years) and the LPs have limited governance mechanisms to influence GP behaviour outside of the terms specified in the management contract. As compensation, the GP receives a management fee stipulated in this contract. This typically follows a "2/20/1" structure as Metrick and Yasuda (2010) explain. This consists of a management fee of 2%, carried interest (profit) of 20% and GP ownership of 1%. Often an 8% hurdle rate is also included as Phalippou (2009) states. A hurdle is the minimum level of return a fund manager must achieve to receive a bonus. This hurdle rate is included to promote yearly growth. Due to the information asymmetry and fixed fund lifespan, GPs could take on risky projects for potential increased personal gain. Chasing the hurdle rate is an example of this. However, reputation plays a mitigating role. Ivashina and Kovner (2011) and Demiroglu and James (2010) present

evidence that successful GPs have cheaper and easier access to capital for future funds. Robinson and Sensoy (2013) find that higher management fees do not lead to decreased performance. Furthermore, they actually find that managers with higher fees deliver higher gross performance. The paper also comments on the agency costs and states that they are an inevitable consequence of the information frictions endemic to agency relationships. Braun et al. (2019) that GPs aim to maximise their compensation by increasing their fund size. They go on to explain that when PE funds grow larger, their relative returns decrease due to the challenges of maintaining high performance at scale. However, despite this drop in relative returns, the gross value added continues to increase. This indicates that larger funds create value that benefits both the GPs and LPs.

2.1.3 Entry, Holding Period and Exit

After the capital has been raised, a GP will evaluate investment opportunities. Gompers et al. (2016) explain that the methods used by GPs to do this are often the internal rate of return (IRR) and the multiple on invested capital (MOIC). They explain that PE firms typically target a 22% IRR on their investments, on average. This return is above the CAPM-based rate. Wilson and Wright (2022) find that the targets of these investments are often in stable industry sectors with lower than average failure rates and are more likely to be a single product (not diversified). The firms also have lower levels of equity and lower than average productivity. For PE firms these characteristics are promising as they can realise performance improvement and therefore, growth.

After the PE firm has acquired the subsidiary, it focuses on one thing: value creation. Achleitner et al. (2010) find that two-thirds of the value created by PE firms comes from operational improvements in their subsidiaries. One-third comes from changes in leverage that boosts the return on equity. Valkama et al. (2013) examine the drivers of holding period returns. They find that the main drivers are the size of the target as well as other acquisitions made during the holding period. The latter referring to the buy-and-build strategy. For this strategy, the PE firm purchases a "platform" company and grows it by acquiring different firms within the same industry, to boost its value. This added value is due to the benefits of economies of scale as Borell and Heger (2013) explain. The average holding period of subsidiaries has increased over time Strömberg (2008) finds. In the 1980's, the average holding period was 6-7 years and in 1995-1999, it was 9 years. This period is only measured for LBOs. Thus, not including transactions done by VC firms, for example. This is in line with this paper's aforementioned definition of PE.

The two main exit strategies are initial public offerings (IPOs) and secondary buyouts. An IPO is an offering of a private company to the public market where its shares are sold. A secondary buyout is when a PE firm sells its subsidiary to a different PE firm. Jenkinson and Sousa (2015) find that the choice between these exit options heavily depend on the conditions of the equity and debt market. They explain that when stock markets are strong, there is a preference for the use of IPOs and when debt is abundant and cheap, there is a preference for secondary buyouts. Both of these strategies have benefits and drawbacks. IPOs could offer vastly higher returns, however these returns are uncertain as they rely on the market. Secondary offerings yield less return. However, the selling PE firm knows exactly what the return will be and will receive the payment instantly. Thus, these transactions provide liquidity for PE firms in an otherwise illiquid sector. Ljungqvist et al. (2019) find that competition in the PE sector also plays a role when deciding to exit or not. If a GP decides to exit during a highly competitive environment, they may receive higher returns; however, they could have more difficulty finding new investment opportunities. Thus, GPs rather wait until they are certain of other investment opportunities when competition is lower.

2.2 Firm Resilience and Patents Filed

2.2.1 Resilience Defined

Perrings (2006) defines resilience as: "the ability of a system to withstand either market or environmental shocks without losing the capacity to allocate resources efficiently". This study uses this definition when talking about firm resilience. Linnenluecke (2015) identifies gaps in the literature and on the resilience of firms. Their paper finds that there are multiple definitions and theories regarding firm resilience. This paper aims to contribute to the missing literature on the resilience of subsidiaries and public firms during times of financial crises. It will do so by following Conz and Magnani's (2020) framework, which proposes three avenues to help develop a better understanding of firm resilience. Firstly, they emphasise taking the temporal dimension into account. This means that researchers should design studies with specific time frames, such as before and after a crisis, which this study will do. Secondly, they suggest exploring capabilities that contribute to resilience. This study investigates whether being backed by a private equity firm enhances resilience. Thirdly, they recommend analysing the outcomes of being resilient. This involves comparing the performance of resilient firms to non-resilient ones. This study will examine the differences between PE backed firms and public firms in term of resilience.

Sabahi and Parast (2019) have done research on the link between innovations and resilience. They find that firms with more innovative environments are more resilient to disruptions, because innovation helps firms strengthen capabilities that increases a firms risk management capability. Their findings indicate that size, age and financial resilience are important for the survival of financial institutions. However, firm innovation plays a particularly crucial role when looking at resilience. Innovation has the greatest explanatory power compared to the other factors. This indicates that innovation significantly enhances resilience, making firms better equipped to deal with challenges and maintain long-term operations.

2.2.2 Patents as a Measure of Innovation

Using patents as a measure of innovation dates back to Scherer (1965). He found that inventive output increases with firm sales and that they are not systematically related to prior profitability and liquidity. Hall et al. (2005) and Atanassov (2013) build on previous studies to confirm that patents filed contain significant information on the market value of firms. Many of these papers also take patent citations into account to get a better measure of R&D success. However, this paper will not take citations into account. The information required for this research needs to be as current as possible. The number of patents filed yearly show the current innovation activity of a company, with the focus being on whether research is still being conducted after a crisis, instead of looking at the quality of the research itself. This paper will use these findings as a base to use patents as an indicator of innovation and, consequently, firm resilience.

2.3 Economic Crises and Innovation

2.3.1 Background on Economic Crises

The COVID-19 Pandemic Economic Crisis (2020) is the crisis this paper will use for its research. This crisis started as a sudden stop crisis due to the sudden economic halt due to lockdown and its restrictions, and triggered a debt crisis because governments borrowed heav-

ily to support economies as Joyce and Prabowo (2020) explain. Another crisis this paper will mention is the Great Financial Crisis (2007-2009). This crisis was a banking crisis caused by the collapse of many major financial institutions due to subprime mortgage losses. This crisis lead to a debt crisis in European countries.

Reinhart and Rogoff (2009) find that asset market collapses are deep and prolonged. Housing prices decline on average by 35% over six years, and equity prices collapse an average of 55% over a downturn of about three and a half years. During banking crises, unemployment rises on average by 7% and this lasts roughly for four years. A huge government debt increase of 86% is also one of their findings. The main cause for this large increase is the cost of bailing out and recapitalising the banking system. Tax revenues contract, contributing to a long lasting large budget deficit.

The COVID-19 crisis was a crisis like no other, Borio (2020) explains. It did not have an economic origin, it was largely influenced by non-economic factors, the crisis was truly global and the policy response has been unprecedented in terms of speed, size and scope. He finds that the crisis lead to large contractions in output and employment, more intense than the Great Depression. He argues that the pandemic accelerated existing trends of higher debt and lower interest rate and might lead to a potential debt trap.

2.3.2 The Impact of Crises on Innovation

Zhang (2015) finds that firms with higher R&D expenditure often have a higher correlated risk due to bad performance. This is due to the risky nature of R&D. They find that R&D firms have an increased chance in defaulting during an economic downturn. However, firms that have previous successes in R&D can mitigate the relationship between R&D and distress risk.

Hingley and Park (2017) find that patents are strongly pro-cyclical, supporting the view that short-term resource constraints affect patenting decisions, even though long-term factors usually determine innovation. This suggests that financial crises could significantly impact the amount of patents filed. Hardy and Sever (2021) expand on these findings by showing that financial crises have a significant impact on innovation. They attribute this to firms not only losing funding for new projects, which reduces patents in the long term, but also to needing to liquidate existing projects to meet immediate financing pressure which reduces patents in the short term. Juergensen et al. (2020) find that during the COVID-19 crisis small and medium-sized enterprises (SMEs), typically the size of subsidiaries as discussed before, also faced increased demand disruptions and logistical challenges.

2.4 Hypotheses Development

In this section the hypotheses are developed which will be used for the analysis. This paper will expand on what Reinhart and Rogoff (2009), Zhang (2015) and Hingley and Park (2017) find on the impact that crises have on R&D and innovation. This research sets out to discover if the COVID-19 pandemic had an adverse effect on innovation by using "patents filed" as a indicator. The pandemic had profound effects on economic activity and thus the expectation is that when companies are in distress their innovation will also decrease.

Making the first hypothesis:

Hypothesis 1: *"COVID-19 had a negative impact on innovation."*

The rationale behind this hypothesis is based on the theory that economic downturns and crises often lead to uncertainty and less financial resources, to, in turn, invest in R&D. Bloom and Reenen (2002) find that during uncertain economic times, firms are less likely to engage in innovating activities. The COVID-19 pandemic proved a very uncertain time for many firms as it not only had a major impact on the financial markets but was also something never seen before. Archibugi et al. (2013) further explore this by conducting research on the 2008 financial crisis. They find that the crisis reduced short-term willingness of companies to invest in innovation and innovation expenditure. This paper expects the COVID-19 crisis to have similar effects on innovation. Reinhart and Rogoff (2009) and Zhang (2015) find that during crises, companies tend to cut back on spending to conserve cash and focus on core business activities. Furthermore, Juergensen et al. (2020) explain that the pandemic caused unique disruptions such as consumer demand, operational challenges and supply chain interruptions, leading to an even larger decline of innovation when measured as patents filed.

Furthermore, this paper will examine if subsidiaries of PE firms have greater resilience during the COVID-19 crisis compared to public firms. Gompers et al. (2016), and Kaplan and Stromberg (2009) show that PE firms enhance operational efficiency and provide strategic support. This may contribute to the resilience of their subsidiaries. PE firms implement strict performance monitoring, a source of additional capital, and offer managerial expertise, all of which can be valuable during times of economic crisis.

The main question for developing this hypothesis is if the PE firms will deplete and liquidate their subsidiaries' assets to minimise financial losses during times of crisis or if they will help their investments navigate these difficult financial times. As discussed before, during crises the focus tends to shift to short-term survival instead of long-term returns. However, intuitively and backed by the literature, a strong argument can be made that PE firms are used to long investment horizons and will not be fazed by short-term economic setbacks. Therefore this study hypothesises that PE firms will offer support to their subsidiaries during the COVID-19 crisis. Furthermore, subsidiaries are typically actively managed by the GP or PE firm that owns them, providing not only capital but also operational guidance, strategic improvements and the access to the PE firm's vast network. During crises, these supporting capabilities can be used to navigate challenges and help maintain business continuity. The alignment of incentives between PE firms and their subsidiaries can also lead to a more aggressive pursuit of growth opportunities, even during downturns. Thus, there is a plausible possibility that the subsidiaries of PE backed firms will be more resilient in times of crisis.

Due to these advantages, it is hypothesised that subsidiaries of PE firms are better equipped to deal with financial crises compared to public firms. The second hypothesis is as follows:

Hypothesis 2: *"Subsidiaries of PE firms are more resilient during a crisis than public firms."*

This hypothesis will be tested using patents filed as a measure of innovation and comparing the impact COVID-19 had on the firms patenting trends.

3 Data

To fully understand the methodology and outcomes of this study, an understanding of the data is required. This chapter provides a detailed description of how the data was collected, transformed and utilised for the research. Additionally, the data will be visualised and analysed, showcasing and discussing insights gained from the collected data.

3.1 Data Collection

3.1.1 Data Source

The dataset used for the study was collected from Orbis, Bureau van Dijk's main company database, which specialises in private company data. Orbis provides extensive information on companies all over the world. With information on around 400 million companies and 170 different data providers, it is one of the main databases used by researchers. This study requires information on company variables as well as the yearly patents filed for public firms and subsidiaries of PE firms.

3.1.2 Data Criteria

For this research, detailed company level data is required. The data extracted from Orbis has not been altered in any way. However, to ensure that only relevant data is included in the analysis, specific criteria have been applied.

Criteria for public firms and subsidiaries:

- *Only active companies*: for this research only active companies are considered. Companies that failed during the data collection period will be excluded. This is mainly done to ensure the data is complete and by excluding companies that might have been vulnerable before the pandemic, there will be a better understanding on how the COVID-19 crisis impacted the two different groups of firms. This first criterion results in over 370 million observations.
- *Data from 2010-2023*: the data spans a 13 year time frame. By starting the data collection in 2010, this study circumvents data that could be distorted by the 2008 financial crisis, ensuring that the analysis is not influenced by this event. The large time frame provides a robust dataset, offering enough data to observe longer term trends and the flexibility to trim the dataset when needed for the analysis.
- *Minimum of 3 patents*: by setting a minimum amount of three filed patents, firms that do not participate in R&D are excluded. This criterion ensures that all companies used contribute to innovation and can provide insights into their resilience during a crisis. This criterion eliminates many observations and results in just over 1 million observations.
- *Companies registered in the United States of America*: to ensure relevance and consistency while comparing companies, this study only includes companies that are registered in the United States of America. By only using extracting companies from one country, cross-country differences like regulatory, economic or market differences are therefore neutralised. Additionally, since the COVID-19 crisis impacted countries differently, analysing companies within the same context eliminates these disparities. Adding this criterion results in 190 thousand observations.

Criteria for public firms:

- *Company must be publicly traded*: this criterion is included to distinguish between publicly traded firms and the subsidiaries of PE firms. This study involves comparing these two groups, making it essential to include a distinguishing term for the only publicly traded companies. This ensures that the effects that the COVID-19 crisis had on both groups can be effectively compared. This criterion brings the total observations down to just over 1000.

Criteria for subsidiaries:

- *Private equity firms as ultimate owners*: this criterion is included to distinguish between publicly traded firms and the subsidiaries of PE firms. By ensuring these companies are owned by PE firms they can effectively be classified as subsidiaries. This distinction is essential for the analysis, as it enables us to compare the two groups effectively. Implementing this criterion results in just over 1100 results.
- *PE target deal between 2010 and 2020*: by including this criterion, it makes sure that the firm was acquired between 2010 and 2020 in a PE deal. By including firms only acquired in this time frame, the data aligns with the study period and makes sure that firms were under the control of a PE firm before and during the COVID-19 pandemic. This allows for an effective analysis of the differences in resilience between public firms and subsidiaries during the pandemic. The final criterion leaves just over 200 observations.

3.2 Data Description

3.2.1 Summary Statistics

Table 1: Combined Summary Statistics of Patents, Turnover, and Employees

Variable	Year	Mean	SD	Min	Max
# of Patents	2010	12.134	26.655	0	255
	2011	13.733	30.582	0	337
	2012	13.495	27.726	0	302
	2013	14.682	30.066	0	272
	2014	17.001	35.990	0	382
	2015	16.69	33.256	0	355
	2016	17.855	35.637	0	292
	2017	18.609	38.507	0	338
	2018	19.094	39.905	0	439
	2019	19.638	50.159	0	1178
	2020	17.278	36.051	0	360
	2021	17.203	40.634	0	468
	2022	12.863	33.077	0	386
	2023	6.513	17.827	0	285
Turnover		2733181.8	20011021	0	3.578e+08
Ln Turnover		10.837	3.645	0	19.695
Employees		4243.099	28440.238	0	529000
Ln Employees		5.655	2.318	0	13.179

Note: Combined dataset has 1155 observations.

Table 1 presents the summary statistics of the combined dataset. It shows that the overall trend of patents has increased from 2010-2019 then staggers and decreases after 2020. This is likely a consequence of the COVID-19 pandemic and its impact on the innovation capabilities of firms. The standard deviation for all years is high. This is not surprising as some firms will be highly active in patent filing while others not. The standard deviation of the variable "turnover" is very high. This can be attributed to the big array of firm sizes included in this dataset. The variable "employees" also has a high standard deviation. This can also be attributed to the wide range of firm sizes in the dataset. The highest values for turnover and employees belong to CVS and FedEx respectively. The main outliers will be dealt with later during the matching process or before using winsorizing.

The summary statistics of all firms provide a foundation for further analysis. The impact of the COVID-19 pandemic on the ability of a firm to innovate is especially noteworthy as it aligns with the general idea of this paper. Additionally, the wide range of the variables "turnover" and "employees" along with their high standard deviations, show the heterogeneity of the firms included in this study.

Table 2 presents the summary statistic for publicly traded firms and the summary statistics for the subsidiaries. It is important to look at these datasets separately as this research relies on comparing the two groups. The split summary statistics show that the mean of patents filed for public firms largely follow the same trend seen in the combined dataset. There is a sizeable increase in patents filed from 2010-2019, followed by a pronounced drop from 2019-2023. This again could be contributed to the COVID-19 crisis. However, subsidiaries do not follow this trend. While there is a sizeable drop after 2019, the mean patents filed reach their peak in 2014. This could be explained by a change in operations after being taken over by a PE firm. If the patents filed are measured comparable to previous years, this discrepancy disappears. This will be discussed later in this paper.

Another insight to be gained from comparing the two tables, is that the public firms have higher means compared to the subsidiaries in all variables. This is not a surprise, as previously discussed subsidiaries are almost always small to medium-sized firms. How the size difference will be compensated for will be discussed later in this paper.

Firm heterogeneity is shown in both groups by analysing the summary statistics. This ensures both groups include a diverse selection of companies for the analysis. The heterogeneity and the impact of COVID-19 on patenting trends create a solid foundation for this paper's research.

3.2.2 The Sectors

For the analysis, matching will be used. As Chabé-Ferret (2015) explains, matching compares the outcome treated units to that of untreated units with the same observed characteristics. To ensure that the firms are matched as relevantly as possible the sectors will be used. The matching process will be explained in greater detail later, this section is used to describe how the sectors have been divided.

The dataset used for this research contains the Standard Industrial Classification (SIC) codes of the included firms. SIC codes are a way of classifying a company's industry using a four digit code. The first two digits of the code show the primary industry group of a company. The third digit specifies the broader industry group and the fourth digit specifies the industry sector. An example of this would be Apple Inc's industry code 3571. The "35" specify man-

Table 2: Summary Statistics of Patents, Turnover, and Employees

Variable	Year	Public				Subsidiaries			
		Mean	SD	Min	Max	Mean	SD	Min	Max
# of Patents	2011	15,285	31,735	0	337	4,741	20,659	0	236
	2012	14,895	28,853	0	302	5,382	18,013	0	178
	2013	16,177	34,255	0	406	6,009	20,162	0	272
	2014	15,835	31,013	0	355	5,941	19,438	0	355
	2015	18,081	34,066	0	261	5,887	21,617	0	382
	2016	20,236	37,165	0	292	5,090	18,972	0	198
	2017	21,215	41,022	0	338	4,877	16,916	0	196
	2018	21,504	40,005	0	357	4,761	16,663	0	193
	2019	22,606	53,726	0	1178	4,247	14,987	0	257
	2020	19,864	43,416	0	468	3,886	15,149	0	178
	2021	19,846	44,059	0	511	3,822	15,250	0	178
	2022	14,751	19,048	0	285	3,546	12,126	0	116
	2023	7,451	19,048	0	285	3,546	12,126	0	116
Turnover		148,756	3,681,999	70	3,000,000,000	31,792,242	2,169,300,003	360	3,588,000,000
Ln Turnover		10.106	2.154	4.263	14.914	10.963	3.831	5.889	19.055
Employees		348,971	676,621	0	4,000	4,915,182	3,077,495	0	52,900,000
Ln Employees		4.433	1.844	0.693	8.294	5.865	2.328	0	13.179

Note: Public group has 985 observations; Subsidiaries have 170.

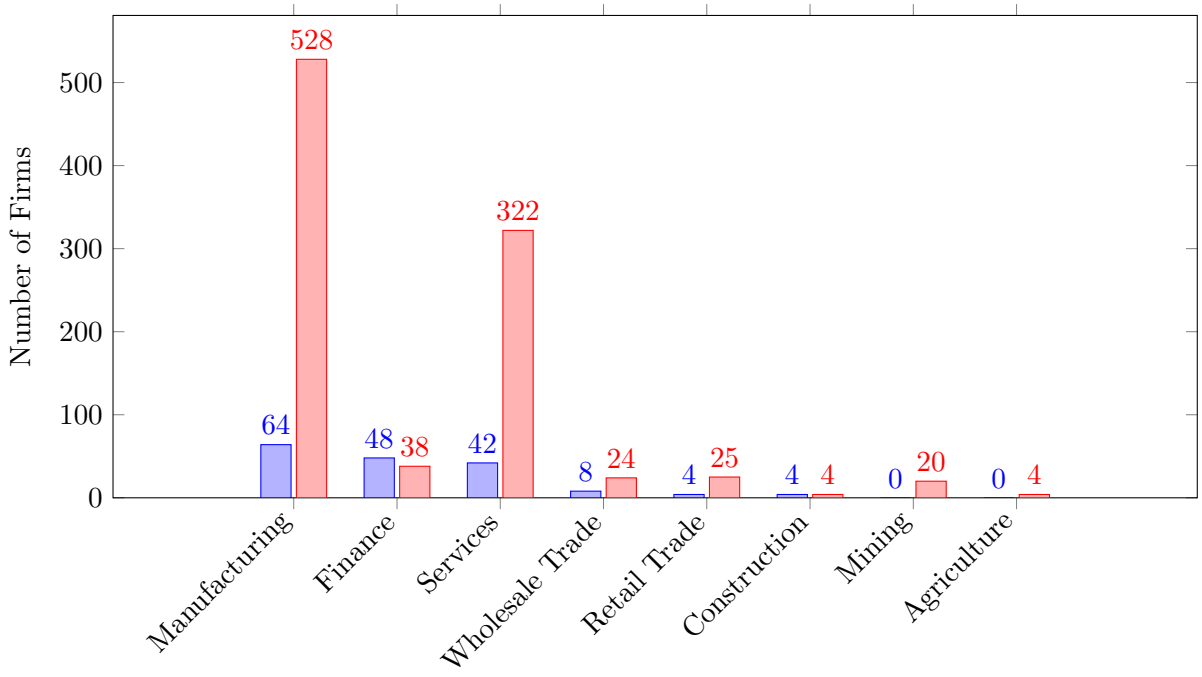
ufacturing, the "7" narrows it down to "Computer and Office Equipment" and finally the "1" classifies it as "Electronic Computers". These codes were all provided by Orbis.

This research will utilise the first two digits of the SIC code, the so called primary industry group. Kahle and Walkling (1996) conduct research using the official Two Digit Major Group classification. Table 16 has been constructed in an identical fashion.

For the analysis, it is important that there are observations in the same sector for both the public firms and the subsidiaries. Figure 1 shows the split of sectors between public firms and subsidiaries. There is an uneven split in the firms of our dataset. This is not problematic and there are interesting insights to be gained from it. For the public firms, the most firms are in the manufacturing and service sector. Most subsidiaries belong to the manufacturing, finance and services sectors. This is not surprising, as firms that operate in those sectors are often targeted by PE firms: the manufacturing sector as it is capital intensive and can benefit from economies of scale, the finance sector as it offers high growth potential, and the services sector as they are often scalable and a PE firm can assist with growth.

For all sectors except for "Finance, Insurance and Real Estate" there are more public firms than subsidiaries. Due to the matching process used in this analysis, it is preferable to have more public firms than subsidiaries to ensure a better match. This will be discussed in more detail in the method section of this paper.

Figure 1: Public (Red) and Subsidiary (Blue) Sector Split



3.3 Data Preparation

3.3.1 Count Data and Solving the Zeros

The dataset contains count data, countable data with non-negative integers, as the amount of patents filed cannot be negative. However, count data often exhibits a strong skew due to the fact that there many zeros in the dataset. This research will use the percentage growth to compare the different firms, and to do this the data must be transformed. The most popular method is the Log1plus method. The Log1plus method consists of transforming all data using $ln(1+y)$ where y is the data. Doing so solves the problem of the zero values in the data. There is some critique when using this method. Jonathan Cohn (2022) finds that it is better to use the Poisson distribution when using zero data. However, using the Poisson regression imposes the restriction that the conditional mean and variance of the outcome are equal. This is not the case as table 15 shows. For this reason the tried and tested Log1plus method will be used.

3.3.2 Log Differences

The last part of the data transformation will alter the measure of patents filed so that firms can be compared easily. As this paper looks at the impact that COVID-19 had on firms, the amount of patents filed are not of importance, only how their patenting trend was altered by COVID-19. This is done by creating a new variable, logdiff_patent. The formula for this is: logdiff_patents $= (patent + 1) - log(lagged_patent + 1)$. This way, any size firms with any amount of patents filed can be directly compared without it distorting our main research objective. This also addresses the presence of outliers as it compresses the range of data and reduces the influence of them. Doing so has fully prepared the data for the coming research.

4 Methodology

This chapter provides a detailed overview of this study's research design. The framework of Conz and Magnani (2020) will be incorporated. The choice of methodology and how it works will be explained in great detail. By providing a thorough explanation, this chapter aims to not only enhance the credibility of the research but also to provide a base for further research to be conducted.

4.1 Research Design

This research will expand on the previously discussed literature on the role that PE firms have in our economy. By looking at the effects of the COVID-19 crisis on subsidiaries and public firms, this research hopes to determine if there are benefits, such as improved resilience, from being part of a PE firm. A Difference -in-Difference event study (DiD) analysis will be performed to compare the effect of the COVID-19 crisis on the groups. The DiD analysis is a fitting way of comparing the firms before and after the COVID-19 crisis. The DiD event study analysis is a quasi-experimental approach that compares the changes between two groups after an event. In this case, the event is the COVID-19 crisis.

The first hypothesis, *COVID-19 had a negative impact on innovation*, will be used as a base for the research. If this relationship cannot be proven, the second hypothesis, *subsidiaries of PE firms are more resilient during a crisis than public firms*, cannot be explored. The theoretical background of this paper establishes the link between innovation and resilience. Without certainty that COVID-19 had a tangible impact on innovation, one cannot determine if subsidiaries or public firms are more resilient during times of crisis. The second hypothesis, *subsidiaries of PE firms are more resilient during a crisis than public firms*, will be the main focus of this paper. By comparing the impact COVID-19 had on patents filed for subsidiaries and public firms, this paper will try to see if the different groups have a different level of resilience.

An important assumption of the DiD analysis is the parallel trend assumption. Wing et al. (2018) explain that before the treatment, the time series outcomes in each group should differ by only a fixed amount in every period and should follow the same trend overall. This is necessary to accurately determine if an event, in this case COVID-19, impacted the different groups differently. If the parallel trend assumption does not hold, one cannot use the DiD to effectively determine the results of a treatment.

To increase the validity of the DiD analysis, matching will be applied. By matching a subsidiary to a public firm using active employees and turnover, this paper hopes to compare firms that function similarly. If this is not done, and all firms are included, the results could be invalid. Comparing large firms and their patenting trends, to small or medium-sized firms' patenting trends, can cause distortion in the regression. By directly comparing firms that operate similarly to one another, this distortion can be avoided.

Multiple DiD regressions will be conducted if significance is found in the main regression. This paper will expand on the base regression to see if adding more variables impacts on the effect or significance of the DiD variable included. Doing so will not only fortify this paper's findings but will also deepen the understanding of the mechanisms that connect the dependant variables to the independent variable.

After the main findings have been presented, a different regression will be run. This second regression will alter the way this research has been done to see if the findings remain the same.

This is a way of exploring the validity of the findings by checking if the result remains the same while using a different research method.

4.2 Parallel Trend Assumption

Wing et al. (2018) state that in the parallel trend assumption, the time series outcomes in each group should differ by only a fixed amount in every period and should follow the same trend overall. The parallel trend assumption should be tested to see if it holds, before conducting a difference-in-difference study. This is due to the fact that the parallel trend assumption demonstrates that observations before a specific event act in the same fashion, establishing a baseline. In an ideal difference-in-difference study, the two lines will diverge after the event, showing that before the event they acted in the same fashion and the event broke that parallel trend.

The analysis of the parallel trend assumption for this study will be done visually using a graph. The graph will be constructed using a 95% confidence interval while using the predicted values for the interaction between the year and the amount of patents filed. This will be done for both the subsidiaries and the public firms. Once the graph has been constructed, the analysis can take place. If both the values for the subsidiaries and the public firms fall in each other's confidence interval for a specific year, it can be concluded that in that specific year (with 95% confidence) there are no ascertainable differences. If they fall outside of each other's confidence interval, the inverse can be concluded. This way, the analysis can be done to see if the parallel trend assumption holds for this study.

4.3 Matching

4.3.1 Nearest Neighbour Propensity Score Matching

This research will utilise matching. Matching ensures that firms with similar characteristics are matched up and compared. Doing so creates the clearest picture of the effects that COVID-19 had on firms and whether the two groups reacted differently. The matching process makes use of specific characteristics to find the closest match. This study uses employees and turnover as the characteristics for the matching process. As PE firms are not required to disclose their financials, information on them is scarce. However, employees and turnover are often reported. These variables will be used to ensure that the firms that are matched up are the same size. This is important as bigger firms often file more patents. The study would not be valid if it would be comparing big firms that file many patents to small firms that file close to none. The matching process also takes care of many outliers by only matching firms that are similar.

There are many different methods of matching. Austin (2014) finds that nearest neighbour propensity score matching produces low bias and variance in the estimates, while remaining a straightforward method. Due to the simplicity, a clear interpretation can be formed when analysing the results of the matching. Making nearest neighbour matching the preferred method for this research.

The first step of this method of matching is to calculate the propensity score. The propensity score describes how likely a unit is to be part of a specific group, in this case the subsidiaries.

$$p(X) = P(T = 1 \mid X) = \frac{e^{\beta_0 + \beta_1 \ln(\text{turnover}) + \beta_2 \ln(\text{employees}) + \beta_3 \text{sector_code}}}{1 + e^{\beta_0 + \beta_1 \ln(\text{turnover}) + \beta_2 \ln(\text{employees}) + \beta_3 \text{sector_code}}}$$

- $p(X)$ is the propensity score.

- $P(T = 1 \mid X)$ is the probability of being a subsidiary (T=1) given the covariates X.
- β_0 is the intercept term.
- β_1,β_2,β_3 is coefficients for the covariates of the log chances of being a subsidiary.

The propensity score is calculated using a logistic regression that uses the natural logarithm of both turnover and employees, while also taking the firms sector into account. The natural logarithm is used to normalise the distribution of variables. This reduces the variability of data and brings the large values closer together, making the data easier to interpret.

Once all the propensity scores have been calculated, the nearest neighbour matching can take place. Using the propensity score, a subsidiary is matched up with a public firm that has the closest comparable propensity score to their own. Doing so ensures that subsidiaries are only compared to public firms with similar characteristics, correcting for differences in patenting behaviour that are caused by firm size and sector. A restriction of no replacement is also applied. This ensures that public firms that have previously been matched to subsidiaries are removed from the available pool, making sure that each subsidiary is matched to a unique public firm.

4.3.2 Testing the Matched Sample

After all the subsidiaries have been matched to public firms, a few checks will take place. The first test will be the Wilcoxon Rank Sum test. This test is a non parametric statistical test used to compare to independent samples. This test shows if two groups have the same continuous distribution, and will be done for the variables "employees" and "turnover" both pre and post matching. The Wilcoxon Rank Sum test states the null hypothesis as: the two populations are identical. Thus, if the null hypothesis is rejected, the two populations are not identical. The results of these tests will determine whether there are differences in the data when comparing the subsidiaries to the public firms. The test will be run before and after the matching, to determine if there are any differences and the matching eliminates them. This test will be part of determining if the matching process was successful.

The second test will help interpret the balance of the two groups in terms of sector representation. As discussed before, firms in different sectors have different patenting habits. If during the matching process an unbalanced group of public firms are matched to the subsidiaries, it may distort the results. For this reason, a similar sector split to the subsidiaries is favourable. To test this, a Pearson Chi-Square test will be used. The null hypothesis for this test states: there are no differences in the distribution of firms across sectors. If this null hypothesis is rejected, the public firms used to match form an unbalanced group when taking sectors into account. If the null hypothesis of the Pearson Chi-Square test cannot be rejected, and the Wilcoxon rank-sum test shows that the firms have been appropriately matched, the matching process is complete.

4.4 Ordinary Least Squares

4.4.1 Assumptions

For the OLS to be a valid way of conducting research, assumptions must hold. As Burton (2021) states, the assumptions are:

- Linearity. This assumption states that the dependent and independent variables share a linear relationship.

- Independence. This assumption is met if the error terms are uncorrelated. When this assumption is violated there is autocorrelation.
- Constant Error Variance. This assumption states that the variation of the variable is constant. When this is not the case it is called heteroskedasticity.
- Normality. This assumption states that the distribution of the residuals must be normally distributed.
- Multicollinearity. For a valid OLS there must be no multicollinearity. Multicollinearity exists when two or more independent variables share a near perfect linear relationship.

Testing to see if these assumptions hold are an important part of an OLS regression and the method to do so will be discussed in the next sections.

4.4.2 Clustering

In OLS analysis, it is important to address potential issues of heteroskedasticity and autocorrelation if they are found, particularly with panel data. These issues can lead to inefficient estimates and biased standard errors, compromising the results of the research. Clustering standard errors is a good solution to these issues as Vogelsang (2012) states. Clustering standard errors is a way to address both heteroskedasticity and autocorrelation by adjusting the standard errors to be the estimates of the correlation within specific groups or clusters. Clustering allows different variances amongst different clusters, solving the heteroskedasticity issue by providing standard errors that do no rely on the assumption of homoskedasticity. Clustering solves the autocorrelation issue on the same principles, by adjusting the standard errors for the predicted correlation of residuals within each separate cluster the autocorrelation does not hold anymore. Thus, if heteroskedasticity and autocorrelation is found in the previously mentioned tests, clustering the standard errors can be used to still be able to validly use an OLS regression for this research.

4.5 Fixed or Random Effects

When analysing panel data, there are two primary approaches to account for unobserved heterogeneity when using an OLS regression: fixed effects and random effects models. Choosing the right model is crucial for robust statistical analysis.

4.5.1 Fixed Effects

Fixed effects models are used to control for unobserved heterogeneity. The fixed effects model does this by including a dummy variable for each firm, doing so controls for all time-invariant characteristics of the firm. Borenstein et al. (2010) explain that the fixed effects model is used to control for heterogeneity when it is constant over time for each cross-sectional unit. They use the example of estimating the mean aptitude score for students at a specific college. When taking two separate groups and assuming the assignment to a group has no impact on the score, then all the groups share a common effect size and thus the fixed-effect model applies. A simplified formula for the fixed effects model is as follows:

$$y_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}$$

- y_{it} is the dependant variable.
- α_i is the firm specific intercept that compensates for the fixed effects.
- βX_{it} is the explanatory variables and their coefficients.

- ϵ_{it} is the error term.

To determine whether this study requires the use of a fixed effects model, the Hausman test will be used. The Hausman test can compare the fixed effects and the random effects estimators to see whether the unobserved effects are correlated with the independent variables. The null hypothesis is as follows: the random term is not correlated with the independent variables. Thus, if the null hypothesis is rejected, a fixed effects model will be the appropriate model for this study.

4.5.2 Random Effects

Random effects models assume that the unobserved firm-specific effects are uncorrelated with the explanatory variables. They can be treated as random variables. Borenstein et al. (2010) explain that the random effects model has a distribution of true effect sizes. An example they use is to estimate the mean aptitude score for any college in a specific state. When comparing two separate groups from two separate colleges, there is a high chance that the true mean differs from college to college. Thus, the fixed effects model does not apply anymore. In this case a random effects model would be used. A simplified formula for the random effects model is as follows:

$$y_{it} = \alpha + \beta X_{it} + u_i + \epsilon_{it}$$

- y_{it} is the dependant variable.
- α is the overall intercept.
- u_i is the random term that corrects for the random deviations from the intercept.
- βX_{it} is the explanatory variables and their coefficients.
- ϵ_{it} is the error term.

To determine whether this study requires the use of a random effects model, the Breusch and Pagan Lagrange Multiplier test will be used. This test evaluates if the variance of the random effects is zero. By doing so, it can be determined whether using a random effects model is appropriate. The Breusch and Pagan test uses the null hypothesis: variance of the random effect is zero. If this hypothesis is rejected, a random effects model should be used.

4.6 The Regressions

In this study, the log difference in patents will be used as the main dependent variable. This will be done to examine the effects that the independent variables have on patenting trends. The main independent variable that will be focused on will be the DiD variable. This variable is used to examine the difference between public firms and subsidiaries after the COVID-19 crisis. There will be four main regressions, all expanding on the previous ones. Each regression will add new independent variables to examine the effects of the new variables on the log difference in patents and the impact on the significance of the DiD variable. The fourth regression integrates all the variables from the previous models and will, therefore, be the focus of discussion in this section.

4.6.1 Difference-in-Difference

This study will use a Difference-in-Difference (DiD) Ordinarily Least Squares (OLS) approach. The DiD method is suited for this study as it aims to identify changes between the patenting trends of subsidiaries and public firms after the event, COVID-19. By including an

interaction term in the OLS regression, the DiD can be determined. An interaction term is the backbone of a DiD regression and, as its name implies, demonstrates the interaction between two variables. The interaction term for this study is as follows:

$$DiD = subsidiary * post_covid$$

This interaction term captures the DiD effect. For this research, the interaction term will determine the difference between subsidiaries and public firms in terms of patenting trends in the post COVID-19 era. If significance is found, and the coefficient for the DiD variable is positive (negative), then it means that subsidiaries filed more (less) patents compared to their public counterparts. Four regressions will be used, each regression adding more variables than the last. This will be done as a robustness check to see if the DiD term stays significant.

4.6.2 The Complete Regression

The fourth and final regression encompasses all variables that are of interest for this study, with the dependent variable being patents_filed. The regression examines if COVID-19 had an effect on patents filed. This is an important step as if this relationship cannot be determined, the rest of the research will not be valid. It also accounts for the difference between being a subsidiary or a public firm by using a dummy variable. The interaction term is the most important variable included in the regression, as it is the variable that will demonstrate the difference between subsidiaries and public firms post COVID-19. Firm specific indicators are also included, the natural logarithm of the number of employees and the natural logarithm of the firm's turnover. This is done to control for the potential effects of a firm's size on patenting trends, and its resilience during a crisis. This model also includes a set of dummy variables representing the sector to which the firms belong. Manufacturing will be used as the baseline, as it includes the most subsidiaries and public firms. This controls for sector specific factors that might influence patent filings. These final variables have been added with the goal of providing a more comprehensive understanding of the determinants of patenting activities and what role being a subsidiary plays in this. The regression also includes a lagged term for the log patents filed. This lagged term accounts for the persistence effect of patenting activities. This is crucial for understanding how previous patenting trends influence current patenting behaviour. By including a term for the lagged log patents filed, a better model can be created on the dynamics of patents filed. Including this term may also alter the significance of the interaction term.

$$\begin{aligned}\text{patents_filed}_{it} = {} & \beta_0 + \beta_1 \text{lagged_patents}_{it-1} + \beta_2 \text{subsidiary}_i + \beta_3 \text{post_covid}_t + \beta_4 \text{DiD}_{it} \\ & + \beta_5 \text{Construction}_i + \beta_6 \text{Finance}_i + \beta_7 \text{Retail Trade}_i \\ & + \beta_8 \text{Services}_i + \beta_9 \text{Transportation}_i + \beta_{10} \text{Wholesale Trade}_i \\ & + \beta_{11} \text{ln_Employees}_i + \beta_{12} \text{ln_Turnover}_i + \epsilon_{it}\end{aligned}$$

- patents_filed is the log difference in patents filed.
- subsidiary is a dummy variable indicating if the firm is a subsidiary.
- post_covid is a dummy variable indicating the post COVID-19 period.
- DiD is the interaction term that incorporates subsidiary * post_covid.
- lagged_patents includes the amount of log patents filed in the previous year.
- sectors is a set of dummy variables that indicate the sector to which the firm belongs. A 0 value indicates the sector manufacturing.

- ln_Employees is the natural logarithm of the number of employees.
- ln_Turnover is the natural logarithm of the firms turnover.

These four models will be used to determine the what influences the amounts of patents filed. Using these four regressions, a clear picture can be formed on what role being a subsidiary has on the amount of patents filed after the COVID-19 crash.

4.6.3 Weighted Regressions

As a robustness check, the four regressions will be run again with an altered dependent variable. The dependent variable will now be weighted, giving more importance to firms with an increased amount of patents filed. This is done to check if divergent results can be found compared to the original four regressions. The idea behind weighting the dependent variable is that there is a possibility that firms with high patenting activities might have a slightly different reaction compared to the average firms. The weighted method is used instead of the clustering method used in the first set of regressions. In ordinary OLS, the goal is to minimise the sum of the squared residuals. The mathematical formula is as follows:

$$\min_{\beta} \sum_{i=1}^{n} (\text{patents_filed}_i - X_i\beta)^2$$

When introducing weights, the formula is altered to:

$$\min_{\beta} \sum_{i=1}^{n} w_i(\text{patents_filed}_i - X_i\beta)^2$$

The w_i is added. This value is the number of patents filed by a specific firm divided by the sum of patents filed by all firms. This ensures that firms with more patents filed, and thus a higher weight, have more influence on the estimation of the regression coefficients. The regressions will further remain exactly the same. This altered approach will be a robust way to compare the two sets of results to see whether the outcomes differ at all. If the results remain the same, even after an altered characteristic of the dependent variable, a stronger case can be made in support of the findings of the original regressions.

4.6.4 Further Testing

Further tests will be conducted if the interaction term is significant to understand what drives the significance. The subsidiary group will be split into two subgroups, one subgroup will contain the subsidiaries that exhibited an increase in patenting activities after the COVID-19 crisis and the other subgroup will include subsidiaries that saw their patenting activities decrease after the crisis. This is done to examine if there are any differences when comparing the most resilient subgroup to the subgroup that saw a decrease in patenting activity. The Wilcoxon Rank Sum test will be conducted to examine if the variables "turnover" or "employees" differ in the two subgroups. If one of the variables is significant, it indicates that the two subgroups have different distributions. Therefore, the turnover of a subsidiary or their employee count could effect their resilience.

The second test that will be conducted is also the Wilcoxon Rank Sum test. The subgroups of this test will be the subsidiaries and the public firms that exhibited an increase in patenting activities after the COVID-19 crisis. This way the firms that exhibited the most resilience after the crisis can be compared. The variables will remain "turnover" and "employees".

Lastly, the split of sectors, comparing the firms that exhibited an increase in patenting activities after the COVID-19 crisis to their counterparts, will be presented. This will be done both for the public firms and subsidiaries. This will be a good way to gauge if the sectors play a significant role in driving the resilience of firms.

5 Results

This section will present the findings of the statistical analysis conducted for this study. The preliminary tests as well as the main findings will be included in this section.

5.1 Parallel Trend Assumption

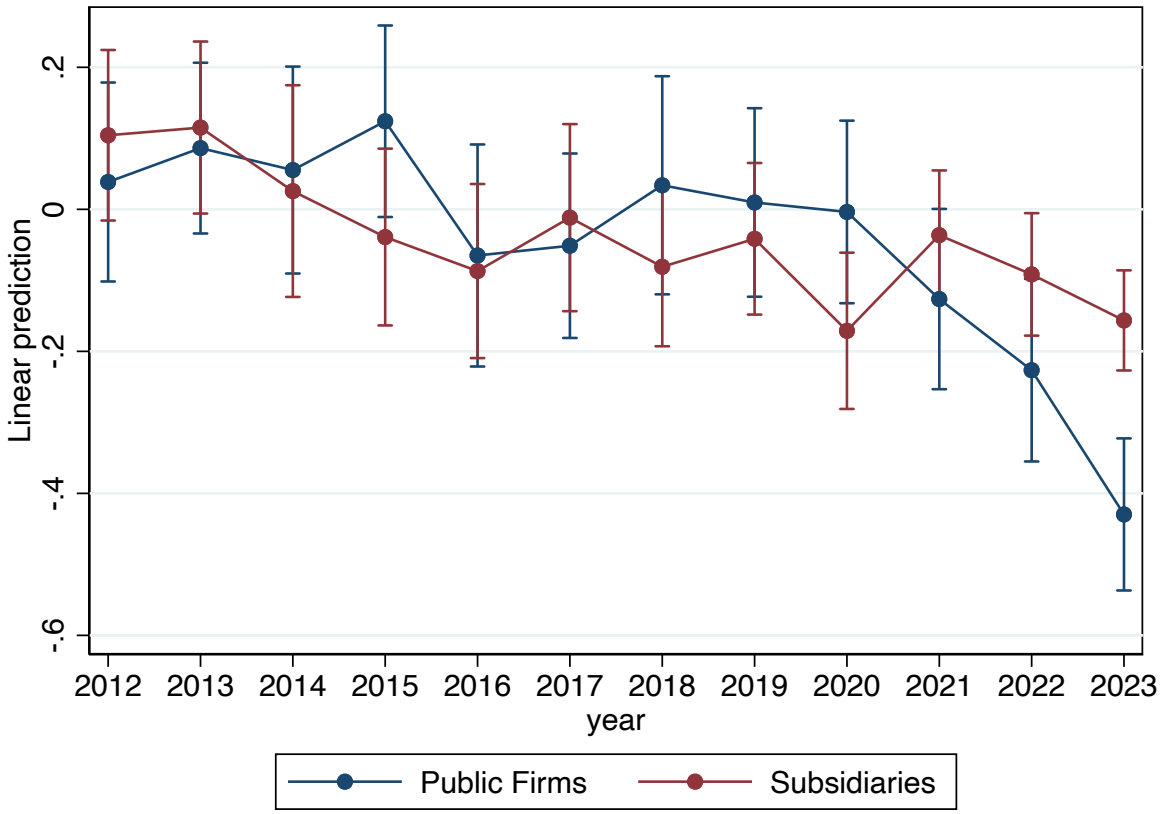


Figure 2: Parallel Trend Assumption, 95% CI

The graph in figure 2 is a visual representation for the parallel trend assumption. The graph shows a parallel trend in all years leading up to the years before COVID-19, with 2015 being the exception. After 2020 there is a clear divergence and shows that public firms and subsidiaries fall out of each others confidence intervals in 2022 and 2023.

5.2 Matching

5.2.1 Wilcoxon Rank Sum Test

Table 3 presents the outcomes of the Wilcoxon Rank Sum test for the unmatched observations. The p-value is 0.000. This indicates a high level of significance. The null hypothesis, that two populations are identical, is rejected. Thus, prior to the matching, the public firms and subsidiaries are not identical for the variable "employees". This can further be seen by the large difference in the median, 344 for public firms and 87.5 for subsidiaries.

Table 4 presents the outcomes of the Wilcoxon Rank Sum test for the matched observations. The p-value is 0.955, indicating non-significance. Thus, the two populations of public firms and

subsidiaries, when looking at the variable "employees", can be deemed identical as the null hypothesis cannot be rejected. This can further be reiterated by the median of both groups, 97.5 for public firms and 97 for subsidiaries.

Table 3: Two-sample Wilcoxon Rank Sum (Mann-Whitney) Test - Unmatched Employees

group	**Obs**	**p25**	**Median**	**p75**
Public	985	65	344	2170
Subsidiary	170	19	87.5	375
Combined	1155	84	431.5	2545
$z = 7.471$	p-value = 0.000			

Table 4: Two-sample Wilcoxon Rank Sum (Mann-Whitney) Test - Matched Employees

group	**Obs**	**p25**	**Median**	**p75**
Public	166	12	97.5	610
Subsidiary	166	20	97	393
Combined	332	32	194.5	1003
$z = -0.057$	p-value = 0.955			

The same tests will be conducted for the turnover variable. Table 5 presents the results of the Wilcoxon Rank Sum test for the unmatched variable "turnover". The p-value is 0.000 indicating a high level of significance. Thus, again for the unmatched set, the null hypothesis is rejected and it can be concluded that there are differences in the populations. This can be confirmed by public firms having a median of 133,590 and subsidiaries having a median of 26,050.

Lastly, table 6 presents the results of the Wilcoxon Rank Sum test for the matched variable "turnover". The p-value being 0.169 shows that there is non-significance and the null hypothesis cannot be rejected. This leading to the conclusion that after matching, for the variable "turnover", there are no significant differences between the populations of public firms and subsidiaries. This once more can be reiterated by comparing the median of the two groups, 31,905 for public firms and 27,458 for subsidiaries.

5.2.2 Pearson Chi-Square Test

The results of the Pearson Chi-Square test are presented in table 7. When looking at the sector split most of the firms have a relatively even split. The biggest difference being 63.64% and 36.36% for retail trade, however this sector only contains 3.3% of all data. The p-value is 0.797 and indicates a high level of significance. The null hypothesis of the Pearson Chi-Square test, there are no significant differences in the distribution of firms across sectors, therefore cannot be rejected. Thus, the distribution of firms between sectors for subsidiaries and public firms do not significantly differ from each other.

Table 5: Two-sample Wilcoxon Rank Sum (Mann-Whitney) Test - Unmatched Turnover

group	Obs	p25	Median	p75
Public	985	9303	133,590	872,198
Subsidiary	170	4990	26,050	112,500
Combined	1155	14,293	159,640	984,698
$z = 5.666$	p-value = 0.000			

Table 6: Two-sample Wilcoxon Rank Sum (Mann-Whitney) Test - Matched Turnover

group	Obs	p25	Median	p75
Public	166	5979	31,905	336,000
Subsidiary	166	5135	27,458	116,000
Combined	332	11,114	59,363	452,000
$z = 1.375$	p-value = 0.169			

5.3 Assumptions

This section will present the results of the tests done to examine if all the assumptions, of an OLS regression, hold.

5.3.1 Residuals and Fitted Values Plot for Linearity

The results of the test for linearity are presented in figure 5. The plot contains the residuals plotted against the fitted values and creates a scatter plot around the zero line. As the figure shows, there is an absence of a systematic pattern, nor a discernible structure in the scatter plot. Thus, by visually inspecting the figure, the conclusion can be made that the relationship between the independent variables and dependent variables is linear.

5.3.2 Wooldrige Test for Autocorrelation

The results for the Wooldridge test for autocorrelation are presented in table 8. The null hypothesis of this test states that there is no first order autocorrelation. As the p-value is 0.00, and thus non significant, the null hypothesis cannot be rejected. This indicates the presence of first order autocorrelation in the data.

5.3.3 Breusch-Pagan/Cook-Weisberg Test for Heteroskedasticity

The results for the Breusch-Pagan/Cook-Weisberg test for heteroskedasticity are presented in the table 9. The null hypothesis states that there is constant variance in the error terms. The p-value is 0.00, and thus non significant. Therefore, the null hypothesis is rejected. This indicates the presence of heteroskedasticity in the model, meaning the variance of the error terms is not constant across observations.

Table 7: Sector Distribution by Group

Sector	**Public Firms**	**Subsidiaries**	**Total**
Construction	4	4	8
	50.00%	50.00%	100.00%
Finance, Insurance, Real Estate	37	48	85
	43.53%	56.47%	100.00%
Manufacturing	64	60	124
	51.61%	48.39%	100.00%
Retail Trade	7	4	11
	63.64%	36.36%	100.00%
Services	41	39	80
	51.25%	48.75%	100.00%
Transportation, Communication	2	3	5
	40.00%	60.00%	100.00%
Wholesale Trade	11	8	19
	57.89%	42.11%	100.00%
Total	166	166	332
	50.00%	50.00%	100.00%

$Pearson\ Chi2(6) = 3.09$ $p - value = 0.797$

Table 8: Wooldridge Test for Autocorrelation

Test	**Result**
H0:	No first-order autocorrelation
F(1, 331)	128.09
Prob > F	0.00

5.3.4 Histogram of Residuals and Q-Q Plot for Normality

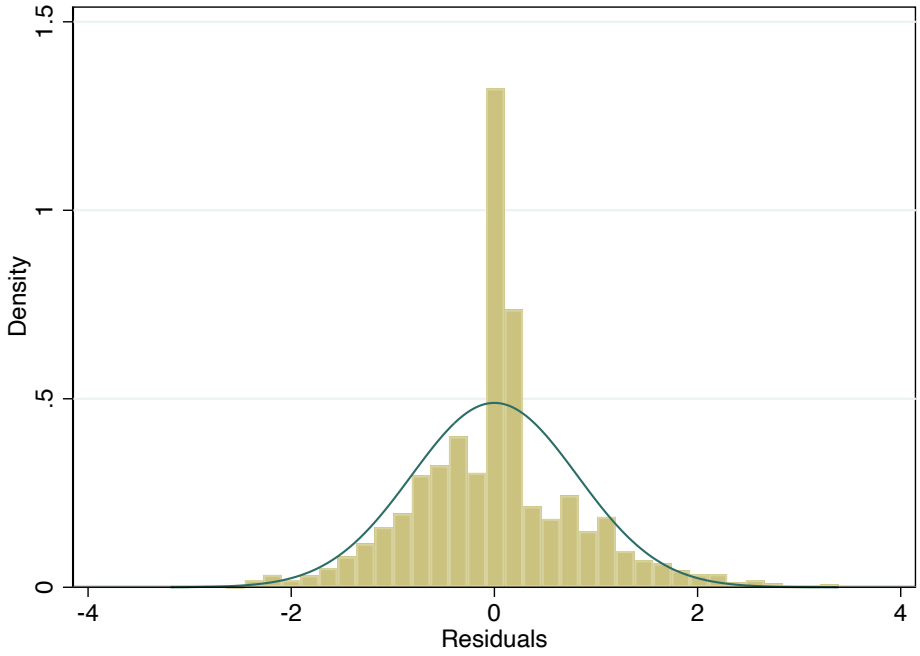
Figure 3 illustrates the distribution of the residuals with a superimposed normal distribution curve. As the figure shows, the residuals are approximately normally distributed. There is a large spike in the middle of the distribution. This suggests a large number of the residuals are concentrated around the mean, which means the residuals are more tightly clustered around the mean when compared to a perfectly normal distribution.

Figure 4 provides further insight into the normality of the data. This Q-Q plot demonstrates that most points lie approximately along the line, with deviations in the tails. This shows that while the residuals are for the most part normally distributed, there are some minor departures from normality in the extremes. When looking at both figures, they display a clear normal shape. Even with the slight departures, overall the combination of the histogram and Q-Q plot suggests the residuals are approximately normally distributed.

Table 9: Breusch-Pagan/Cook-Weisberg Test for Heteroskedasticity

Test	**Result**
Variable:	Fitted values of `logdiff_patent`
H0:	Constant variance
chi2(1)	16.06
Prob > chi2	0.00

Figure 3: Histogram of Residuals with Normal Curve



5.3.5 Variance Inflation Factor Test for Multicollinearity

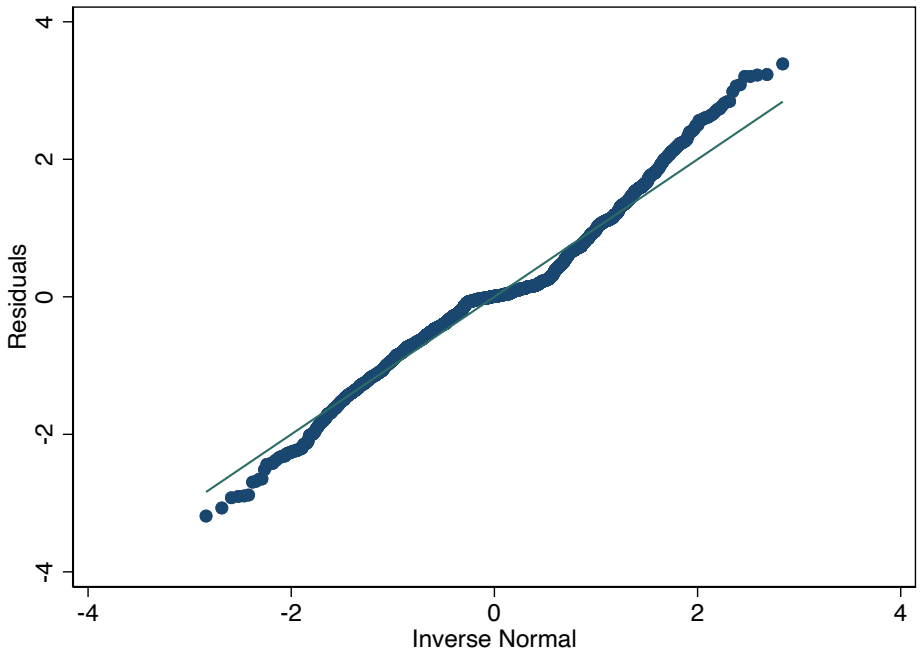
Table 10 shows the results of the variance inflation factor (VIF) test. All of the values are below 5. The highest values being ln_employees and ln_turnover, being 3.87 and 3.96 respectively. Due to all the values being under 5, the conclusion can be drawn that multicollinearity is not a significant issue in this regression model. The mean VIF of 1.56 further supports this claim. The inverse of the VIF is included as another way of interpreting the results, with the value of the inverse representing the % of the variance that is not explained by other predictors. Only ln_employees and ln_turnover have a lower tolerance than 81%, again supporting that there is no multicollinearity in the data.

5.4 Fixed or Random Effects

5.4.1 Hausman Test for Fixed Effects

Table 11 presents the results of the Hausman test for fixed effects vs. random effects. The test has a p-value of 1.00. This means that the p-value is significant and the null hypothesis, the difference in coefficients is not systematic, cannot be rejected. The result of this test states that a random effects model is preferred when compared to a fixed effects model.

Figure 4: Q-Q Plot of Residuals Against Normal Distribution



5.4.2 Breusch and Pagan Test for Random Effects

Table 12 demonstrates the results of the Breusch and Pagan test for Random Effects. The p-value of 1.00 indicates that the null hypothesis, the variance of the random effect is zero, cannot be rejected. This result indicates that the variance of the random effects does not significantly diverge from zero. This means a random effects model is not appropriate.

5.5 Regression Results

5.5.1 Main Regressions

Table 13 presents the results of the main regressions of this research paper. In the first model the variables included are "subsidiary" and "post_covid". The coefficient for "subsidiary" is positive, however it is not significant. The coefficient for "post_covid" is negative (-0.136) and significant at the 1% level. This indicates that being a subsidiary does not have a significant effect on the amount of patents filed, however the COVID-19 crisis significantly decreased the percentage change in patents filed compared to the previous year by 13.6%.

In the second model, the interaction term, between "subsidiary" and "post_covid, DiD is added. After the addition of this variable, the coefficient of "subsidiary" changes to negative (-0.028) and significant at the 10% level. The coefficient of "post_covid" is slightly more negative (-0.189) and remains highly significant at the same level. The coefficient of the newly added interaction term DiD is positive (0.105) and significant at the 1% level. This indicates that subsidiaries experience a 2.8% decrease in the percentage change of patents filed when compared to public firms. The COVID-19 crisis is associated with an 18.9% decrease in the percentage change of patents filed. However, being a subsidiary increased the percentage change of patents filed by 10.5% when compared to public firms.

The third model includes the variable "lagged log_patent". The inclusion of this variable

Table 10: Variance Inflation Factor (VIF) Analysis

Variable	**VIF**	**1/VIF**
logdiff_patent	1.02	0.981
L1.post_covid#subsidiary		
0 1	1.15	0.872
1 0	1.15	0.872
1 1	1.10	0.908
sector		
Mining	1.02	0.976
Construction	1.23	0.810
Wholesale Trade	1.01	0.989
Retail Trade	1.17	0.851
Finance	1.01	0.993
Unknown	1.03	0.970
ln_employees	3.87	0.258
ln_turnover	3.96	0.253
Mean VIF	1.56	

Table 11: Hausman Test for Fixed vs. Random Effects

Coefficient	**Fixed**	**Random**	**Difference**	**Std. Err.**
post_covid	-0.189	-0.189	2.39e-15	0.011
DiD	0.105	0.105	-2.43e-15	0.0156
H0: Var(u) = 0				
Chi-Square (df = 2)			0.00	
Prob > Chi-Square			1.000	

alters the coefficients of the other variables. The coefficient of "subsidiary" is still negative but no longer significant. The coefficient of "post_covid" increases its negative effect (-0.225) and remains significant at the 1% level. The interaction term DiD increases its positive effect (0.113), however decreases to a significant level of 5%. The newly added variable of "lagged log_patent" is negative (-0.392) and is significant at the 1% level. The new variable shows that for a 1% increase in the number of patents filed in the previous year, it is associated with a 0.392% decrease in the percentage change of patents filed in the current year.

The fourth model expands by including more information about the firms. It includes "ln_employees", "ln_turnover" and dummy variables for the relevant sectors. The variable "subsidiary" remains insignificant and the coefficient for "post_covid" remains exactly the same. The coefficient of the interaction term remains the same, however, the standard error slightly increases from 0.047 to 0.048. This change in standard error has no effect on the significance and the interaction term remains significant at the 5% level. The newly added variables "ln_employees" and "ln_turnover" are both insignificant. When looking at the added sector

Table 12: Breusch and Pagan Test for Random Effects

Coefficient	**Fixed**	**Random**
logdiff_patent	0.807	0.898
residual	0.859	0.927
random effect	0	0
H0: Var(u) = 0		
chibar2(01)	0.00	
Prob > chibar2	1.000	

variables, the coefficient of the dummy variable "Services" is positive (0.058) and significant at the 1% level, "Transportation" is positive (0.059) and significant at the 1% level, and "Wholesale Trade" is positive (0.077) at the 1% level. Thus, when adding these explanatory variables the main dependent variable of this study, the interaction term DiD, remains constant and significant. The added sector variables demonstrate that when a firm belongs to the services, transportation or wholesale trade sector they experience a 5.8%, 5.9% and 7.7% increase in the percentage change of patents filed when compared to the base sector manufacturing.

Table 13: Regression Results

VARIABLES	(1) logdiff_patent	(2) logdiff_patent	(3) logdiff_patent	(4) logdiff_patent
lagged log_patent			-0.392***	-0.393***
			(0.015)	(0.015)
subsidiary	0.004	-0.028*	-0.030	-0.030
	(0.011)	(0.017)	(0.023)	(0.024)
post_covid	-0.136***	-0.189***	-0.225***	-0.225***
	(0.019)	(0.030)	(0.038)	(0.038)
DiD		0.105***	0.113**	0.113**
		(0.037)	(0.047)	(0.048)
Construction				0.061
				(0.041)
Finance, Insurance, and Real Estate				0.027
				(0.021)
Retail Trade				0.067
				(0.046)
Services				0.058***
				(0.019)
Transportation				0.059***
				(0.017)
Wholesale Trade				0.077***
				(0.026)
ln_employees				0.008
				(0.007)
ln_turnover				-0.007
				(0.006)
Constant	0.000114	0.016	0.020	0.025
	(0.011)	(0.014)	(0.019)	(0.046)
Observations	4,316	4,316	3,984	3,984
R-squared	0.005	0.006	0.166	0.167
r2_a	0.004	0.005	0.165	0.165

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

5.5.2 Weighted Regressions

Table 14 presents the results of the weighted regression models. The first model includes the variables "subsidiary" and "post_covid". The variable "subsidiary" is insignificant while the coefficient of the variable "post_covid" is negative (-0.352) and significant at the 1% level. This indicates that when using a weighted approach to patents filed, the COVID-19 crisis significantly decreased the percentage change in patents filed compared to the previous year by 35.2%.

The second model adds the interaction term DiD. This addition impacts the variable "subsidiary", its coefficient is now negative (-0.109) and significant at the 1% level. The coefficient of the "post_covid" variable is slightly more negative (-0.406) and remains at the same level of significance. The newly added interaction term has a positive (0.201) coefficient and is significant at a level of 1%. The second regression, when using a weighted approach to patents filed, implies that subsidiaries experience a 10.9% decrease in the percentage change of patents filed when compared to public firms. This model also demonstrates that the COVID-19 crisis is associated with an 40.6% decrease in the percentage change of patents filed. However, being a subsidiary increased the percentage change of patents filed by 20.1% when compared to public

firms.

The third model includes the variable "lagged log_patent". The inclusion of this variable has an impact on the significance of the variables "subsidiary" and DiD. The coefficient of both variables decreases, -0.096 and 0.154 respectively, and is significant at the 5% level. The coefficient of the variable "post_covid" decreases slightly to -0.431 and retains the same level of significance. The newly added variable "lagged log_patent" is negative (0.323) and is significant at the 1% level. This indicates that for a 1% increase in the number of patents filed in the previous year is associated with a decrease of 0.323% in the percentage change of patents filed in the current year.

The fourth model expands on the previous models by including more information about the firms. It includes the variables "ln_turnover" and "ln_employees" as well as dummy variables indicating different sectors the firm can operate in. The variables "lagged log_patent", "subsidiary", "post_covid" and the interaction term "DiD" all remain almost identical. The coefficient for the variable "ln_employees" is positive (0.034) and significant at the 1% level. The coefficient for the variable "ln_turnover" is negative (-0.029) and significant at the 5% level. When looking at all the newly added sector dummy variables, only one is significant. The coefficient of the dummy variable "services" is positive (0.106) and significant at the 1% level. The fourth model builds upon the other three, as it now states that ln_turnover and ln_employees both play a part in patents filed by firms. For a 1% increase in employees, the percentage change of patents filed in the current year increases by 0.034%, while the same increase in turnover will lead to a 0.029% decrease in the percentage change of patents filed in the current year. This model also finds that being part of the service sector leads to 10.6% more patents filed than a firm that operates in the manufacturing sector.

5.5.3 Further testing

Table 17 presents the results of the first Wilcoxon Rank Sum test. The less resilient firms have a higher median, however, the p-value of "turnover" is insignificant. This means that there is no significant difference between the most and less resilient subgroups in terms of turnover. The same result holds for the variable "employees". The less resilient firms have a higher median, however, the p-value is insignificant. When looking at table 18, again, no significance can be found for the variables "turnover" and "employees". This means that when comparing the most resilient subsidiaries to the most resilient public firms, there is no significant difference between the variables in the two groups.

Table 19 and table 20 present what sectors the most resilient and less resilient firms are in. There are no big outliers in the tables. However, there are five subsidiaries that are part of the wholesale trade sector and only one public firm.

Table 14: Weighted Regression Results

VARIABLES	(1) logdiff_patent	(2) logdiff_patent	(3) logdiff_patent	(4) logdiff_patent
lagged log_patent			-0.323***	-0.327***
			(0.015)	(0.015)
subsidiary	-0.047	-0.109***	-0.096**	-0.098***
	(0.032)	(0.038)	(0.038)	(0.038)
post_covid	-0.352***	-0.406***	-0.431***	-0.432***
	(0.031)	(0.036)	(0.034)	(0.034)
DiD		0.201***	0.154**	0.154**
		(0.069)	(0.066)	(0.065)
Construction				0.096
				(0.145)
Finance, Insurance, and Real Estate				-0.047
				(0.036)
Retail Trade				0.123
				(0.123)
Services				0.106***
				(0.037)
Transportation				0.165
				(0.293)
Wholesale Trade				-0.017
				(0.095)
ln_employees				0.034***
				(0.013)
ln_turnover				-0.029**
				(0.011)
Constant	0.020	0.037*	0.001	0.137
	(0.019)	(0.020)	(0.020)	(0.084)
Observations	4,290	4,290	3,960	3,960
R-squared	0.030	0.032	0.134	0.139
r2_a	0.030	0.032	0.133	0.137

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

6 Discussion

This section discusses the results and their implications, focusing mainly on the key findings from the regression results and the various tests conducted to validate the regression models.

6.1 Parallel Trend Assumption

The graph in figure 2 is a visual representation of the parallel trend assumption. The firms follow a parallel trend in years leading up to 2020 and diverge in the years 2022 and 2023. This divergence suggests that the impact of the COVID-19 crisis had a significantly different impact on both groups. This significant divergence justifies the use of an interaction term in the subsequent analyses in order to capture these differential effects. However, in 2015 there is a slight deviation from the parallel trend assumption. This slight deviation is not particularly concerning for this study for several reasons. Firstly, the magnitude of the deviation is small, as can be observed, it only narrowly falls outside of the confidence intervals and still largely follows the trend. Secondly, the deviation is temporary. In 2016 the trend resumes immediately and continues from there. This suggests that the deviation does not indicate a sustained difference in trends. Lastly, this study mainly focuses on the period during and after the COVID-19 crisis. This temporary diversion in 2015 does not strongly impact the main results of this study. These terms provide this study with enough confidence to use an interaction term to analyse the effects of the COVID-19 crisis on patenting trends.

6.2 Matching

The Wilcoxon Rank Sum tests have been conducted to compare the distributions of the variables "employees" and "turnover" between the public firms and subsidiaries, both before and after matching. Before the matching, significant differences can be observed. This indicates that the groups initially are not comparable. This can be seen by both the big differences in median ranks and the highly significant outcome of the test. After matching, the median rank values are almost identical and the result of the rest is insignificant, indicating that the matching process is deemed successful and that both groups now have a compatible distribution and are ready for the matched regression analysis.

The Pearson Chi-Square test is used to determine the differences in distribution of firms across sectors between public firms and subsidiaries. A visual inspection and the result of the test indicate that there are no significant differences in the distribution, meaning there is an even split of firms amongst the sectors. This further supports the conclusion that the matching process was successful.

6.3 Assumptions

The linearity is a fundamental assumption for an OLS regression. The results of the test for linearity is presented in figure 5. This figure plots residuals against fitted values to create a scatter plot around the zero line. The figure shows there is no systematic pattern. This suggests that the relationship between the independent variables and dependent variable is linear. By visually inspecting this figure, it can be concluded that the linearity assumption does in fact hold.

The assumption for autocorrelation is tested with a Wooldrige test in table 8. The null hypothesis states that there is no first order autocorrelation. The result indicates that the null hypothesis is rejected and that autocorrelation is found in the data. This is an important finding because autocorrelation is a violation of the OLS assumptions. This violation can lead to biased standard errors and can invalidate the findings. To address this, clustered standard errors have been used in the regression to account for the correlation within firms over time. The use of clustered standard errors is vital to be able to draw reliable conclusions from the results.

To test for heteroskedasticity the Breusch-Pagan/Cook-Weisberg test was used. The null hypothesis states that there is constant variance in the error terms. The results indicate that the null hypothesis is rejected and thus, heteroskedasticity is present. This means that the variance of the error term is not constant across observations, another important assumption of an OLS regression. This violation can lead to inefficient estimates and biased standard errors. Again, clustering will be utilised to deal with this violation. By clustering the standard errors, the heteroskedasticity can be addressed and thus reliable conclusions can be drawn from the results.

The figure 3 presents the distribution of the residuals with a superimposed normal curve, and figure 4 plots a Q-Q plot. Both of these figures are used to determine normality in the data set. The first figure shows that the residuals follow an approximate normal distribution. However, there is a large spike that can be observed in the middle of the data. This spike indicates a large concentration of residuals around the mean. Thus, there is tighter clustering than in a perfectly normal distribution. This spike could be attributed to the many zero values included in the data. When working with patenting data this is common and thus is not of any concern for violating the normality assumption. The second figure demonstrates that most points approximately lie along the line. There are deviations at the ends of the figure, suggesting deviations in the tails. This means that there are slight departures from normality in the extremes. Despite the spike in figure 3, and the deviations in the tails in figure 4, the combination of the two graphs suggests overall that the residuals are approximately normally distributed.

The results of the VIF test for multicollinearity are presented in table 10. Multicollinearity can inflate the standard errors of the coefficients. This can make it hard to determine the significance of individual predictors. When looking at the table, all the values are below 5, indicating no significant multicollinearity in the regression model. The highest values of "ln_employees" and "ln_turnover" being 2.87 and 3.93 respectively, both not even approaching a concerning level. The mean VIF of 1.56 further supports this conclusion. When looking at the inverse VIF, also called the tolerance, which represents the percentage of variance not explained by other predictors, it also supports the idea of multicollinearity not being substantial. Only "ln_employees" and "ln_turnover" have a tolerance lower than 81%. All of these factors confirming the fact that the assumption of multicollinearity is not violated.

The various tests conducted to examine the assumptions of the OLS regression, confirm that almost all assumptions hold for this study. The violated assumptions, autocorrelation and heteroskedasticity, have been compensated for using clustered standard errors. The thorough examination and adjustments of the violations of the assumptions is a way to ensure the validity of the regression results. By conducting these vigorous tests, the confidence in the findings of this study is strengthened. This provides a strong foundation to draw valid and meaningful conclusions on the impact of the COVID-19 crisis on patenting trends when comparing subsidiaries to public firms.

6.4 Fixed or Random Effects

Two tests are conducted to determine whether a fixed effect (FE) or random effect (RE) model should be used: the Hausman test in table 11 and the Breusch and Pagan test in table 12. It is important to determine whether fixed or random effects are present in the data because these then need to be accounted for. The null hypothesis of the Hausman test is that the difference in coefficients between the fixed and random effects model is not systematic. The result of the test indicates that the null hypothesis cannot be rejected, meaning there is no significant difference when comparing the coefficients when using a FE model and a RE model. Therefore, because a RE model is more efficient, an RE model is preferred.

The null hypothesis of the Breusch and Pagan test is that variance of the random effect is zero. Table 12 presents the results of this test. The results indicate that the null hypothesis cannot be rejected and thus the variance of the random effect is zero. This indicates that the random effects model does not include any significant explanatory power when compared to a model without accounting for random effects. This indicates that a RE model might not be appropriate for this dataset. Due to the findings of these models a regular OLS will be used, without accounting for FE or RE.

6.5 Regressions

The result of the main regressions will provide clarity on the effect that COVID-19 had on innovation, as a measure of patents filed and the difference of resilience of subsidiaries and public firms. The analysis utilises the framework of Conz and Magnani (2020), which suggests taking the temporal dimension, contributions to resilience, and analysing the outcomes of being resilient, into account.

Hypothesis 1: *"COVID-19 had a negative impact on innovation."*

The first hypothesis is an important pillar of this research. Table 13 shows that in all models COVID-19 had a negative impact on the amount of patents filed and is significant at the 1% level. The first model indicates a 13.6% decrease in the percentage change of patents filed compared to the previous year. This negative effect becomes even stronger in the subsequent models, reaching a 22.5% decrease in the third and fourth model. This suggests that COVID-19 significantly had a negative impact on innovation.

Hypothesis 2: *"Subsidiaries of PE firms are more resilient during a crisis than public firms."*

The second hypothesis is tested using the interaction term "DiD". The interaction term is introduced in the second model and is significant at the 1% level. This indicates that COVID-19 had a lesser impact on subsidiaries when compared to public firms. The second model suggests that subsidiaries file 10.5% more patents than public firms after the COVID-19 crisis. In the third and fourth model this positive effect remains, although now significant at the 5% level. The effect also increases to 11.3%. These results support the hypothesis that subsidiaries of PE firms are more resilient during a crisis than public firms.

This study captured the impact of the COVID-19 crisis by comparing the periods before and after the crisis. This is in line with the framework of Conz and Magnani (2020). Furthermore, this study examines whether being backed by a PE firm enhances resilience. The significant interaction term indicates that subsidiaries exhibit greater resilience during times of crisis when comparing them to public firms, thus concluding that being backed by a PE firm contributes to a firms resilience. Lastly, this study shows that being resilient has perks when operating in

times of crisis. Using this framework to analyse the resilience of firms will hopefully provide a more comprehensible and uniform understanding of its mechanics and benefits.

The fourth model includes the sectors. The services, transportation and wholesale trade sectors are all positive and significant at the 1% level. This indicates that these sectors file more patents than the base sector manufacturing. Specifically, firms in the services sector saw an increase of 5.8%, firms in the transportation sector saw an increase of 5.9% and the those in the wholesale trade sector saw an increase of 7.7%. These sectors may also be more resilient in times of crisis, however this claim requires further research to validate.

Table 14 presents the results of the weighted regression. This is done to check if other results emerge if the manner in which the research has been done, changes. Mostly, the main results remain the same. When checking the first hypothesis, the COVID-19 crisis has a significant impact at the 1% level for all four models. The first model indicates that the COVID-19 crisis decreased the percentage change of patents filed compared to the previous year by 35.2%. In the last model this effect increases to 43.2%. Thus, when increasing the influence of firms with high patenting activity, the negative effect of COVID-19 increased. In this weighted model the first hypothesis still holds.

The interaction term for the weighted regression remains significant in all models it is included in. The interaction term is included for the first time in the second model. The second model suggests that subsidiaries file 20.1% more patents than public firms after the COVID-19 crisis and is significant at the 1% level. The third and fourth model both indicate that subsidiaries file 15.4% more patents than public firms after the COVID-19 crisis. These models are both significant at the 5% level. The second hypothesis still holds in this altered regression. The main effects this paper focuses on have become even stronger and are in line with the hypotheses.

There are other interesting variable coefficients to note. The weighted regression shows significance in the second, third and fourth model for the effect of being a subsidiary. The effect is negative and significant at the 1%, 5% and 1% levels, respectively. This means that when putting more importance on firms with higher patenting activities, there is a significant negative effect for being a subsidiary. The new models also demonstrate significance at the 1% level for operating in the service sector. However, the sectors transportation and wholesale trade are no longer significant. The variables that account for employees and turnover are significant in this new model. The variable that accounts for employees shows that for a 1% increase in employees, the percentage change of patents filed in the current year increases by 0.034%. This effect is significant at the 1% level. The variable that accounts for turnover demonstrates that for a 1% increase in turnover, a 0.029% decrease in filed patents in the current year will follow. This effect is significant the 5% level.

The main regression and the weighted regression provide evidence supporting the hypotheses of this paper. This research expands on the discussion whether PE firms are beneficial or harmful to the finance world. During a crisis, maintaining previous levels of innovation could potentially boost firms when in the recovery phase. This research indicates that PE firms contribute to less detrimental economic downturns by supporting firms that otherwise would have difficulty in navigating these hard times. These findings can also be useful to different groups of financial players. The results can help investors make decisions about how to allocate resources and choose which portfolio companies to invest in. Policymakers can utilise these findings to learn how PE firms boost the resilience of its subsidiaries. Therefore, these findings could influence economic policies aimed at creating a more resilient and innovative business environment. Investigating the resilience of PE-backed firms is not only academically interesting but also

applicable in practice. This newfound knowledge can contribute to real financial stability and long-term growth.

The further tests aim to further explain the significance of the interaction term. When comparing the most resilient subsidiaries to less resilient subsidiaries, there are no significant differences when looking at the firms turnover and employees. This means that turnover and employee count are not significant drivers of resilience during the COVID-19 pandemic. Furthermore, when comparing the most resilient subsidiaries to the most resilient public firms, again, no significance can be found. Thus, it can be concluded that the significance of the interaction term is driven by factors beyond the firm's size and financial capacity. The sectoral analysis demonstrates that some sectors, like wholesale trade, may have a higher concentration of resilient subsidiaries when comparing them to public firms. However, this will need to be further explored. There are far too few observations to say anything concrete about this. These findings point to the need for a new study on what the drivers are of firm resilience and how being owned by a PE firm enhances it. Including information about the management style, the amount of capital injections the subsidiary has had and the size of the PE firm are examples of ways to explore the drivers of the significance further.

6.6 Limitations and Future Research

This research uses the COVID-19 crisis as a way of examining the resilience of firms during crises. This can be a limitation as only this crisis was used and other crises were not taken into account. This can also be an avenue for further research as it would be interesting to see whether the same results would be obtained when using this method of research on different financial crises. Another limitation is that the data was primarily gathered manually. Despite rigorous double checks being conducted, there remains a possibility that some observations were not properly recorded. The possibility of human error always remains a threat. This could be avoided in the future by applying more advanced automated data collection methods. Another limitation could be the variables included in the models. As this research focuses on PE firms, not much data is freely available. More research could be conducted using more information on the firms and using different variables. Other interesting avenues for future research would be to separate different sized PE firms, to see whether bigger PE firms provide an increased amount of resilience to their subsidiaries compared to boutique PE firms. Furthermore, redoing this research in a few years' time, it would be interesting to see if other results would be obtained when more data would be available about the longer-term effects of the COVID-19 crisis.

7 Conclusion

This study investigates the impact of the COVID-19 crisis on innovation, specifically the amount of patents filed, within public firms and subsidiaries of private equity firms. This is done to compare the resilience of these two groups during times of crisis. The analysis utilises the framework proposed by Conz and Magnani (2020), which emphasises researching firm resilience by looking at resilience through temporal dimensions, capabilities that contribute to resilience and the outcomes of resilience.

The study is conducted by using an OLS regression analysis. The subsidiaries of private equity firms are matched to public firms by using propensity score matching. An interaction term is included to examine whether there is a difference in resilience between the two groups. The main findings of this research confirm the first hypothesis: "COVID-19 had a negative impact on innovation". The result demonstrates a significant decrease in patent filings after the COVID-19 crisis. The second hypothesis: "Subsidiaries of PE firms are more resilient during a crisis than public firms", can be confirmed as well. The significant interaction term finds that subsidiaries exhibited a relatively smaller decrease in patent filing compared to public firms. This implies that subsidiaries of private equity firms demonstrate greater resilience during the COVID-19 crisis compared to their public counterparts. This aligns with the second hypothesis.

In conclusion, this research examines the negative impact that COVID-19 had on innovation while focusing on the relative resilience of subsidiaries backed by private equity firms. These findings contribute to the discussion, "do private equity firms have an overall positive or negative impact on the financial ecosystem", and broaden the understanding of firm resilience during a times of crisis.

8 Appendix

Table 15: Descriptive Statistics for logdiff_patent and patent

Statistic	**logdiff_patent**	**patent**
Mean	-0.0397243	6.8443
Std. Dev.	0.8984576	20.09742
Variance	0.8072261	403.9062
Skewness	0.064062	7.08979
Kurtosis	4.530755	75.10706
Observations	4,316	4,316

Table 16: SIC Sector Codes Classification

Division	**Title**	**Two Digit Major Group**
A	Agriculture, Forestry and Fishing	01-09
B	Mining	10-14
C	Construction	15-17
D	Manufacturing	20-39
E	Transportation, Communication, Electric, Gas and Sanitary Services	40-49
F	Wholesale Trade	50-51
G	Retail Trade	52-59
H	Finance, Insurance and Real Estate	60-67
I	Services	70-89
J	Public Administration	91-97

Figure 5: Residuals vs. Fitted Values Test for Linearity

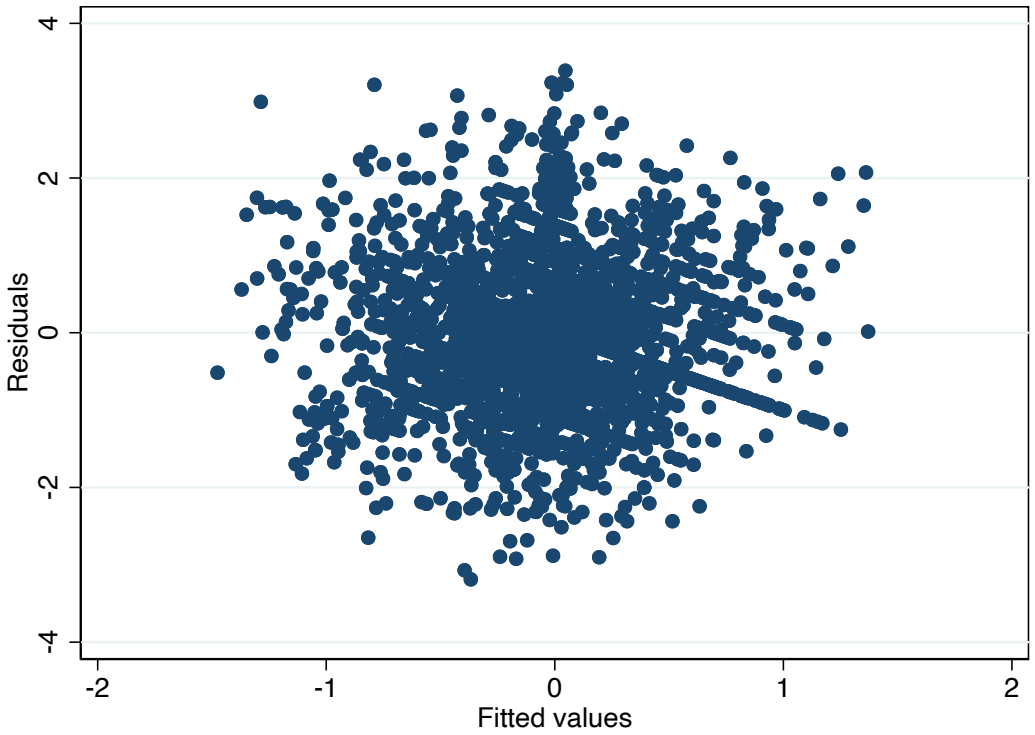


Table 17: Comparing Most to Least Resilient Subsidiaries

	N	p25	Median	p75
Turnover				
Less Resilient	111	5,110	37,500	142,000
Most Resilient	55	5,135	23,400	75,600
p-value: 0.223				
Employees				
Less Resilient	111	20	100	450
Most Resilient	55	17	80	301
p-value: 0.557				

Note: The p-values are based on the Wilcoxon rank sum test.

Table 18: Comparison of Resilient Firms

	N	p25	Median	p75
Turnover				
Public	49	2,060	25,000	186,995
Subsidiary	55	5,135	23,400	75,600
p-value: 0.780				
Employees				
Public	49	8	53	267
Subsidiary	55	17	80	301
p-value: 0.327				

Note: The p-values are based on the Wilcoxon rank sum test.

Table 19: Sectors of Public Firms

Sector	**Less Resilient**	**Most Resilient**	**Total**
Construction	3	1	4
	75.00%	25.00%	100.00%
Finance, Insurance, Real Estate	27	10	37
	72.97%	27.03%	100.00%
Manufacturing	42	22	64
	65.63%	34.38%	100.00%
Retail Trade	7	0	7
	100.00%	0.00%	100.00%
Services	27	14	41
	65.85%	34.15%	100.00%
Transportation, Communication	1	1	2
	50.00%	50.00%	100.00%
Wholesale Trade	10	1	11
	90.91%	9.09%	100.00%
Total	**117**	**49**	**166**
	70.48%	**29.52%**	**100.00%**

Table 20: Sectors of Private Firms

Sector	**Less Resilient**	**Most Resilient**	**Total**
Construction	4	0	4
	100.00%	00.00%	100.00%
Finance, Insurance, Real Estate	33	15	48
	68.75%	31.25%	100.00%
Manufacturing	42	18	60
	70.00%	30.00%	100.00%
Retail Trade	3	1	4
	75.00%	25.00%	100.00%
Services	23	16	39
	58.97%	41.03%	100.00%
Transportation, Communication	3	0	3
	50.00%	50.00%	100.00%
Wholesale Trade	3	5	8
	37.50%	62.50%	100.00%
Total	**111**	**55**	**166**
	66.87%	**33.13%**	**100.00%**

Table 21: Two-sample Wilcoxon Rank Sum Test Raw Output - Unmatched Employees

group	**Obs**	**Rank sum**	**Expected**	**Median Rank**
Public	985	599335.5	569330	6902.5
Subsidiary	170	68254.5	98260	1190.5
Combined	1155	667590	667590	
Unadjusted variance		16131017		
Adjustment for ties		-311.81653		
Adjusted variance		16130705		
		$z = 7.471$ Prob $> \|z\| = 0.000$		

Table 22: Two-sample Wilcoxon Rank Sum Test Raw Output - Matched Employees

group	**Obs**	**Rank sum**	**Expected**	**Median Rank**
Public	166	27589.5	27639	1162.5
Subsidiary	166	27688.5	27639	1162.5
Combined	332	55278	55278	
Unadjusted variance		764679.00		
Adjustment for ties		-107.82		
Adjusted variance		764571.18		
		$z = -0.057$	Prob $> \|z\| = 0.955$	

Table 23: Two-sample Wilcoxon Rank Sum Test Raw Output - Unmatched Turnover

group	**Obs**	**Rank sum**	**Expected**	**Median Rank**
Public	985	592084.5	569330	6895.5
Subsidiary	170	75505.5	98260	1190.5
Combined	1155	667590	667590	
Unadjusted variance		16131017		
Adjustment for ties		-672.81766		
Adjusted variance		16130344		
		$z = 5.666$	Prob $> \|z\| = 0.000$	

Table 24: Two-sample Wilcoxon Rank Sum Test Raw Output - Matched Turnover

group	**Obs**	**Rank sum**	**Expected**	**Median Rank**
Public	166	28841	27639	1162.5
Subsidiary	166	26437	27639	1162.5
Combined	332	55278	55278	
Unadjusted variance		764679.00		
Adjustment for ties		-5.52		
Adjusted variance		764673.48		
		$z = 1.375$	Prob $> \|z\| = 0.169$	

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