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Dynamic Portfolio Allocation

Navigating through Inflationary Times with Commodity Investments

In this thesis, I examine whether adding commodities to an all-equity portfolio enhances its performance. Using S&P 500 and GSCI Commodity Index data from 2014 to 2024 for mean-variance optimization, I find that the tangency portfolio significantly improves the monthly Sharpe ratio by 0.362 when commodities are included in the portfolio. The tangency portfolio invests in commodities only in 2016 (1.6%), 2021 (26.9%), and 2022 (100%), while the global-minimum variance and Black-Litterman portfolios consistently include commodities. The tangency portfolio also earns a positive monthly real return of 0.8%, indicating hedging properties. However, investing equally in equities and commodities leads to a monthly real return of 0.05%, suggesting an effective hedge, as a real return of 0% is desired. Although a 1% inflation increase leads to an insignificant 1.444 percentage point rise in commodity allocation, this influence reverses to -3.942 points during high inflation (2021/2022). Nevertheless, these findings lack statistical significance, suggesting cautious application.

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Preface and Acknowledgements

A project usually falls short of its expectation unless guided by the right person at the right time given opportunities to explore and learn. Not only was this Thesis insightful from a finance perspective, but also it has taught me how to properly examine research. It has also taught me time management and efficiency. Therefore, I am indebted to Dr. Jan Lemmen for guiding me through difficulties and providing the necessary assistance if needed. Furthermore, remaining consistent and critical, as Dr. Jan Lemmen was, helped me to overcome hurdles and master them in an effective way. Additionally, I want to thank my family as well as my partner for continuously believing and supporting me.

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1 Introduction

On March 17, 2022, the US Federal Reserve System increased interest rates by 25 basis points, from 0.25% to 0.50%, trying to tackle heightened inflation. This triggered a series of interest rate hikes in response to rising prices in energy, wheat, fertilizer, and particularly, commodity prices (Bonacina & Civitella, 2023). The main driver of the inflation thereby was the invasion of the Ukraine by Russia, compelling an incremental total raise of interest rates from 0.25% to 4.5%. This recent trend of increasing interest rates led to increasing volatility in the markets. Hence, equity investors seek various ways to diversify their equity portfolio to reduce risk, hedge against inflation and its accompanied uncertainty. Less recent studies highlight an inverse relationship between high-inflationary periods and equity returns (Bodie, 1979; Fama, 1981; Feldstein, 1983). Consequently, investors are concerned about high inflation and seek for strategies to hedge against potential real losses in their portfolios resulting from hightened inflation. While including bonds in the portfolio is seen as a traditional hedge against losses in stock returns, this approach may be dismissed due to a positive correlation between stocks and bonds during high inflationary periods (Andersson et al., 2008). Generally, the correlation between stocks and bonds is negative. However, Cai et al. (2023) find that the negative correlation between stocks and bonds appeard to reverse in the wake of the pandemic during low inflation. Hence, there seems to be confusion around the traditional strategy of hedging against inflation. As an alternative, a more effective strategy might involve diversification by partially investing in commodities or commodity futures. Notably, Bodie (1983) emphasizes the potential effectiveness of commodity futures during times of high unexpected inflation. Other reasearch shows that commodity future returns are comparable to equity returns over the long run, providing diversification benefits without dismissing returns (Gorton & Rouwenhorst, 2006; Jensen et al., 2002; Jensen et al., 2000). Additionally, some studies show that commodity futures are suitable in providing a robust hedge against inflation in an equity portfolio during high-inflation periods (Bhardwaj et al., 2015; Conover et al., 2010; Gorton & Rouwenhorst, 2006). More specifically, Conover et al. (2010) emphasize that a 15% allocation to commodities is the most effective hedge, however this study seems to be outdated. Hence, adding commodities to ones portfolio may enhance diversification benefits without dismissing returns and further hedges against inflation during high-inflationary times.

Despite extensive literature on commodities and equity, there is a lack of research specifically examining the time period of heightened inflation during 2021 and 2022. Thus, this thesis aims to fill that gap by making a more specialized contribution to the literature of the relationship between commodities and equity during inflationary pressure. Moreover, not only do I provide theoretical relevance, but also do I shed light on the societal perspective by providing optimal solutions to retail as well as institutional investors in difficult times.

Building upon prior research, my primary focus is to analyze daily US stock and commodity returns, encompassing the period from the 1st of January 2014 to the 1st of January 2024. By doing so I fulfill the statistical properties to significantly state a conclusion as I conduct over 120 monthly portfolio weights. I will specifically delve into daily information related to stock and commodity market performances, inflation, and other control variables including volatility, liquidity, and market sentiment. Furthermore, I use FED interest rates to compute excess returns. Employing the SPDR S&P500 ETF Trust (Ticker: SPY) as a suitable proxy for the S&P500 and the iShares S&P GSCI Commodity-Indexed Trust (Ticker: GSG) for a commodity index, I perform the mean-variance optimization according to Markowitz (1952) to construct the monthly tangency and the global-minimum variance (GMV) portfolio. Additionally, I follow the approach of Black and Litterman (1992) to further incorporate market views and confidence levels of investors. Hence, the purpose of this research inquiry is to investigate the fluctuations of portfolio weights for different portfolios over time depending on variations of inflation, particularly focusing on the surge of inflation in 2021/2022. Therefore, I pose the following research question: *"How can equity portfolio weights be strategically optimized by adding commodity futures to hedge effectively against inflation during high-inflationary times?"*

The literature reveals interesting results when it comes to commodity investments as a complement to equity portfolios. As commodities correlate positively with inflation, I expect the allocation of commodities to increase for heightened inflation. Moreover, I expect the allocation of the tangency portfolio to commodities to be around 5% to 15%, as these weights offer the best diversification benefit according to recent studies. Nevertheless, the empirical results shed interesting light on the portfolio weights across different types of portfolios. First, on a yearly basis, the tangency portfolio invests only in three out of ten years in commodities. Two of the three years fall into the period of 2021 and 2022. In numbers, the tangency portfolio allocates 1.6% (2016), 26.9% (2021) and 100% (2022) into commodities. On the other hand, the GMV invests continuously in commodities over the sample period with an increased contribution in 2020 and 2022. Complementary, the Black-Litterman portfolio invests each year into commodities reaching from 2.7% allocated to commodities (2023) to 9.5% allocated to commodities (2016). The Sharpe ratios significantly improve only for the tangency portfolio over the sample period showcasing an improvement of the Sharpe ratio by 0.362 per month. Furthermore, the weights for commodities in the tangency portfolio react positively (1.444 percentage points) to an increased inflation of 1%, whereas the GMV reacts negatively (-2.687 percentage points). The reversed pattern can be found for the unexpected component of the inflation, as the tangency portfolio decreases its allocation to commodities by 0.665 percentage point for every 1% increase in unexpected inflation. On the other hand, the GMV portfolio increases its allocation by 0.613 percentage points for every 1% increase in unexpected inflation.

This thesis is outlined as follows: First, I delve further into the state of the art concerning modern portfolio theory and inflation dynamics (Chapter 2). I complement Chapter 2 with developing the relevant hypotheses. Second, I present the relevant data and its limitations in Chapter 3. Third, I introduce the methodology (Chapter 4) applied including the mean-variance spanning, the Black-Litterman portfolio, the studentized and circular block bootstrap methods as well as the regression analysis conducted to test the hypotheses. Fourth, I present the main empirical results in Chapter 5. Lastly, I follow up the thesis with an elaborate discussion around the main findings, which can be found in Chapter 6. To get more insights into the empirical results, I refer to the Appendix.

2 Theoretical Framework

2.1 Literature Review

This section presents the relevant literature for the reader. The literature review expands on the modern portfolio theory and their extensions, inflation dynamics, and comprehensive knowledge regarding commodities, including their diversification benefits and their capacity to hedge against inflation. The information gathered in this section aims to encompass all the necessary knowledge for understanding the theoretical background of this thesis. Following the literature review, I develop several hypotheses that need to be tested throughout the inquiry.

2.1.1 Modern Portfolio Theory

Portfolio management has been subject to extensive scrutiny and analysis by researchers in recent decades. Central to this discourse is the inquiry into the most effective investment strategy and asset allocation to optimize returns while mitigating risk. Markowitz's modern portfolio theory (Markowitz, 1952) is a significant milestone in finance, shaping optimal portfolio construction, asset allocation and investment diversification. Mainly focusing on the so-called "second stage" of a conventional portfolio selection process, the author discusses the beliefs of an investor of future performances based on past observations and experiences. Consequently, he defined several crucial assumptions: the first assumption is that an investor always maximizes expected returns, whilst for the second assumption an investor favors expected returns while he/she dislikes variance of the returns. Subsequently, Markowitz defines the concept of the mean-variance efficiency as a combination of N-securities in a portfolio. The spectrum of these combinations varies with the weights assigned to each asset in the portfolio and the covariances amongst the assets. Hence, the mean-variance model helps an investor to find the optimal asset allocation with the highest expected return subject to a specific level of risk or, equivalently, minimizing risk subject to earning a certain level of return. Markowitz (1952) further demonstrates, that if assets are not perfectly correlating ($\rho < 1$), the combination of any possible portfolio weight allocation forms a curve, also known as the efficient frontier. The difference in standard deviation between two perfect correlating assets and two non-perfect correlating assets equals the diversification benefit, given a certain level of return. Including portfolio weight constraints, assets and other factors, the efficient frontier can be altered accordingly. Another view of diversification benefits gives the alteration of the position of the efficient frontier: A shift to the left in the mean-variance world means reducing risk while still maintaining the expected return. Additionally, an investor seeks to move his/her efficient frontier to the upper left corner in the mean-variance world, where expected returns is high while risk maintains low. Markowitz's groundbreaking theory not only reshaped the landscape of portfolio management but also initiated a marathon of scholarly exploration in modern portfolio theory.

The process of investment choice following the mean-variance approach can be delineated into two stages: firstly, an investor can choose an unique optimum of a combination of risky assets, or secondly, an investor can invest into a combination of funds that incorporates risky assets and a risk-free asset (Tobin, 1958). This approach allows investors to leverage their position in a risky asset by short selling the risk-free asset. In addition to Markowitz's (1952) framework and as a response to Tobin (1958), Sharpe (1964) further develops modern portfolio theory. In equilibrium, where asset prices have adjusted, an investor that follows the principles of diversification can attain any desired outcome on a *"capital market line"*. Hence, an investor obtains higher returns only by incurring additional risk. Consequently, Sharpe (1964) finds a way to extend the models of past research and construct a market equilibrium theory of assets under conditions of risk. This contribution was named the Capital Asset Pricing Model (CAPM). Despite its relevance in portfolio selection and understanding the pricing of capital assets under risk, as well as the behavior of stock-market prices, the mean-variance approach has not been extensively utilized in evaluating mutual fund performance. Building upon the work of Treynor (1961), Sharpe (1966) proposes a theoretical performance measure for mutual funds that comprises expected returns and risk combined, also known as the Sharpe ratio. For this, the excess return of an asset is put in relation to its risk, measured in standard deviation. Hence, the result is a number greater than zero (if not, the investor would initially invest all into the riskless asset as the return of the risk-free asset exceeds the return of the risky asset). The Sharpe ratio tells us what additional unit of return we can expect for incurring one unit of additional risk. The groundbreaking work of Markowitz (1952), Tobin (1958), Treynor (1961) and Sharpe (1966) lay the foundation of this paper as I mainly focus on the approach of Markowitz and test its performance with the Sharpe ratio, while checking for consistency using the Treynor Ratio.

However, the methodology of finding the optimal portfolio using a mean-variance approach has recently received much criticism by researchers. For example, Bai et al. (2009a) find that the estimated optimal portfolio permanently overestimates the expected returns, especially when the portfolio consists of a large number of assets. The authors provide an improved estimator that is computed using a bootstrap method to address the issue of parameter uncertainty, which is also illustrated by Zhu et al. (2019). Consequently, they apply a scaled covariance estimator $w(c)$ with a constant optimal parameter c, which minimizes the loss of utility and the mean square error. Complementing the work of Bai et al. (2009a), Bai et al. (2009b) estimate the optimal portfolio for self-financing portfolios. Nevertheless, these scholars are mainly focusing on adjustments in the approach and the corresponding methodology. However, not only the approach and methodology are crucial for the mean-variance optimization but also the consideration of potential portfolio constraints.

For example, Markowitz (1952) already implements short selling constraints where portfolio weights cannot be negative. Ever since, many other approaches have been proposed or further elaborated on to improve the performance of the mean-variance model by constraining portfolio weights. For instance, Best and Grauer (1991) also apply a non-negativity constraint and they find that it improves the performance, remaining consistent with the findings of Markowitz (1952). Furthermore, Grauer and Shen (2000) demonstrate that introducing constrains lead to appreciably more diversification and less realized risk, but only in exchange for a lower expected return. Jagannathan and Ma (2003) delve deeper into Markowitz's (1952) short selling constraints and explain why short selling constraints can reduce risk in estimated optimal portfolios.

Recent advancements have significantly advanced modern portfolio theory. Markowitz (1952) establishes the groundwork by determining optimal portfolio weights in an N-portfolio scenario. These models including the mean-variance approach as well as the CAPM model, offer the possibility of portfolio optimization from a rational point of view. However, each investor has different views and confidence levels about the relative performance of an asset. Hence, it is important to incorporate these views into the optimization process to align more closely with real-world practices.

2.1.2 Investor views

As mentioned earlier, Sharpe (1964) identifies an equilibrium model that allows investors to base their investments on a neutral stance, known as the market equilibrium view. According to this approach, if investors do not have specific expectations or perspectives on asset returns, they can simply align with the market equilibrium expectations. However, when investors' views on asset performance differ from the market's, they can adjust equilibrium values based on their own views. Additionally, investors can control the impact on portfolio weights based on their confidence in these views. They start by using the CAPM equilibrium - the market portfolio - as a reference point. Then, they adjust their positions towards the most favorable assets in the portfolio through appropriate long or short positions. Black and Litterman (1992) proposed a model that integrates investors' views and their corresponding confidence levels into the investment process. By addressing the challenges of estimating expected returns and the sensitivity of optimal portfolio asset weights to return assumptions, this model distinguishes between various views and ambiguous assumptions. Moreover, it offers a framework for combining the forecasts of an equilibrium model with forward-looking manager views, enhancing portfolio allocation decisions.

Despite extending modern portfolio theory, the model proposed by Black and Litterman (1992) has seen limited adoption among professional portfolio managers (Drobetz, 2001). Furthermore, their model assumes accurate specification of equilibrium and expert views, which may not always hold true and could lead to significant efficiency losses if misspecified (Chen & Lim, 2020). Chen and Lim (2020) propose a generalized model to address such misspecifications, along with a method to assess expert views using a Bayesian model estimated from historical returns.

While Black and Litterman (1992) offer valuable insights into combining equilibrium portfolios with investor views, market perspectives and expectations on returns are not the sole factors influencing portfolio weights during optimization. One influence that genuinely worries investors is inflation and its unexpected changes.

2.1.3 Inflation Dynamics

The concept of inflation refers to increasing prices of goods and services over time. While sometimes prices fall, the majority perceives a tendency of increasing prices (O'Neill et al., 2017). A primary tool for measurements of inflation is the Consumer Price Index (CPI) which is the "general level of prices" elucidating the average of prices of consumer goods and services relative to a reference period. Recent research on how to measure inflation has been intense, but there is no consensus yet. However, most researchers agree that measurements of inflation are dynamic and do not stand still (O'Neill et al., 2017).

Of interest for this inquiry is the split of the overall inflation into expected and unexpected inflation. Consequently, I make use of the computations of expected and unexpected inflation according to Neville et al. (2021). Firstly, they derive expected inflation from professional forecasts. Secondly, the compute unexpected inflation as the difference between the realized level of inflation and the expected inflation.

One way to fight unusual inflation is a tightened monetary policy introduced by the central banks. This process involves the tool of increasing short-term interest rates, among which the FED acts according to the so-called 'Taylor rule'(Federal Reserve, 2024). In general, central banks use monetary policy to control demand-driven inflation spike. The unprecedent monetary policy and fiscal stimulus implemented by the government are the primary cause for the inflation spike since 2021(Bonacina & Civitella, 2023). However, Bonacina and Civitella (2023) also argue that the inflation spike is not solely the result of the actions implemented by the central banks. For example, many central banks emphasize an explicit inflation target rate that is 2% and considered to be the international standard across central banks worldwide (Bonacina & Civitella, 2023; Diwan et al., 2020). Everything above the international standard of 2% is looked at as unusual and demands actions. However, it is not reasonable to expect central banks to keep the inflation rate exactly within the target rates (Bonacina & Civitella, 2023). Particularly, central banks stress that it is difficult to shift their monetary policy, even a drastic one, for short-term purposes. Accordingly, they stress the purpose of policies and that it can be anticipated that the inflation rate will return to the desired target rate.

Over the cycle of 2022, US inflation had increased by over 5% reaching the highest level since the 1970s (Eickmeier & Hofmann, 2022). One of the most important questions thereby is to which extent the surge of inflation can be explained by demand or supply. While the former refers to monetary and fiscal policy tightening, the latter would be associated with policy trade-offs. Multiple scholarly articles try to answer this question. However, no consensus has yet been found about the interplay of demand or supply during high-inflationary times. For example, Phillips (1958), the father of the famous Phillips-curve, explains that the rate of change in money wage rates can be explained by the level of employment and the rate of change in unemployment. Nevertheless, the most recent research has found that the US inflation surge in 2022 can mostly be explained by unusual expansionary demand conditions and tightening in supply (Eickmeier & Hofmann, 2022). Further, their analysis provides insights on how tightened monetary policy reduces demand and hence brings back inflation to the desired rate. Anyhow, delving deeper into the drivers of inflation is out of the scope of this research paper. More interestingly, though, is the question of the influence of inflation on investors and their demands. According to Bonacina and Civitella (2023), alongside Eckmeier and Hofmann (2022), the escalation in inflation during 2021 can largely be attributed to monetary policy and quantitative easing. However, they argue that this surge can be attributed to several other factors, including: 1) government initiatives such as wage subsidies and income transfers to households in 2020-21, 2) significant disruptions in the global supply chain due to the COVID-19 pandemic, and 3) the Russia-Ukraine conflict, which led to immediate spikes in commodity prices, particularly in energy, wheat, and fertilizer (Bonacina & Civitella, 2023). However, despite the general trend of higher consumer prices resulting from increased commodity prices due to the Russia-Ukraine conflict, evidence suggests that commodity prices and consumer price levels may not be co-integrated, and the hypothesis of a long-run relationship between them may be rejected (Boughton & Branson, 1988). Nevertheless, Boughton and Branson (1988) note a tendency for consumer prices to increase when commodity prices rise, indicating a lack of clear consensus on whether commodity prices directly impact consumer price levels. This highlights the importance of acknowledging that while central banks and governments can influence demand-driven inflation, other factors such as supply chain disruptions and geopolitical conflicts also exert significant influence. Understanding these inflation drivers is crucial for grasping concepts related to asset pricing, portfolio management, and investment sentiment. Additionally, analyzing investor reactions to inflation surges and their perceptions of different types of inflation, whether short-term or long-term, is essential for informed decision-making.

Evidence suggests that investors care mostly about constant movements in inflation and less about temporary fluctuations (Cieslak & Pflueger, 2022). However, an increasing inflation rate means more uncertainty seen as increased risk for an investor. Hence, investors demand a risk premium for increased inflation resulting in higher expected returns. According to Feldstein (1980) and controversially to Cieslak and Pfluger (2022), economic theory would predict that all real assets would hold their real value if the price level rises. However, this is not the case, and investors change their portfolio choices accordingly. The paper of Feldstein (1980) shows that changes in inflation expectations alters the real value of land, while a continuous inflation rate leaves the real value of land untouched, which contradicts the view of Cieslak & Pflueger (2022). This is mainly the case due to uncertainty of another unexpected change in inflation rate, which is not liked by any type of investor (Marotta, 2022). More general and to complement both papers mentioned above, the relation between common stocks and inflation rates has substantial influence on optimal portfolio allocations, as found by Katzur and Spierdijk (2010). The study conducts several types of investors with different magnitudes of risk-aversion, including a benchmark and an agnostic investor, and various investment horizons.

A useful metric for understanding the magnitude of the increased risk and its impact on asset prices is to use the inflation-beta, which refers to the price of one unit of additional risk incurred by exposure to inflation. The inflation-beta illustrates the behavior of an asset if inflation fluctuates. Boons et al. (2020) assess the inflation-beta exposure using a regression analysis where they regress excess returns on inflation in a consumption-based asset pricing model. Marrotta (2022) finds that commodities are extremely sensitive to inflation, indicating a high inflation-beta. Moreover, due to rising energy prices in 2022 (Ari et al., 2022), it is crucial to disentangle the effects of core and energy inflation on the overall inflation. Fang et al. (2022) compute the core and energy beta for different asset classes such as stocks, bonds, commodities, and real estate. They find that the core beta, covering the effect of the core inflation on asset returns, turns out to be negative across all assets. Consequently, these assets are not a hedge against core inflation. On the other hand, energy shocks appear to not incorporate a compensation premium. However, this does not mean that energy shocks are irrelevant to asset prices (Meltzer, 2005).

In general, Fama (1981) finds that stock returns correlate negatively with both components of inflation, the expected and unexpected components. This theorem is supported by the article of Schwert (1981). Furthermore, stocks and inflation are strongly negatively related to measures of future real activity. Hence, Fama's (1981) findings are consistent with the rational expectations view in which markets set prices based on the forecasts of relevant real variables. The same pattern of negative correlations can be observed for bonds. Although, these statements suggest a general understanding of correlations between assets and inflations, one needs to notice, that the relation is heavily dependent on the type of inflation (Eickmeier & Hofmann, 2022).

In conclusion, inflation dynamics are crucial for every portfolio. For some assets (stocks and bonds) inflation seems to be negative, whereas for others (commodities) inflation seems to be positive. The following sections discuss commodity futures, their diversification benefits and how investors can successfully hedge against inflation by including them into the portfolio.

2.1.4 Commodity Futures

This thesis uses the concept of commodity futures instead of commodities, as spot prices of commodities are not listed and thus, it is difficult to retrieve available data. A significant distinction between commodities and their corresponding futures lies in the division of price quotations within the commodity market: spot prices and future prices. The former refers to an immediate delivery of a raw material whereas the latter relates to the delivery in the future. Hence, a commodity future is a contract to buy or sell a specific quantity of a commodity, for a specific price, at a specific time in the future. As a consequence, the value of the contract at the time of closing is zero, as there is no payment at this time (Black, 1976). Another distinction between commodities and their appropriate futures is the consideration of storage costs. From now on, this thesis applies commodity futures as the appropriate asset class. Hence, any use of commodities refers to commodity futures.

2.1.4.1 Commodity Futures: Diversification

The correlation coefficient between a portfolio and the new asset determines the diversification benefit by adding the asset to the portfolio. A correlation coefficient $\rho < 1$ between assets lead to diversification benefits. Many academic scholars have been investigating the correlation behaviour of assets particularly focusing on equity and bonds. However, recent studies explored the correlation amongst commodities with other asset classes and therefore referring to diversification opportunities. For instance, Gorton and Rouwenhorst (2006) as well as Bhardwaj et al. (2015) find that commodity futures are effective in diversifying equity and bond portfolios, suggesting a non-perfect positive correlation between commodities and equity. Touching upon the correlation mentioned between commodities and equity, Bhardwaj and Dunsby (2013) find that the correlation between equity and commodities is almost zero over the long horizon. However, this relationship varies over time.

In the context of the mean-variance approach, how can these correlations between stocks and commodities be used as an advantage for any investor? As mentioned above, any portfolio consisting of non-perfect correlating assets ($\rho \neq 1$) comprises diversification benefits. Therefore, adding commodities to an all-equity portfolio should diversify the portfolio and hence, reduce its implied risk. If the correlation coefficient between two assets is perfectly negative ($\rho = -1$), then the diversification benefit is fully utilized. Interestingly, an early work of Bodie and Rosansky (1980) investigates the behavior of an all-stock portfolio when adding commodity futures. They find that commodity futures exhibited around the same average return, over the period of 1950 to 1976, as stocks did. Further, they state that commodity futures perform well in times when stocks perform badly and vice versa, indicating hedging opportunities of stock returns with commodity futures. Subsequently, they suggest a portfolio allocation of 60/40 in stocks and commodity futures, respectively, to reduce the aggregate portfolio risk by one-third without sacrificing any return. Additionally, based on a sample period of 36 years, Conover et al. (2010) shed insightful light on the substantial benefits of adding commodity futures to a portfolio. They come across three interesting results: First, it requires an asset allocation to commodity futures greater than 5% to obtain significant diversification benefit. Second, an asset allocation of 15% to commodity futures yield the highest reduction in portfolio risk. Third, diversification benefits of adding commodity futures to one's portfolio depend largely on the monetary policy of the central bank. On the one hand, adding commodity futures to the portfolio increases returns significantly and decreases risk when central banks are raising interest rate. On the other hand, when central banks are decreasing interest rates, adding commodity futures to a portfolio decreases the risk of the portfolio as well as the return. Furthermore, the findings of Jensen et al. (2002) and Wang et al. (2022) strengthen the general view of commodity futures enhancing portfolio performance for investors. In particular, they differentiate between managed and unmanaged commodity futures, suggesting managed futures to provide the largest diversification benefit to a portfolio.

Despite being a great tool to diversify portfolios in tightened monetary policy times, commodity futures also encounter criticism. For example, Erb and Harvey (2006) rail against the use of past performance as an indicator of a complete future prospect. However, they only consider a long-only portfolio for commodity futures. On the other hand, over the sample period of 1991 to 2015, Yan and Garcia (2017) find that including commodity products in a portfolio does little to improve the Sharpe ratio of a portfolio. Consistent with Yan and Garcia (2017) are Lean et al. (2023), who find that including commodity futures in a portfolio does not always enhance the risk-return tradeoff, except for gold. Additionally, Marotta (2022) finds that commodities-based futures are certainly helpful to mitigate inflation, otherwise they only increase the risk of a portfolio while remaining the expected return on the same level. In the meantime, an academic scholar finds that the attainable diversification benefit of adding commodities to the portfolio are much smaller than suggested (Bessler & Wolff, 2015).

In conclusion, adding commodity futures to an equity portfolio has been subject to extensive research. Many studies have found arguments in favor of commodity futures, but there has also been disagreement. In the context of diversification and as of my knowledge, there has been paid little attention to time variation of portfolio weights. In particular, there has not been conducted a study, where optimal portfolio weights and its variation over time have been assessed for the period of the increased inflation in 2022. Hence, I contribute to the theory of portfolio optimization by investigating optimal portfolio weights over the period of 2022 and its surge in inflation. The subsequent section investigates commodity futures and their abilities to hedge against inflation.

2.1.4.2 Commodity Futures: Hedging against Inflation

As discussed above, investors fear the occurrence of unexpected inflation. On the one hand, we discussed commodities (and accordingly commodity futures) and their diversification benefits to a portfolio. On the other hand, stocks/bonds are negatively correlated to inflation whereas commodities are positively correlated to inflation, implying inflation-hedging properties. Hence, commodities are not only considered to be a diversification tool but also a hedge against inflation. For instance, the extraordinary US inflation in 2022 was mainly due to inclined oil prices caused by the Ukraine-Russian conflict (Kilian & Zhou, 2022). Consequently, this led to a decline in stock and bond prices as a response to the increased inflation. In order to hedge against these losses, an investor could have added oil futures to the portfolio and benefit from the increased inflation as commodities are positively correlated to inflation. Therefore, a loss in real terms could have been hedged by including oil futures into the portfolio, aligning with the interests of an investor that cares only in real terms. However, to continue the literature on commodity futures, we first need to precisely define hedging and what it means to hedge against inflation.

Hedging refers to the concept of reducing the risk exposure of one asset by taking offsetting positions in a derivative or other asset. One of the main risks that households face is income risks.

Therefore, if the income risk is not insurable and non-diversifiable it may be optimal for households to participate in the stock market, depending on the correlation between income risk and stock market returns (Bonaparte et al., 2014). This way, potential losses in the income of a household can be compensated by potential gains on stock returns. If gains equal losses, the hedge is perfect. For instance, potential loss of purchasing power due to high-inflationary times could have been hedged by investing in oil barrels. Hence, a widely adopted view in the economic literature is that a portfolio represents a good hedge against inflation if it holds true to the Fisher's hypothesis (Katzur & Spierdijk, 2010). With this definition in mind, we continue the literature review on commodities.

Extending his research of commodities and commodity futures, Bodie (1983) shows that commodity futures are not only used to diversify portfolios but they are also used to hedge against inflation. In particular, he delves into the real values of basic agriculture and industrial commodities that alter the consumption of a household if prices fluctuate. Aligning with Bodie (1983), Gorton and Rouwenhorst (2006) find that contrary to the S&P500, commodity future returns are positively correlated to inflation, signaling a suitable hedge against inflation and its impact on stock returns in a portfolio. More insights on hedging against inflation are given by Spierdijk and Umar (2014), showing that commodities have significant abilities in hedging US inflation. They mainly focus on commodity futures in the energy market, industrial metals, and live cattle, which offer the most favorable hedge against inflation. However, they also find that they experience a significant tradeoff between the real portfolio variance and the real portfolio expected return, as they add commodities to the portfolio. To back up the findings of the named scholars, Podkaminer et al. (2021) find the perfect hedge against inflation, using a diversified portfolio consisting of real assets including TIPS, real estate, commodities and certain assets to pass increasing costs over to the costumers. However, they mention that commodities are only represented sparingly in this diversified portfolio. To back up Podkaimer et al. (2021), Brière and Signori (2012) establish four different portfolios with a variety of investment horizons. They find that the longer the investment horizon, the greater the portfolio weights of precious metals reaching from 1% for a 2-year investment horizon up to 15% for a 30-year investment horizon. They do so for different regimes, marking different macroeconomic volatility and procyclical inflation, and for different short-fall probabilities.

Interestingly, but rather unimportant for this inquiry are the findings of Briere and Signori (2011) who investigate emerging markets, suggesting a portfolio allocation of 70/30 between domestic assets and foreign assets, respectively, offers an optimal hedge against inflation. In addition to that, Zaremba et al. (2021) investigate 50 different commodities across 80 different countries as suitable inflation-hedge. They support the former literature by concluding that commodities incorporate hedging capacities. However, they add that these hedging capabilities vary over time, vary across countries and that not all commodities offer the same level of protection.

Controversially, more recent studies investigated inflation hedging properties of commodities and whether they are consistent with what the past has taught us. For instance, Liu et al. (2023) find that total commodity futures are not a hedge against inflation. However, industrial and precious metal are a suitable hedge against inflation and hence adding these commodities can be a valuable component to a portfolio. Inconsistent with Liu et al. (2023), however, is Taylor (1998) who argues that precious metals (gold, silver and platinum) could not be used as a hedge against inflation instead the period before 1939 and around the second OPEC oil shock in 1979. Nevertheless, Liu et al. (2023) are consistent with Zaremba et al. (2021) referring to a variation in hedging capabilities over time. Additionally, commodities are considered to not be an inflation hedge in the long term, emphasizing on short-term inflation hedging properties of commodities (Findsen, 2023). Consistent with Findsen (2023) is Futerman and Sarjanovic (2022) who find that commodities combined in a short-term frame and an in-out game plan can be a good hedge against high inflationary times. However, in any other scenario of low inflation, commodities seem to be an expensive hedge. Also, mitigating the risk of inflation by adding commodity-based futures to one's portfolio only increases the risk while leaving the expected return unchanged (Marotta, 2022). Furthermore, an observed negative consequence of futures hedging is revealed by Bessembinder (1992) who finds that agriculture futures vary with the net holdings of a hedger. This mainly implies that after correcting for risk, the market segment and hedging pressure determine the risk premiums demanded.

The scholar of hedging remains a well-discussed and well-researched topic. To conclude, this thesis focuses on adding commodity futures to a portfolio consisting of US equities. Hereby, I mainly follow the approach of Markowitz (1952) and Sharpe (1960).

2.2 Hypothesis Development

2.2.1 Research Question

Given the knowledge that has been outlined in the literature review, it remains challenging to predict the future behavior of portfolio weights in relation to a surge in inflation. However, the past and its information offer an insightful perspective and allow to draw conclusions on a potential optimal investment behavior, especially with respect to the portfolio allocation between commodities and equity. Or as Charlie Munger noted: *"There is no better teacher than history in determining the future."* (Dimson et al., 2024: 5). Taking this into consideration, my objective is to retrospectively optimize the portfolio allocation during the spike in inflation in 2022. For this, I build upon the portfolio optimization of Markowitz (1952) and complement my approach with the concept of Sharpe (1966) and Sharpe (1994) using the Sharpe ratio. Hence, the purpose of this research inquiry is to investigate the fluctuations of portfolio weights over time depending on variations of inflation, particularly focusing on the surge of inflation in 2022. Therefore, I pose the following research question: *"How can equity portfolio weights be strategically optimized by adding commodity futures to hedge effectively against inflation during high-inflationary times?"*

2.2.2 1st Hypothesis

To answer the research question, I test several hypotheses. As we have seen in the literature review, commodity futures offer diversification benefits as well as hedging capabilities. Therefore, it may be interesting to investigate whether these promises on hedging and diversification benefits of commodities still apply to a recent artificially created equity-commodity portfolio. Bhardwaj and Dunsby (2013) find a zero-correlation between equity and commodities. Hence, following the concept of Markowitz (1952), adding commodity futures to an equity portfolio should lead to reduced risk due to diversification and thus, the efficient frontier should move to the left. More specifically, the global-minimum variance portfolio and the tangency portfolio should lower its volatility given the same level of expected return. Therefore, the first hypothesis goes as follows:

$\mathbf{H_{0.0}}$: Adding commodity futures does not lead to a shift to the left of the efficient frontier, and hence does not offer diversification benefits. Equivalently, there is no improvement in the risk-return tradeoff.
$\mathbf{H_{1.0}}$: Adding commodity futures does lead to a shift to the left of the efficient frontier, and hence does offer diversification benefits. Equivalently, there is an improvement in the risk-return tradeoff.

The first hypothesis can also be expressed differently by using the concept of the Sharpe ratio by Sharpe (1996). Thereby, I change the phrasing to the following:

$H_{0.1}$: Adding commodity futures does not increase the Sharpe ratio of the tangency portfolio and hence does not lead to a superior risk-return tradeoff.
$H_{1.1}$: Adding commodity futures does increase the Sharpe ratio of the tangency portfolio and hence does lead to a superior risk-return tradeoff.

2.2.3 2nd Hypothesis

The first part of my hypotheses considers diversification benefits. However, according to the literature, commodities also offer hedging opportunities. Gorton and Rouwenhorst (2006) find that commodities and inflation are positively correlated. In other words, if inflation goes up the price of commodities go up as well and hence, the nominal return of commodities increases. Supported by Spierdijk and Umar (2014), I hypothesize that adding commodities to an all-equity portfolio hedges against high-inflationary times. Therefore, I expect that the portfolio allocation to commodity futures increase if inflation increases, as an investor wants to compensate for the losses in returns of equity. Hence, my second hypothesis is:

$H_{0.2}$: Portfolio allocation to commodity futures does not increase during a high-inflationary time as we have experienced during 2021/2022 and thus, commodity futures are not a suitable hedge against inflation.
$H_{1.2}$: Portfolio allocation to commodity futures does increase during a high-inflationary time as we have experienced during 2021/2022, and thus, commodity futures are a suitable hedge against inflation.

2.2.4 3rd Hypothesis

Thirdly, investor care about constant expected movements in inflation rather than short-term fluctuations (Cieslak & Pflueger, 2022). However, unexpected movements in inflation means higher risk and therefore requires a risk-premium as a compensation for the additional incurred risk. If the excess return remains constant whilst risk (standard deviation) increases, consequently the Sharpe ratio of the asset decreases. Ultimately, this decreases the portfolio weights as another asset might exhibit a higher Sharpe ratio and thus delivers a better risk-return tradeoff. In fact, this showcases that investors do not like unexpected events such as unexpected inflation. Hence, I hypothesize that unexpected inflation has a more pronounced impact on the portfolio allocation to commodity futures as the excess return of equity goes down whereas the corresponding risk goes up, leading to a lower Sharpe ratio. Consequently, I test the following hypotheses:

$H_{0.3}$: Unexpected inflation does not have a greater influence than expected inflation on portfolio weights.

$H_{1.3}$: Unexpected inflation does have a greater influence than expected inflation on portfolio weights.

2.2.5 4th Hypothesis

Given the importance of inflation expectations in guiding investment decisions, I hypothesize that changes in inflation expectations will influence portfolio allocation. Building upon the work of Stulz (1986) who emphasizes the impact of expected inflation on asset pricing, I further hypothesize that when inflation forecasts rise, there will be a corresponding increase in portfolio allocation towards inflation-hedging assets, such as commodities. Hence, my fourth and last hypothesis is:

$H_{0.4}$: Changes in inflation expectations do not significantly affect portfolio allocation towards commodity futures.
$H_{1.4}$: Changes in inflation expectations do significantly affect portfolio allocation towards commodity futures.

With the help of the hypotheses stated, I aim to answer the research question posed above. For each hypothesis, I test and try to reject H0 in favour of H1. The next chapter discusses the data and focuses on the indices used for the analysis as well as the descriptive statistics of the data. Furthermore, the data section is complemented by the methodology section which delves deeper into how the hypotheses are tested.

3 Data

This section serves as the foundation of the thesis and its analysis, aiming to offer readers deeper insights into the data used to test the hypotheses outlined in the previous chapter. To begin, I introduce the proxies employed for the S&P 500 equity index and the GSCI Commodity Index. Specifically, I utilize two different investment indices: the SPDR S&P 500 ETF Trust as a portrayal of the S&P 500, and the iShares S&P GSCI Commodity-Indexed Trust as a representation of the GSCI Commodity Index. The combination of these two indices depicts the artificial portfolio consisting of an equity and commodity part that I create over the sample period. Additionally, I add limitations and concerns about the data. Finally, I provide descriptive statistics for the collected data, focusing on the quantile distributions of returns, including minimum, 25th quantile, median, 75th quantile, and maximum values.

In this thesis, I use daily returns spanning from January 2014 to January 2024. This 10-year sample period provides robust statistics, incorporating over 100 monthly returns and more than 2000 daily returns. Furthermore, I fixate on high-inflationary times such as the time between 2021 and 2022. Thereby, I argument from the perspective of an US investor. More specifically, I focus hereby on the period between the 1st of March 2021, and the 1st of December 2022, which is marked by an abnormal high inflation that is an inflation rate above the proposed rate of 2% by the central bank. Moreover, as previously mentioned, I employ the SPDR S&P 500 ETF Trust as a proxy for an US equity index, thereby enhancing reliability and real-world applicability. To cover commodities and commodity futures, I employ the GSCI Commodity Index as a proxy, encompassing the most relevant commodities in the market. Additionally, I use 3-month US treasury bills as a proxy for the risk-free rate, sourced from Federal Reserve Economic Data (FRED). The risk-free rate fluctuates monthly, serving as a benchmark that is the US market. With that, I follow the approach outlined by Erb and Harvey (2006). Furthermore, I retrieve US inflation data incorporating realized inflation rate and inflation expectations from the US bureau of Labor Statistics. The unexpected inflation rate is constructed by deducting the realized inflation rate from the expected inflation rate. Henceforth, the unexpected inflation is assessed rather than retrieved. As a result, a potential occurrence of estimation uncertainties might make the data not completely reliable. Moreover, as mentioned earlier, there is an ongoing debate on which measurement is the correct measurement of the inflation indicator. The US inflation data retrieved from the US bureau of Labor Statistics relies on the Consumer Price Index (CPI) and depicts monthly changes in price for all urban consumers. Thus, I remain consistent in assessing the inflation rate with many other research scholars (Bryan & Cecchetti, 1993; Dougherty & Van Order, 1982; Gorton & Rouwenhorst, 2006). Moreover, I incorporate various control variables, such as monthly volatility, measured by the Volatility Index (VIX), monthly market sentiment, measured by the Consumer Confidence Index (CCI), and monthly liquidity of the mentioned indices, captured by the trading volume of each ETF. Nevertheless, I delve deeper into the control variables by the end of this chapter.

3.1 SPDR S&P 500 ETF Trust

The SPDR S&P 500 ETF Trust represents an exchange-traded fund (ETF) designed to mirror the performance of the S&P 500 index and hence reflect the broader US market performance. To improve clarity and readability, this thesis uses the abbreviation S&P 500, while consistently referring to the SPDR S&P 500 ETF Trust. This ETF is part of the SPDR State Street Global Advisors fund family and is traded on the New York Stock Exchange. With an annual net expense ratio of 0.09%, the ETF offers a relatively low cost compared to the category average of 0.78%, which stands as one of its key advantages. The ETF primarily consists of 99.75% stocks, rendering it eligible to serve as a representation of an all-equity index. Additionally, the ETF allocates its investments across various sectors, including technology (30.58%), healthcare (12.45%), financial services (12.68%), and others. This diversified investment approach across sectors ensures a well-diversified portfolio. The data pertaining to the SPDR S&P 500 ETF Trust is retrieved from yahoo.finance. The ETF is a total return ETF where not only capital gains are included but also dividends. Figure 1 depicts the performance of the ETF over time, indicating that an initial investment of $100 in 2014 would have grown to $159 by 2024. Moreover, the ETF exhibits an average annualized return of 5.06%, coupled with an annualized standard deviation of 7.63%, which is considered relatively low on average.

Figure 1: SPDR S&P 500 ETF Trust

Notes: Depiction of the return of the SPDR S&P 500 ETF Trust standardized to 100. In other words, how would a $100 look like today if it would have been invested into the ETF on the 1st of January 2014[1]. Source: Own creation.

Consequently, the SPDR S&P 500 ETF Trust serves as the equity part of the artificially constructed portfolio for this thesis. However, the commodity part of the portfolio is represented by the iShares S&P GSCI Commodity-Indexed Trust which is introduced in the next paragraph.

[1] Find the data on yahoo.finance: https://finance.yahoo.com/quote/SPY?.tsrc=fin-srch

3.2 iShares S&P GSCI Commodity-Indexed Trust

The S&P GSCI is the world's first major commodity index that can be invested into and plays a major role in my thesis following the S&P GSCI methodology from December 2023. The index is a world-production index that represents the relative significance of each commodity in a global context. A noteworty point here is that the index only consists of physical commodities and commodity futures rather than other financial products like securities or currencies. Each commodity is assessed by the average production over the last five years. In addition to that, the index offers liquidity to investors by including the most liquid commodity futures on the market, namely 24 exchange-traded commodity futures, whilst still maintaining diversification benefits through low correlations among other asset classes. Generally, there is no restriction on how many future contracts can be included into the S&P GSCI index. However, three eligibility criteriums that need to be fulfilled in order to be included in the index. The first criterium states that the contract must be specified in expiration, delivery and settlement. Second, the contract must be traded at any point in time at least five months prior to expiration. Third, the trading exchange must allow market participants to execute spread transactions. These criteriums enhance the ability of sustaining liquidity, investibality and tradability of the S&P GSCI Index and hence, are desired by investors.

Table 1: Composition of the iShares S&P 500 GSCI Commodity-Indexed Trust

Sub-Index	Weight	Included Commodities
Energy	57.77%	Crude Oil and Natural Gas
Livestock	7.81%	Lean Hogs, Live Cattle and Feeder Cattle
Agriculture	17.65%	Wheat, Corn, Soybeans, Coffee, Sugar, Cocoa and Cotton
Precious Metals	5.71%	Gold and Silver
Industrial Metals	11.07%	Aluminum, Copper, Lead, Nickel and Zinc

Notes: The composition of the Sub-Index is based on the Factsheet of iShares by Blackrock as of the 31st of December 2023. Source: Own creation.

Moreover, the S&P GSCI index is categorized into five main subcategories: energy, industrial metals, precious metals, agriculture, and livestock. However, direct investments in the S&P GSCI Index are not possible. Instead, investors require index-linked products that facilitate investing in the S&P GSCI. One such product is the iShares S&P GSCI Commodity-Indexed Trust (Ticker: GSG), which replicates the S&P GSCI index, enabling investors to invest indirectly into the index. For enhancing readability, I use GSCI as a short form for the iShares S&P GSCI Commodity-Indexed Trust. Table 1 provides an overview of the GSCI composition across its subcategories, including sub-index weights and the commodities represented within the sub-indices.

What particularly catches attention in the GSCI composition is its heavy reliance on the energy sub-index, with a focus on crude oil and natural gas. This raises concerns about the index's diversification and preparedness in the face of geopolitical tensions like the Ukraine-Russia conflict. It is also surprising to note the relatively low weights assigned to precious metals (5.71%), despite gold usually being considered a diversification tool due to its safe-haven properties. The overall weight distribution prompts questions about its optimality. However, delving into the optimal weights for the GSCI sub-index might be an intriguing research topic for other scholars, as it impacts my findings though falls outside the scope of this thesis.

As the GSCI Index tries to replicate the commodity market through commodity futures, the index never holds futures until maturity as this would inherent direct delivery of the commodity. Hence, in the beginning of the expiration month, the GSCI Index sells the future contracts that are about to expire this month and substitutes them by future contracts with the next applicable expiration date. This exchange process of selling the expiring future contracts and substituting them with other contracts happens in a monthly 5-day roll period between the fifth and ninth business day.

There are two different ways to calculate the return on the index: The total return (TR) and the excess return (ER). This thesis mainly focuses on the total return of the index as this makes the index comparable to stocks as well as bond investment as the future contracts within the GSCI Index are fully collateralized. Meaning, that hypothetical payments of the underlying are invested into treasury bills. According to the methodology of the S&P GSCI from December 2023, the total return of the GSCI index is computed using three different concepts: First, the Contract Daily Return which is the ratio of Total Dollar Weight Obtained (TDWO) of a business day t divided by the Total Dollar Weight Invested (TDWI) of business day t-1. Second, the methodology applies a risk-free rate which is assessed by the daily compounding German Bubill Rate (GBR) on day t, which is the equivalent to a German treasury bill. Third, the risk-free rate is set to the power of delta, that is the number of non-business days since the preceding business day. Formally:

$$GSCI\ TR_t = (CDR_t + Rf_t) * (1 + Rf_t)^{delta} \tag{1}$$

Where the total return of the GSCI is denoted in euros. Furthermore, the CDR is formally computed as follows:

$$CDR_t = \frac{TDWO_t}{TDWI_{t-1}} - 1 \tag{2}$$

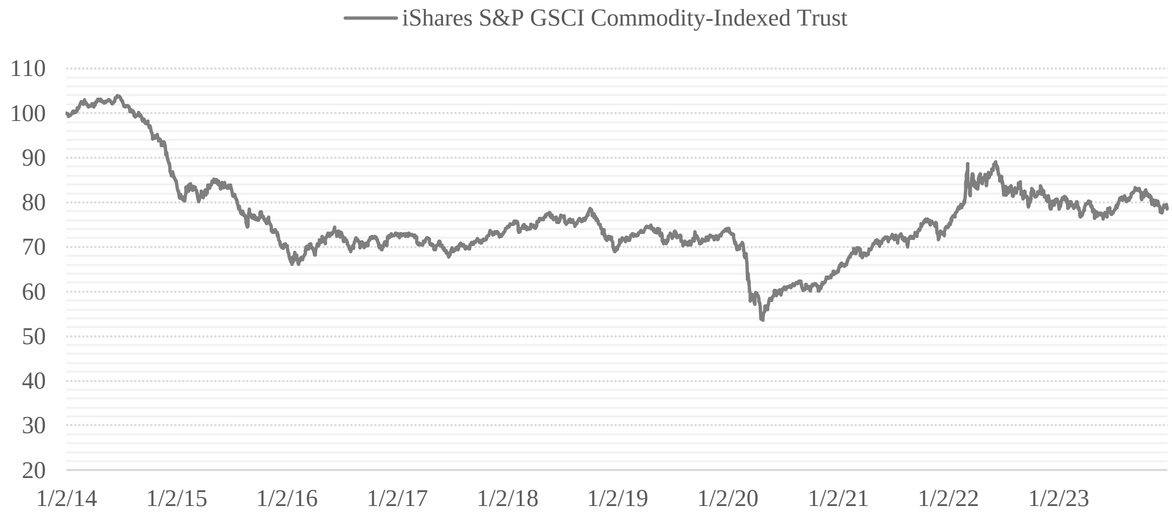
The Rf represents the risk-free rate and is the equivalent to the German Bubill Rate (GBR).

$$GBR_t = \left[\frac{1}{1 - \frac{91}{360} * SGBR_{t-1}}\right]^{\frac{1}{91}-1}$$

(3)

Where the German Bubill Rate (GBR) on day t is assessed by incorporating the Simple German Bubill Rate on day t-1 ($SGBR_{t-1}$) which represents the discount rate of a 3-month German treasury bill that is effective on the preceding business day[2].

Figure 2: iShares S&P GSCI Commodity-Indexed Trust



Notes: Depiction of the return of the iShares S&P GSCI Commodity-Indexed Trust standardized to 100[3].

Figure 2 illustrates the performance of the GSCI Index throughout the sample period. Notably, the return appears to be negative. Specifically, the expected daily return of the GSCI Index is calculated at -0.01% with a daily standard deviation of 0.61%. When converting to annualized returns, the Index exhibits a negative annualized return of -1.92% with a standard deviation of 9.72%. In comparison to the S&P 500, it is observed that the equity portion of the constructed portfolio performs better, with a lower annualized standard deviation of 7.63%. Conversely, the commodity portion of the portfolio exhibits negative performance. Additionally, it is noteworthy and somewhat surprising that the GSCI Index displays negative performance despite the positive inflation rate since 2014. However, the increased inflation rate since 2021 might have had an impact on the GSCI Index, as the Index has risen ever since 2021.

[2] These computations are all according to the methodology of the S&P GSCI December 2023. One need to note that these assessments are dynamic and vary over time. For further total return index calculations, I refer to other sections on the S&P Dow Jones Indices Commodity Index Mathematics Methdology.
[3] Find the data on yahoo.finance: https://finance.yahoo.com/quote/GSG?.tsrc=fin-srch

3.3 Descriptive Statistics

Table 2 presents descriptive statistics for the data used throughout the analysis, which is segmented annually to gain insights into variable variations. The variables include mean, standard deviation, minimum, 25th quantile, median, 75th quantile, and maximum values. It is crucial to mention that the examination is built upon log returns. Therefore, this thesis refers to log returns when speaking about returns.

Table 2: Descriptive statistics for daily return distribution

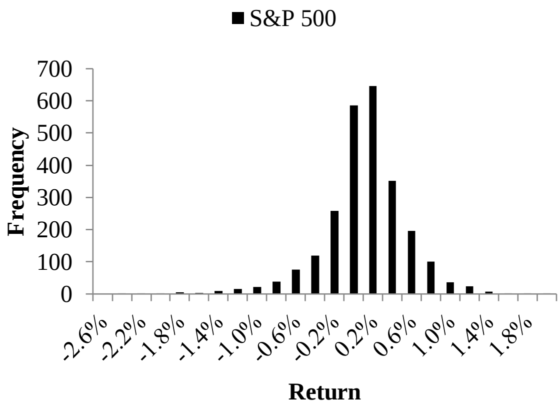
Date	Mean	Standard Deviation	Min	25th quantile	Median	75th quantile	Max
SPDR S&P500 ETF Trust (SPY)							
2014	0.02%	0.31%	-0.99%	-0.11%	0.04%	0.21%	1.06%
2015	0.00%	0.42%	-1.87%	-0.20%	-0.01%	0.24%	1.64%
2016	0.02%	0.36%	-1.59%	-0.12%	0.02%	0.20%	1.05%
2017	0.03%	0.18%	-0.78%	-0.05%	0.02%	0.11%	0.60%
2018	-0.01%	0.47%	-1.86%	-0.20%	0.02%	0.24%	2.14%
2019	0.05%	0.34%	-1.33%	-0.10%	0.05%	0.26%	1.43%
2020	0.03%	0.92%	-5.03%	-0.27%	0.10%	0.44%	3.77%
2021	0.05%	0.35%	-1.07%	-0.15%	0.06%	0.27%	1.04%
2022	-0.04%	0.66%	-1.93%	-0.45%	-0.08%	0.42%	2.32%
2023	0.04%	0.36%	-0.88%	-0.18%	0.03%	0.28%	0.98%
iShares S&P GSCI Commodity-Indexed Trust (GSG)							
2014	-0.06%	0.38%	-2.20%-	-0.25%	-0.04%	0.17%	1.16%
2015	-0.07%	0.65%	-2.08%	-0.48%	-0.10%	0.32%	2.00%
2016	0.02%	0.62%	-1.55%	-0.39%	0.00%	0.37%	1.93%
2017	0.01%%	0.40%	-1.23%	-0.21%	0.03%	0.25%	1.28%
2018	-0.03%	0.48%	-1.80%	-0.30%	0.03%	0.27%	1.27%
2019	0.02%	0.48%	-1.65%	-0.23%	0.08%	0.28%	2.85%
2020	-0.05%	0.89%	-5.59%	-0.44%	0.04%	0.38%	2.45%
2021	0.06%	0.57%	-2.92%	-0.24%	0.10%	0.43%	1.52%
2022	0.04%	0.88%	-4.92%	-0.40%	0.07%	0.53%	2.55%
2023	0.00%	0.53%	-1.47%	-0.35%	0.00%	0.38%	1.27%

Notes: This table shows the daily return distribution split up in years, including minima, 25th quantile, median, mean, 75th quantile and maxima. The sample period ranges from 2014 until 2024. Source: Own creation.

Upon initial examination, a notable observation from Table 2 is the lower standard deviation of daily returns for the S&P 500 compared to the GSCI, suggesting lower daily risk for the former based on the assumption that standard deviation measures risk. Additionally, the S&P 500 experienced mean negative daily returns in two years (2018 and 2022) within the observed period, while the GSCI had mean negative daily returns four times (2014, 2015, 2018, 2020). Notably, the GSCI recorded its lowest daily return in 2020 at a significant -5.59%, contrasting with the S&P 500's lowest point of -5.03% in the same year amid the COVID-19 crisis. Despite this, while the GSCI displayed the lowest minimum compared to the S&P 500, it did not exhibit the highest maximum. Specifically, the S&P 500 achieved a daily return of 3.77% compared to the GSCI's 2.85%. When considering the average daily returns of both indices, the GSCI showed an overall negative daily return of -0.01%, whereas the S&P 500 had an average daily return of 0.02%. Consequently, the mean daily return distribution for the GSCI index appears to be closer to zero than that of the S&P 500 index.

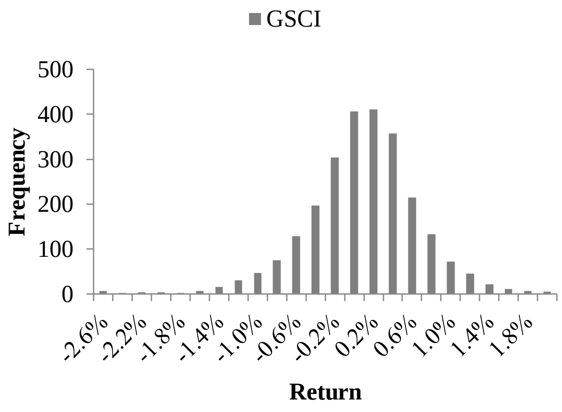
Figure 3 and Figure 4 display the daily return distributions for both indices. Notably, the GSCI's return distribution exhibits a smaller kurtosis of 7.66 compared to the S&P 500's kurtosis of 13.96. Kurtosis measures the peakiness of a distribution in relation to a normal distribution (Kurtosis = 3). Consequently, an increases kurtosis result in greater tails that lead to higher probability of extreme events (both large and small returns) for the S&P 500 compared to the GSCI index. Furthermore, regarding skewness, both indices' distributions are negatively skewed, with values of -0.80 for the S&P 500 and -0.90 for the GSCI. A negative skewness suggests that positive events occur more frequently than negative events. Thus, the heavier tails in the positive region of the return distribution indicate a greater likelihood of positive events, which is desirable for investors. However, since the mean for the GSCI is negative, negative events are more pronounced than positive ones, resulting in a negative daily return of -0.01%. This is evident in the descriptive statistics, as the GSCI records the lowest minimum but fails to reach the highest maximum.

Figure 3: Daily return distribution of the S&P 500



Notes: Daily return distribution of the S&P 500 with returns on the x-axis and the frequency on the y-axis. The timeframe reaches from the 1st of January 2014 until the 1st of January 2024. Source: Own creation.

Figure 4: Daily return distribution of the GSCI



Notes: Daily return distribution of the GSCI with returns on the x-axis and the frequency on the y-axis. The timeframe reaches from the 1st of January 2014 until the 1st of January 2024. Source: Own creation.

As previously discussed in the literature review, the cruciality of portfolio optimization lies in the correlation among assets within the portfolio. Table 3 presents the correlations between the S&P 500 and the GSCI, along with those for the inflation rate. Furthermore, I include the expected inflation rate, which are forecasts, as well as the unexpected inflation rate. An intriguing observation is the positive correlation between the GSCI Index and the S&P 500 (0.398). However, this correlation is not perfectly positive, indicating potential diversification benefits when combining these indices, as suggested by theoretical frameworks. Additionally, the GSCI shows a positive correlation (0.128), while the S&P 500 exhibits a negative correlation (-0.123) with the inflation rate. These non-perfect positive correlations between equity and commodities align with previous studies by Gorton and Rouwenhorst (2006) and Bhardwaj et al. (2015). Furthermore, the data is consistent with findings on the correlations between equity/commodities and inflation (Bhardwaj et al., 2015; Bodie, 1983; Feldstein, 1983; Gorton & Rouwenhorst, 2006). Literature has typically found a negative correlation between equity and inflation, whereas commodities tend to exhibit a positive correlation with inflation. Hence, the constructed portfolio comprising the S&P 500 and the GSCI offers diversification benefits due to their non-perfect positive correlation (0.398) and serves as a hedge against inflation due to a positive correlation (0.128).

Table 3: Correlation matrix

Correlations					
Index	**GSCI**	**S&P 500**	**Inflation**	**Expected Inflation**	**Unexpected Inflation**
GSCI	1.000				
S&P 500	0.398	1.000			
Inflation	0.128	-0.123	1.000		
Expected Inflation	0.159	-0.052	0.890	1.000	
Unexpected Inflation	0.098	-0.151	0.968	0.745	1.000

Notes: Correlation Matrix. Source: Own creation.

3.4 Black-Litterman model

In Chapter 4, I employ the Black-Litterman model (Black & Litterman, 1992). To do so, I replace the entire S&P 500 index with the top four assets that carry the most weight across different industries within the index, using this data as input for the model. This approach maintains the focus on an all-equity portfolio while dividing the S&P 500 into individual stocks. I do so because of two reasons: First, portfolios (including the S&P 500) comprise an estimate of the underlying population distribution that is the assets included in the portfolio. Second, the return distribution of the portfolio may be viewed as a direct result of many repetitions of a random asset selector. This approach is backed by Breen and Savage (1968) who compute the return distribution of the portfolio based on ten randomly selected securities. In my case, the selected stocks for the Black-Litterman model are Microsoft (MSFT), Berkshire Hathaway (BKB-B), JP Morgan (JPM), and UnitedHealth Group (UNH), representing the IT, Retail, Banking, and Healthcare sectors respectively. Historical stock data for these assets was sourced from yahoo.finance, covering the period from January 1st, 2014, to January 1st, 2024.

Table 4 presents descriptive statistics for stocks MSFT, BKB-B, JPM, and UNH, emphasizing annualized daily returns, annualized standard deviation, and the beta relative to the benchmark that is the S&P 500. Additionally, I provide an estimated return derived from the CAPM model, utilizing log returns as input. This expected return subsequently serves the Black-Litterman optimization process. In Table 4, the first row presents the annualized log returns of the assets, with MSFT displaying the highest annualized return at 10.42%. The second row represents the standard deviation of the returns, indicating the total risk for each asset, with MSFT exhibiting the greatest risk magnitude at 11.75%. Furthermore, the market beta, displayed on the third line, indicates return movements relative to the benchmark, which in this case is the S&P 500, represented by SPY. Finally, the expected return is assessed according to CAPM and can be found on the last row. Notably, the CAPM returns are somewhat smaller for all assets, indicating an underestimation of the returns over the sample period. The risk-free rate used for the CAPM calculations is the logarithmic average of the 3-month US Treasury bill over the sample period, which stands at 0.55%.

Table 4: Descriptive statistics of the stocks used for the Black-Litterman optimization

	SPY	**MSFT**	**BRK-B**	**JPM**	**UNH**
Return	3.25%	10.42%	3.09%	4.53%	8.59%
Standard deviation	7.64%	11.75%	8.36%	11.68%	11.04%
Market beta	1	1.21	0.89	1.14	0.90
CAPM return	3.25%	3.83%	2.94%	3.61%	2.97%

Notes: This table shows the annualized return, standard deviation, market beta and the estimated CAPM expected return. The risk-free rate is the average of the 3-month treasury bill over the sample period of 10 years resulting in 1.28%. Source: Own creation.

3.5 Control Variables

The selected control variables encompass volatility, liquidity, and market sentiment. To begin, the volatility control variable is represented by the CBOE Volatility Index (VIX)[4], which indicates overall market volatility in the US market. The data for the VIX is retrieved from yahoo.finance. By choosing the VIX, I account for fluctuations in market conditions that might have an impact on investment strategies and its chosen portfolio weights. Thereby, high volatility suggests increased risk which results in different investment behaviors. Furthermore, liquidity, crucial in asset trading, is chosen as a control variable due to its influence on portfolio weights. Liquidity is represented by the trading volume of each index, with relative changes computed from day t-1 to t [5]. The data for trading volume is retrieved from yahoo.finance. Generally, higher trading volume results in lower transaction costs as the bid-ask spread

[4] Find the data on yahoo.finance: https://finance.yahoo.com/quote/%5EVIX/
[5] The trading volume of both indices can be found in former footnotes. Thereby, I refer specifically to the footnotes of the GSG and SPY indices.

tends to be smaller. Therefore, I aim to account for liquidity as this might also change investment strategies or the choice of assets. Finally, market sentiment is evaluated using the US Consumer Confidence Index (CCI) from the OECD database[6], indicating consumer willingness to consume. By doing so, I again aim to account for consumer perception and its impact on financial markets and hence potential influences on portfolio weights.

[6] Find the data on OECD: https://data.oecd.org/leadind/consumer-confidence-index-cci.htm

4 Methodology

This section focuses on the methodology I applied to test my hypotheses and to answer the research question. First, I explain key concepts such as different portfolio types including their calculations that are important to my hypotheses. In particular, I focus on the concept of the Sharpe ratio constructed by Sharpe (1994) and the tangency (or optimal) portfolio that comprises the highest possible Sharpe ratio in the risk-return dimension. Additionally, I define the global-minimum variance, the naive-diversification and the Black-Litterman portfolio using mean-variance and Black-Litterman approach (Black & Litterman, 1992; Markowitz, 1952). Once I define these concepts, I delve into the main analysis, which includes using Markowitz's (1952) mean-variance optimization approach to assess the out-of-sample efficient frontier of two risky assets: the S&P500 and the GSCI. Moreover, I apply the Black-Litterman model (Black & Litterman, 1992) which assesses the optimal portfolio that maximizes the expected return given an objective risk tolerance. This approach allows to combine individual views regarding the relative performance of an asset with the market equilibrium resulting in a diversified portfolio. From there, I identify the tangency portfolios for each month and their relevant Sharpe ratios. To determine whether the change in Sharpe ratio resulting from adding commodity futures to an all-equity portfolio is significant, I employ the Jobson Korkie test (Jobson & Korkie, 1981) that was adjusted by Memmel (2003) but then reviewed and corrected by Ledoit and Wolf (2008). The application of the adjusted Jobson Korkie test helps to test the first hypothesis $H_{0.0}$ and $H_{0.1}$. Subsequently, for the second hypothesis $H_{0.2}$, I conduct a regression analysis in order to see whether the changes in inflation in 2021/2022 have a significant impact on portfolio weights. Further, I compute real returns to see whether commodities offer a perfect and hence effective hedge against inflation. Meanwhile, I regress the nominal returns on the inflation to offer robustness of the hedging properties. Subsequently, I assess the third ($H_{0.3}$) and fourth ($H_{0.4}$) hypothesis by also employing a regression analysis. I do the mentioned steps above particularly for portfolio weights of the tangency portfolios. Nevertheless, I complement my findings by incorporating the global-minimum variance portfolio, the equally weighted portfolio and the Black-Litterman portfolio optimization. To enhance robustness, I add multiple robustness checks, elaborated in the last paragraph of this section. It is important to mention that all the computations made in this chapter are consistent with Brooks (2019), especially with regards to the regression analyses. The reason for this stems from the application of Brooks (2019) methodology in many universities. The analysis is carried out using Microsoft Excel, for the efficient frontier analysis, and R Studio, for the regression analysis, the bootstrap method based of Jobson and Korkie (1981) as well as the appropriate robustness checks.

4.1 Concepts: Mean-variance portfolio optimization

4.1.1 Sharpe Ratio

Sharpe (1994) invented the Sharpe ratio which serves as a relative performance measure that puts the excess return of an asset computed with the risk-free rate in relation to its standard deviation, which is the total risk of the asset. Hence, Sharpe (1994) computes the Sharpe ratio as follows:

$$Sharpe\ Ratio_i = \frac{r_i - r_f}{\sigma_i} \tag{4}$$

Where r_i and σ_i represent the return that asset i earns and its respective incurred risk measured as standard deviation. r_f is the risk-free rate.

The Sharpe ratio reflects a simple number that explains how many units of return are required to take on one more unit of risk. Furthermore, the Sharpe ratio allows to compare two or more assets with each other due to relative performance properties. Therefore, using the Sharpe ratio in this thesis enables to explore the difference in performance for an all-equity portfolio (S&P 500) and a portfolio that combines the components of equity (S&P 500) and commodities (GSCI).

4.1.2 Equally Weighted Portfolio

The equally weighted portfolio, as its name suggests, distributes weights equally across all asset classes. This procedure is also known as naïve diversification or the 1/N portfolio strategy (DeMiguel et al., 2009). Consequently, it serves as a benchmark rather than a portfolio optimization strategy, providing a reference against which other optimized portfolios can be evaluated. In its simplest form, the initial scenario allocates 100% to equities, while in the second scenario, a 50/50 split is assigned between equities and commodities. This serves as a foundational starting point for the optimization process and is therefore essential to mention in this context.

4.1.3 Tangency Portfolio

The tangency portfolio, also known as the maximum Sharpe ratio portfolio, aims to optimize the Sharpe ratio of a portfolio, representing the optimal tradeoff between risk and return. An efficient portfolio achieves the highest return possible for a given level of standard deviation across various weight allocations. Consequently, my focus lies solely on the upward-sloping segment of the efficient frontier, as the downward-sloping portion is deemed inefficient. This preference stems from the fact that the upward slope promises greater expected returns for a given level of standard deviation compared to the downward slope. The global minimum variance portfolio marks the transition point between the efficient and inefficient sections of the "hyperbola," showcasing the lowest standard deviation across all portfolio weight combinations. Therefore, the tangency portfolio resides on the efficient frontier, tangential to the Capital Allocation Line (CAL). The CAL, originating from the risk-free rate and terminating at the tangency portfolio, exhibits a slope equivalent to the Sharpe ratio, reflecting the incremental increase in excess return relative to the incremental increase in standard deviation. Consequently, the tangency portfolio, being tangent to the CAL, represents the portfolio on the efficient frontier with the highest achievable Sharpe ratio.

$$\max\ Sharpe\ Ratio = \frac{r_p - r_f}{\sigma_p}$$
$$s.t. \sum_{i}^{n} \omega_i = 1$$
$$0 \leq \omega_i \leq 1 \tag{5}$$

Where r_p and σ_p represent the return of the portfolio and its standard deviation. Furthermore, r_f denotes the risk-free rate, which I calculate in my case as the average monthly return of a 3-month US Treasury bill over the sample period.

Moreover, there are constraints imposed on the equation. Specifically, the sum of the portfolio weights must equal 100%, and leveraging is prohibited. These constraints hold true across all portfolios and throughout the whole analysis.

4.1.4 Global-Minimum Variance Portfolio

The global minimum variance portfolio (GMV) marks the portfolio with the lowest possible standard deviation and hence, as mentioned above, represents the distinction point between the efficient and inefficient part of the efficient frontier. Furthermore, the GMV solely depends on the standard deviation and the correlation among the assets within the portfolio. Hence, the objective of the portfolio is to minimize the standard deviation without any further restrictions on the expected return. However, I rely on the same restriction as for the tangency portfolio that is no short-selling and no leveraging.

$$\min \sigma_p^2 = \sum_{i}^{n} \sum_{j}^{n} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$$
$$s.t. \sum_{i}^{n} \omega_i = 1$$
$$0 \leq \omega_i \leq 1$$

Where σ_p^2 is the portfolio variance, ω_i is the portfolio weights of asset i and j, σ_i is the standard deviation of asset i and ρ_{ij} is the correlation coefficient between asset i and asset j that cannot be greater than 1 and lower than -1.

4.1.5 Black-Litterman Portfolio

In my final portfolio analysis, I utilize the Black-Litterman approach (Black & Litterman, 1992), where I replace the S&P 500 with four individual stocks: Microsoft (Ticker: MSFT), Berkshire Hathaway (Ticker: BKB-B), JPMorgan Chase & Co (Ticker: JPM), and UnitedHealth Group Inc (Ticker: UNH). This shift is supported by Breen and Savage's (1968) notion that the return distribution reflects underlying population parameters, justifying the use of individual securities over the S&P 500. If the S&P 500 would not have been replaced by the named individual stocks, the Black-Litterman would result in extreme long or short positions caused by preferences for one asset over the other, which results in neglecting the existing assumptions of no short selling and no leveraging. The Black-Litterman model builds upon modern portfolio theory, incorporating investors' views to adjust portfolio weights beyond mean-variance optimization. Following Merton's (1972) methodology but slightly adjusted for efficient portfolio frontier construction in the presence of risky assets and a risk-free asset, I perform several steps annually from 2014 to 2024 to derive the Black-Litterman optimized portfolio.

Initially, I assess expected returns using the CAPM estimation approach. This involves calculating market beta, which, when multiplied by the market premium (the S&P 500 return minus the risk-free rate), adjusted by the risk-free rate, yields the expected return. Since asset and market returns are annualized, I use the annual risk-free rate.

$$\beta_i = \frac{cov(r_i, r_M)}{var(r_M)} \tag{6}$$

Where β_i represents the beta of each asset to the market, $cov(r_i, r_M)$ is the covariance between the market and asset i, and $var(r_M)$ is the variance of the market.

$$r_i = r_f + \beta_i(r_M - r_f) \tag{7}$$

Where r_i is the expected return according to the CAPM estimations, β_i the market beta, r_M the market return and r_f the risk-free rate.

Secondly, the covariance matrix and its inverse of all four assets including the GSCI is computed. Thirdly, I compute the vectors e, r, h and g, that are needed for the subsequent step. Vector e is a unity vector consisting of only the number 1 that allows matrix calculations.

$$e = (1,1,\ldots,1) \in \mathbb{R} \tag{8}$$

$$h = eC^{-1} \tag{9}$$

where C^{-1} is the inverse of the covariance matrix and e the unity vector.

$$g = rC^{-1} \tag{10}$$

where r is the expected return according to CAPM (Eq. 7), and C^{-1} the inverse of the covariance.

Fourthly, I use the vectors calculated to assess the variables α, β, γ and Δ. The computations for these variables look as follows:

$$\alpha = eh^T \tag{11}$$

$$\beta = eg^T \tag{12}$$

$$\gamma = rg^T \tag{13}$$

$$\Delta = \alpha\gamma - \beta^2 \tag{14}$$

Fifthly, I assess the target return R_t of the tangency portfolio according to the Black-Litterman portfolio optimization as well as the portfolio μ and λ . Here, μ represents the value of the tangency portfolio if the risk-free rate equals zero, and λ the value of the minimum-variance portfolio.

$$R_t = \frac{\gamma - \beta r_f}{\beta - \alpha r_f} \tag{15}$$

Where R_t represent the target return of the tangency portfolio at time t, γ, β, α the variables assessed in Eq. 11,12,23, and r_f the risk-free rate.

$$\mu = \frac{\alpha R - \beta}{\Delta} \tag{16}$$

$$\lambda = \frac{\gamma - \beta R}{\Delta} \tag{17}$$

Finally, I arrive at the optimal portfolio weights of the Black-Litterman optimization by adding the multiplication of the portfolio λ with the variable h to the the product of μ and g.

$$w^* = \lambda h + \mu g \tag{18}$$

Where w^* represents the optimal portfolio weights.

4.2 Hypothesis testing all portfolios

After carefully examining the portfolio weights for the S&P 500 and the GSCI, the subsequent steps is to test the Sharpe ratio changes for significancy. Furthermore, the remaining hypotheses are examined in the following sections.

4.2.1 Test for changes in Sharpe ratio (1st Hypothesis)

For the first hypothesis ($H_{0.1}$), I test whether the inclusion of commodities in the portfolio improves the risk-return tradeoff of the portfolio as measured by the Sharpe ratio. Therefore, I estimate the monthly Sharpe ratios for two portfolios: one that consists only of the S&P 500 and one that allocates capital to the S&P 500 and the GSCI. To do this, I apply the Sharpe Ratio calculations mentioned above, where I subtract the risk-free rate from the total return of the portfolio and divide the excess return by the standard deviation of the portfolio. Regarding the risk-free rate, I apply a dynamic rate that changes monthly (Chapter 3). In terms of the constructed portfolio, I focus on creating the tangent portfolio by maximizing the Sharpe ratios for the example period mentioned. Prior to computing the Sharpe ratios, I use the approach of Markowitz (1952) to construct the efficient frontier and its efficient portfolios, in particular the tangency portfolio. To do so, I use return data mentioned earlier and create monthly covariance-variance matrix based on daily returns. Moreover, I compute expected return and variance for both assets. I do this for each month using historical return data for the period from January 2014 to January 2024. After evaluating all Sharpe ratios for the different portfolios, I count 120 Sharpe ratios for the equity-only portfolio and 120 Sharpe ratios for the mixed portfolio. I then calculate the true difference of the Sharpe ratios of the two portfolios for each month.

$$\Delta_t = SR_{i,t} - SR_{j,t} \tag{19}$$

Where Δ_t equals the true difference in Sharpe ratios. $SR_{i,t}$ and $SR_{j,t}$ represent the Sharpe ratio for the mixed portfolio i at time t, and the all-equity portfolio j at time t, respectively.

In addition, I calculate the Sharpe ratio estimations for both portfolios using the approach of De Roon et al. (2012), which build their model based on the estimations of Lo (2002). By doing so, I enhance accuracy for the true difference by comparing them with the estimated value of the Sharpe ratios for all portfolios.

Subsequently, I perform a comparative analysis in which I check whether the true differences in the Sharpe ratios are statistically significantly different from zero. Therefore, I perform a two-sided test for $H_0: \Delta = 0$ at significance level α by carrying out a studentized bootstrap methodology, which is a resampling method. Therefore, I propose to test $H_0: \Delta = 0$ by inverting a bootstrap confidence interval. This means that I construct a two-sided bootstrap confidence interval at the nominal level of $1 - \alpha$. If the confidence interval does not contain the value of 0, then H_0 is rejected at nominal level α.

I thus follow the approach of Jobson and Korkie (1981), which was adjusted by Ledoit and Wolf (2008). The original version of Jobson and Korkie (1981) is not robust against heavy tails unequal to the normal distribution and to time series data (Leidoit and Wolf, 2008). Furthermore, according to the authors, the studentized bootstrap outperforms the "standard" inference. The advantage of the studentized bootstrap method is that one can easily draw a new sample from the observed data. In particular, I propose to approximate the studentized statistics via the bootstrap method as follows:

$$\varsigma\left(\frac{|\hat{\Delta}-\Delta|}{s(\hat{\Delta})}\right) \approx \varsigma\left(\frac{|\hat{\Delta}^{*}-\hat{\Delta}|}{s(\hat{\Delta}^{*})}\right) \tag{20}$$

Notably, Δ represents the true difference in Sharpe ratios, $\hat{\Delta}$ the estimated difference in Sharpe ratios computed from the data, $s(\hat{\Delta})$ is the standard error for the estimated $\hat{\Delta}$, $\hat{\Delta}^{*}$ is the estimated difference computed by the studentized bootstrap method with the standard error $s(\hat{\Delta}^{*})$ also computed with the original data. Finally, $\varsigma()$ denotes the distribution of a random variable.

Furthermore, to test the monthly differences in Sharpe ratios for the tangency, the GMV and the equally weighted portfolio, I employ a circular block bootstrap method. The reason for this can be found in the distribution of returns that do not resemble a normal distribution, as well as the time series nature of the data used. In employing the circular block bootstrap method, resampling is conducted by forming circular blocks of observed pairs in the time series data, rather than resampling all the data randomly. Each block, in this case, represents a year with a block length of 12, corresponding to 12 months per year. This approach aligns with the recommendation of Leidoit and Wolf (2008) to use a block size of $b \geq 1$. To ensure robustness and accuracy in the analysis, it is essential to generate a sufficient number of pseudo sequences for each block size and confidence interval. Following the guidelines provided by Leidoit and Wolf (2008), a minimum of 1000 pseudo sequences is recommended for bootstrap sampling; however, I have opted to employ 2000 pseudo sequences to further enhance accuracy and reliability. This increase in pseudo sequences contributes to improved accuracy, particularly as the bootstrap estimate of the sampling distribution is centered around the bootstrap mean rather than the total sample mean. An advantage of the circular block bootstrap method is its ability to capture the temporal dependence structure inherent in time series data while providing reliable inference for statistical analysis.

In addition to the Studentized bootstrap method for the mean difference of the Sharpe ratios and the circular block bootstrap method, I also evaluate the standard error of the heteroskedasticity and autocorrelation consistent (HAC) kernel estimator, which takes heteroscedasticity and autocorrelation into account and thus also provides robust statistics. In this way, I calculate the statistical significance of the difference for the overall mean and for the monthly differences.

4.2.2 Commodities as a hedge for inflation in 2022 (2nd Hypothesis)

The second hypothesis ($H_{0.2}$) of this paper examines how portfolios are allocated to commodities during periods of high inflation and whether commodities offer hedging properties against inflation. Specifically, the period of high inflation from 2021 to 2022 is examined. Nevertheless, even though 2023 was also marked as high-inflationary times, I only focus on the years 2021 and 2022. By doing so, I test for the immediate effects of the COVID-19 crisis as well as the invasion of Ukraine by Russia. The inflation in year 2023, however, might only be the aftereffects of the events in 2021 and 2022. To test the hypothesis, I use a regression analysis where the portfolio weights assigned to the GSCI are the dependent variable. The GSCI, which is calculated to test the primary hypothesis mentioned above, is meant to explain the portfolio weights of the GSCI relative to the inflation rate. There, I use the inflation rate as an independent variable. Furthermore, I complement the regression equation with the monthly interest rate as this might also explain a change in portfolio weights. In addition, I introduce a dummy variable that covers the period from March 1st, 2021, to December 1st, 2022. As mentioned in Chapter 3, this period is characterized by an unusually high inflation rate that exceeds the central banks' target rate of 2%. Moreover, I include control variables such as the volatility index, market sentiment and trading liquidity for both the GSCI and the S&P 500 in the regression equation. By doing so, I align with the practices of several studies that explore the impact of liquidity, volatility and market sentiment on returns and its implications of asset pricing and return predictions (Amihud & Mendelson, 1986; Bekaert & Hoerova, 2014; Mascio et al., 2023). To assess whether the independent variables have affected the portfolio weighing in the period from 2021 to 2022, I form interaction terms between the dummy variable and the independent variables. Hence, I perform a two-sided t test to analyze the impact of each coefficient on to the dependent variable in the regression. Nevertheless, this hypothesis focuses on inflation and its impact on portfolio weights, thereby indicating β_1 and β_8 as the coefficient of interest. The remaining coefficients display the influence of the control variables on to the portfolio weights. Therefore, the regression equation looks as follows:

$$GSG = \alpha + \beta_1 Inflation + \beta_2 Rf + \beta_3 D_{Inflation} + \beta_4 VIX + \beta_5 Sentiment + \beta_6 Liq_{GSG} + \beta_7 Liq_{SPY} + \beta_8 D_{Inflation} * Inflation + \beta_9 r_{GSG} + \beta_{10} r_{SPY} + \varepsilon \tag{21}$$

$$D_{Inflation} = \begin{cases} 1, & if\ t\ falls\ between\ March\ 1st, 2021, and\ December\ 1st, 2022 \\ 0, & elsewise \end{cases} \tag{22}$$

Where GSG is the dependent variable and represents the portfolio allocation to commodity futures, α equals the intercept of the regression equation. β_i represents the coefficient estimation for each independent variable. $D_{Inflation}$ is the dummy variable that equals 1 if the date falls within the range of the 1st of March 2021 and the 1st of December 2022. Furthermore, inflation is the inflation rate measured by the CPI, Rf represents the risk-free rate, VIX the volatility index, Sentiment the market sentiment and Liq_{GSG} the liquidity of the GSCI index and Liq_{SPY} the liquidity of the S&P 500, respectively, measured by trading volume. r_{GSG} and r_{SPY} are the returns of the GSCI and S&P 500. Lastly, ε denotes the part that explains the dependent variable GSG yet is not observed through the independent variables.

Furthermore, I test whether commodities and commodity futures offer hedging opportunities against an increasing inflation rate throughout the period between 2021 and 2022. Thus, I calculate the real return of the GSCI index to see whether the returns fluctuate in the same direction with the same magnitude as the inflation rate does. I do so for the whole sample period and for the period between the 1st of March 2021 until the 1st of December 2022.n For this, I use nominal returns as well as the inflation rate and hence, I compute the real return according to the Fisher equation:

$$R_{i,t} = \frac{1 + r_{i,t}}{1 + \pi_t} - 1 \tag{23}$$

Where $R_{i,t}$ is the real return of the asset i at time t and $r_{i,t}$ is the total nominal return of asset i at time t, respectively. Furthermore, π_t represents the inflation rate of the US at time t.

If and only if the nominal return equals the inflation rate, the real return remains the same over t for every asset i. However, if the nominal return exceeds inflation rate, then the real return would be greater than 0. Vice versa, the real return would be lower than 0. Hence, if the GSCI offers perfect hedging properties, then the real return $R_{i,t}$ would remain the same, equaling 0, even though inflation fluctuates. To further enhance robustness, I regress the nominal return of the GSCI onto the inflation rate.

4.2.3 Implication of unexpected inflation on portfolios (3rd Hypothesis)

According to Cieslak and Pfluger (2022), investors are more concerned with the expected movements in inflation rather than unexpected short-term fluctuations. Therefore, I test the third hypothesis ($H_{0.3}$) that unexpected inflation does not have a greater impact on portfolio weights and the allocation to commodity futures. Once again, I employ regression analysis to investigate the influence of both unexpected and expected inflation. To assess the rates of unexpected and expected inflation, I remain consistent with the methodology used by Neville et al. (2021). The regression equation is constructed using the GSCI portfolio weights of the artificial portfolio as the dependent variable. Furthermore, I utilize expected and unexpected inflation as explanatory variables. In order to explore the impact of the high-inflationary period during 2021 and 2022, I include a dummy variable, as previously done. This allows me to focus on the interaction between the dummy variable and both expected and unexpected inflation. By employing this approach, I decompose total inflation into two components, enabling a deeper understanding of the drivers of portfolio weights over time. Here again, I focus on a two-sided t test to observe the statistical influence of each regressor on the dependent variable GSG. Particularly, this regression focuses on β_1 , β_2, β_3 and β_4 as coefficient of interest.

$$GSG = \alpha + \beta_1 Inflation_{expected} + \beta_2 Inflation_{unexpected} + D_{Inflation} * \begin{bmatrix} \beta_3 \, Inflation_{expected} \\ \beta_4 Inflation_{unexpected} \end{bmatrix} + \beta_5 r_{GSG} + \beta_6 r_{SPY} + \varepsilon \tag{24}$$

Where α represents the intercept of the regression analysis, β_i the coefficient estimates of the appropriate explanatory variables and $D_{Inflation}$ the dummy variable for the period between the 1st of March 2021, and 1st of December 2022. The expected and unexpected inflation is denoted as the variables $Inflation_{expected}$ and $Inflation_{unexpected}$, respectively. r_{GSG} and r_{SPY} stand for the return of both indices. Once again, ε represents the part that partly explains GSG yet cannot be observed.

4.2.4 Changes in inflation forecasts and its implications for the portfolio (4th Hypothesis)

The 4th Hypothesis ($H_{0.4}$) states that an increase in inflation forecasts, similarly inflation expectations, leads to an increasing portfolio allocation towards commodities. Therefore, I test whether changes in expected inflation have an impact on portfolio weights of the GSCI within the artificial portfolio. To do so, I apply a regression analysis to explore the relationship between the portfolio weights and changes in inflation forecasts. Using the inflation forecast at time t and t-1 allows for an assessment of changes of the inflation expectations. Hence,

$$\Delta_t = \frac{Expectations_t}{Expectations_{t-1}} - 1 \tag{25}$$

Where Δ_t denotes the relative change in inflation expectations at time t. Furthermore, Inflation expectations in time t and t-1 are $Expectations_t$ and $Expectations_{t-1}$.

Additionally, I create the interaction term between the dummy variable from earlier to see whether there is a different reaction throughout the mentioned time period between 2021 and 2022 for a change in inflation expectations. Thus, the coefficient of interest for this regression is β_1 and β_2, where I perform a two-sided t test to see their impact on GSG. The regression equation run to test the 4th hypothesis is:

$$GSG = \alpha + \beta_1 \Delta_{inflation} + \beta_2 \Delta_{inflation} * D_{Inflation} + \beta_3 r_{GSG} + \beta_4 r_{SPY} + \varepsilon \tag{26}$$

Notably, GSG stands for the portfolio weights of the GSCI within the artificial constructed portfolio. $\Delta_{inflation}$ represents the relative changes in inflation expectations following Equation 12.

Henceforth, all four hypotheses are tested following the methodology outlined in this chapter. I perform all tests for all portfolios including tangency, global-minimum variance, equally weighted and the Black-Litterman portfolio. Moreover, the regression analysis demands enhancing robustness in order to improve the statistical significance of the findings. Hence, I perform the hypothesis tests for portfolio allocations constructed according to the minimum-variance, naïve diversification and the Black-Litterman approach. Later, I discuss robustness checks that are being performed to improve significance of the findings.

4.3 Robustness Check

This section focuses on improving the robustness of the empirical results coming from the analysis. All calculations are performed in the R Studio programming language and utilize the built-in robustness checks. First, I address potential trends in the explanatory variables, defining a trend as a persistent deviation beyond a fixed horizontal range. Trends are identified by inspecting time series plots, and the explanatory variables are transformed with the logarithm for further validation. Regression diagnostics are then performed to test the assumptions of homoscedasticity, error correlation and normality for each regression used in the methodology. To assess heteroscedasticity, robust standard errors are calculated using the consistent covariance matrix estimator and coefficient tests that are performed based on these robust standard errors. If the null hypothesis of homoscedasticity is rejected, in favor of heteroskedasticity, the robust standard errors are immediately used in the subsequent procedures. In addition, if autocorrelation is present, HAC standard errors are used, whereby the Breusch-Godfrey test is used to detect error correlations. To further improve validity and reliability, robust Newey-West standard errors are used, which are also valid for heteroscedasticity and autocorrelation. In addition, tests for multicollinearity between the explanatory variables and correlation between regressors and residuals are carried out. In case of multicollinearity, I apply a ridge regression. The exogeneity of the interest rates is assessed taking into account the potential correlation over time. After a thorough examination and potential rejection of the assumptions, the results are summarized in the following chapter, where the main findings of the paper are presented together with the results of the bootstrap method mentioned earlier.

5 Empirical Results

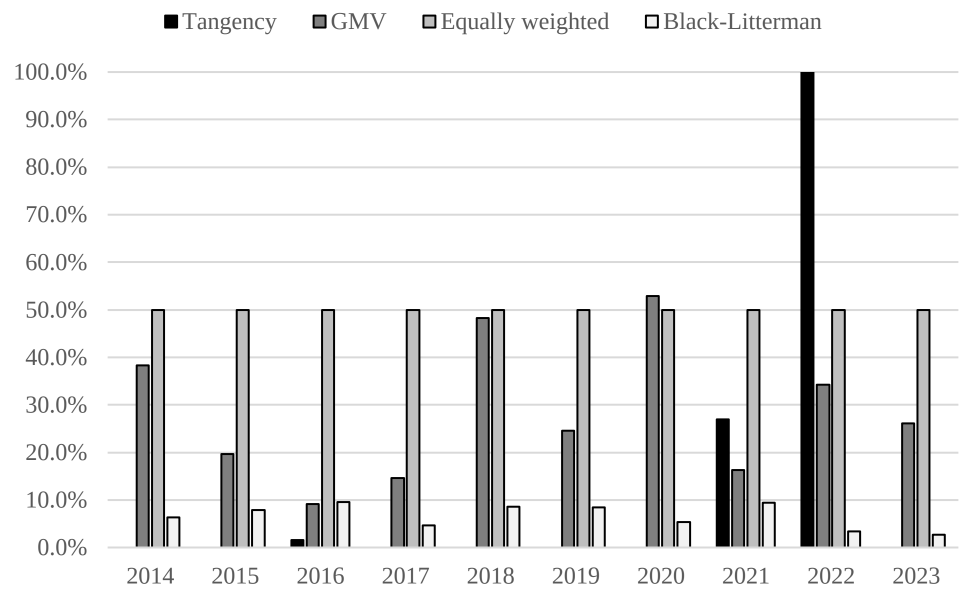
This chapter highlights the main empirical results based on the methodology outlined in Chapter 4. First, I present the out-of-sample tangency, the GMV, the 1/N and the Black-Litterman portfolios over the sample period. Thereafter, I outline significancy tests of the difference in Sharpe ratios of the constructed portfolios consisting of the S&P 500 and the GSCI, and an all-equity portfolio only consisting of the S&P 500, which constitutes the benchmark. This is done for all the portfolios except the Black-Litterman portfolio. Subsequently, I present the main findings of the regression analysis for the 1st hypothesis. Furthermore, I also add the real return of the constructed portfolio to assess hedging properties of the portfolio against inflation, providing insights on the total realized inflation and its distinction between expected and unexpected inflation. Hence, I can test and answer the 3rd hypothesis that aims to explain the impact of unexpected inflation on portfolio weights. Finally, I also account for changes in inflation forecast and its implications on the portfolio allocation. It is crucial to mention that this chapter only outlines the main findings while it neglects to go into depth regarding an elaborate discussion. Consequently, I further present an elaborate discussion of the main findings in Chapter 6.

5.1 Mean-variance portfolio optimization

First, I present the portfolio weights for the portfolio optimization with the constructed portfolio consisting of the S&P 500 and the GSCI. I focus on the tangency, the global-minimum variance, the equally weighted and the Black-Litterman portfolio. As outlined in Chapter 4, the portfolio optimization is assessed using the mean-variance and the Black-Litterman approach that uses the CAPM expected return as a point of reference.

Figure 5 illustrates the annual allocation of weights within the constructed portfolio. A closer look reveals that the tangency portfolio only invests in the GSCI in 2016, 2021 and 2022, with a weighting of 1.6%, 26.9% and 100% respectively. In contrast, the GMV portfolio, which aims to minimize variance and thus portfolio risk, invests yearly in the GSCI. The highest allocation to the GSCI for the GMV portfolio is in 2020, with 52.8% of the portfolio invested in commodities. The Black Litterman portfolio also follows a continuous pattern of allocating capital to commodities each year. The peak allocation for the Black-Litterman model occurs in 2016 and 2021, with allocations of 9.5% and 9.4%, respectively. Over the entire sample period, the tangency portfolio invests an average of 12.86%, the GMV portfolio an average of 28.4% and the Black-Litterman on average 7% into commodities. Nevertheless, bearing in mind that for the Black-Litterman optimization, the S&P 500 index is replaced with individual stocks.

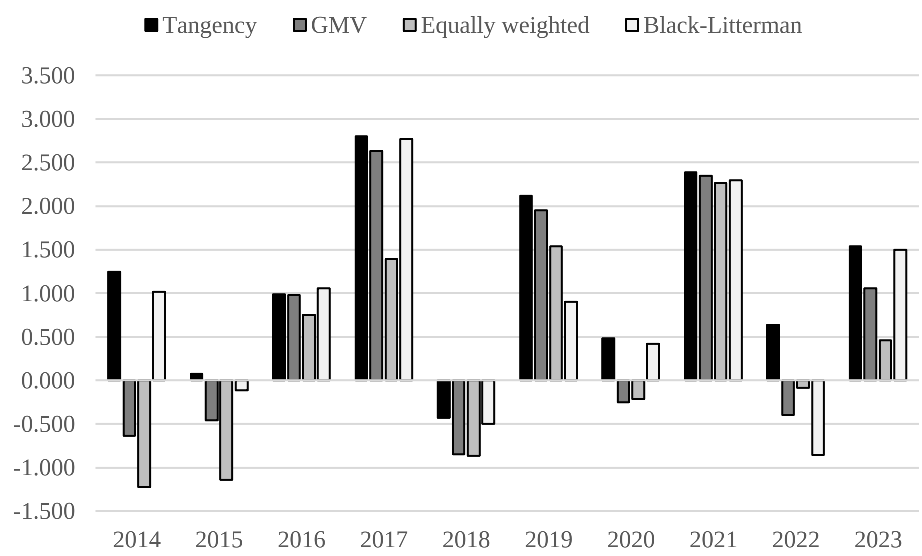
Figure 5: GSCI weights for various types of portfolios over the sample period



Notes: This figure depicts the weights of the GSCI index in the constructed portfolio after portfolio optimization. There, I focus on the tangency portfolio, the global-minimum variance portfolio, the equally weighted portfolio and the Black-Litterman portfolio. Source: Own creation.

These different patterns of allocations come with different Sharpe ratios for each portfolio. Figure 6 shows the Sharpe ratios in accordance with each portfolio, namely the tangency, the GMV, the equally weighed and the Black-Litterman portfolio.

Figure 6: Sharpe ratios for various types of portfolios over the sample period



Notes: This figure depicts the Sharpe ratios for the relevant portfolios, namely the tangency, the GMV, the equally weighed and the Black-Litterman portfolio. Source: Own creation.

It is interesting, but not surprising, that the tangency portfolio almost always has positive Sharpe ratios over the entire sample period, except for 2018. In 2018, the tangency portfolio has a negative Sharpe ratio of -0.43. However, not only the tangency portfolio has a negative Sharpe ratio in that year, but also all other portfolios, which is due to an annualized negative return of -2.31% for the S&P 500 and -6.51% for the GSCI. Also noteworthy are the years 2020 and 2022, in which extremely low Sharpe ratios can be observed compared to all other years. For 2020, only the Tangency and Black-Litterman portfolios show a positive Sharpe ratio of 0.48 and 0.42 respectively. A possible reason for the low Sharpe ratios in 2020 could be the market disruptions of COVID-19, which ultimately led to a high standard deviation of 14.64% for the S&P 500 and 14.28% for the GSCI, but still high returns for the S&P 500 of 7.15%, while the GSCI had a negative return of -11.40%. In 2022, increasing interest rates in combination with low returns for the S&P 500 (- 8.57% annually) could be potentially the reason for the lower Sharpe ratios. An extraordinary year with regards to the risk-return tradeoff is observed for the years 2017 and 2021 for all portfolios. On average, the different portfolios showcase an annual Sharpe ratio of 1.18, 0.64, 0.29 and 0.85 for the tangency, the GMV, the equally weighted and the Black-Litterman portfolio. For more insights with regards to the monthly average Sharpe ratios for the tangency, the GMV and the equally weighted portfolio, I refer to the Appendix.

5.2 Changes in Sharpe ratios (1st Hypothesis)

This section examines the statistical significancy for the differences in Sharpe ratios for the tangency, the GMV and the equally weighted portfolio with respect to the all-equity portfolio. Therefore, I test $H_{0.1}$: Adding commodity futures does not increase the Sharpe ratio of the tangency portfolio and hence does not lead to a superior risk-return tradeoff. The same hypothesis can be rephrased, $H_{0.0}$: Adding commodity futures does not lead to a shift to the left of the efficient frontier, and hence does not offer diversification benefits. Equivalently, there is no improvement in the risk-return tradeoff.

Table 5: Mean monthly Sharpe ratio tests over the sample period

	Tangency	**GMV**	**1/N**	**Equity**
SR	0.474	0.091	-0.110	0.112
$\widehat{SR}$	0.521	0.175	0.078	0.218
Δ	0.362***	-0.022***	-0.222***	
$\hat{\Delta}$	0.300	-0.059	-0.140	
$s_{Studentized}(\Delta)$	0.041	0.086	0.118	
$s_{HAC}(\Delta)$	0.066	0.049	0.076	
$p_{\ studentized}$	0.000	0.000	0.000	

*Notes: This table shows the mean monthly Sharpe ratios for the tangency, the GMV, the 1/N and the equity portfolios. Furthermore, the table also reveals the estimated monthly Sharpe ratio as well as the differences in true value and estimated value. The standard errors for the studentized bootstrap method and the HAC standard error can be found on line five and six, respectively. The last line depicts the p-value of the test. *** p < 0.01; ** p < 0.05; * p < 0.1. Source: Own creation*

Table 5 displays the average monthly Sharpe ratios for these portfolios. It is worth noting that the tangency portfolio shows the highest average monthly Sharpe ratio at 0.474 over the sample period of ten years. The GMV portfolio has a relatively smaller Sharpe ratio, nearly approaching zero, while the equally weighted portfolio's Sharpe ratio is negative (-0.110). In Table 5, column 4 presents the average monthly Sharpe ratio for the all-equity portfolio, represented by the S&P 500, which stands at 0.112. Rows three and four of Table 5 present the true and estimated differences in Sharpe ratios between each portfolio and the all-equity portfolio, denoted as Δ and $\hat{\Delta}$. The tangency portfolio exhibits the largest difference in Sharpe ratios compared to the all-equity portfolio, with values of 0.362 for the true difference and 0.300 for the estimated difference. Interestingly, both the GMV and equally weighted portfolios show negative differences for both true and estimated values. Additionally, rows five and six show the standard error for both the studentized bootstrap method and for the heteroscedasticity and autocorrelation standard error case.

These findings are expected since none of the Sharpe ratios for the portfolios are zero on a monthly average basis, which inherently leads to no zero value within the confidence interval. Therefore, the null hypotheses $H_{0.0}$: $\Delta = 0$ and $H_{0.1}$: $\Delta = 0$ are not true. Consequently, the p-value for all three portfolios is zero, leading to the rejection of $H_{0.0}$ and $H_{0.1}$ in favor of $H_{1.0}$ and $H_{1.1}$.

On the contrary, I examine the monthly differences in Sharpe ratios among all portfolios when compared to the all-equity portfolio. To achieve this, I employ a circular block bootstrap method as outlined in Chapter 4. The empirical findings differ from the monthly average Sharpe ratios presented in Table 5. This discrepancy arises because, in certain months, the portfolios invest entirely in the S&P 500, resulting in no variation in Sharpe ratios. Consequently, zero values are observed within the corresponding confidence interval, confirming $H_{0.0}$ and $H_{0.1}$. The results indicate that there is not enough evidence to reject the null hypothesis that there is a difference in Sharpe ratios on a monthly scale. These results can be found in Table 6.

Table 6: Statistical outcomes for monthly differences in Sharpe ratios

	Tangency	**GMV**	**1/N**
$s_{block}(\Delta)$	0.062	0.045	0.070
p_{block}	0.477	0.667	0.496

Notes: This table presents the statistical outcomes for monthly differences in Sharpe ratios for all portfolios when compared to the all-equity portfolio. Thereby, the first row depicts the standard error for the circular block bootstrap method and the second row the appropriate p-value. Source: Own creation.

In conclusion, the analysis reveals that on average basis, adding commodities significantly improves the tradeoff between risk and return for the tangency and the GMV, but not for the equally weighted portfolio. Therefore, the $H_{0.0}$ and $H_{0.1}$ can be rejected in favor of $H_{1.0}$ and $H_{1.1}$.

However, if the monthly differences are analyzed, the result reveal that the differences in Sharpe ratio are not significantly different from zero. Hence, from this perspective, I confirm $H_{0.0}$ and $H_{0.1}$, indicating that adding commodities to an all-equity portfolio does not significantly improve the tradeoff between risk and return for all portfolios.

5.3 Commodities as a hedge for inflation in 2022 (2nd Hypothesis)

This section focuses on examining the coefficient estimates derived from the regression analysis to test the 2nd hypothesis $H_{0.2}$: Portfolio allocation to commodity futures does not increase during a high-inflationary time as we have experienced during 2021/2022 and thus, commodity futures are not a suitable hedge against inflation.

Table 7: Coefficient estimates of regression analysis for hypothesis 2

	Dependent variable:	
	GSCI weights	
Variables	**Tangency**	**GMV**
Constant	0.421***	0.273 ***
	(0.069)	(0.031)
$Inflation$	1.444	-2.687
	(4.443)	(2.205)
Rf	-5.319	-3.126
	(7.082)	(4.314)
$D_{Inflation}$	0.103	-0.300
	(0.211)	(0.184)
VIX	0.203	0.058
	(0.325)	(0.160)
$Sentiment$	-4.969	-0.225
	(3.892)	(4.021)
Liq_{GSG}	-0.117	0.118
	(0.156)	(0.091)
Liq_{SPY}	-0.196	0.334*
	(0.299)	(0.190)
$D_{Inflation} * Inflation$	-3.942	2.584
	(4.864)	(3.408)
r_{GSG}	10.835***	0.671
	(1.349)	(0.847)
r_{SPY}	-11.313***	-2.687*
	(2.114)	(1.424)
Observations	120	120
Adj. R-squared	0.406	0.143

*Notes: This table shows the output of the regression analysis conducted for hypothesis 2. Hence, the dependent variable is the asset weight of the GSCI in the constructed portfolio consisting of S&P 500 and the GSCI. The first column consists of the explanatory variables of the regression equation. The second column represent the coefficient estimate for the explanatory variables of the first column for the tangency portfolio. The second column represent the coefficient estimate for the explanatory variables of the first column for the GMV portfolio. The robust standard errors are in parentheses. *** p < 0.01; ** p < 0.05; *p <0.1. Source: Own creation.*

The dependent variable on the left-hand side of the regression equation is the asset weight for commodities represented by the GSCI, that is within the constructed portfolio comprising GSCI and the S&P 500. The explanatory variables, on the right-hand side of the regression equation, are detailed in the first column, with the second column representing the coefficient estimates for the tangency portfolio, and the third column for the GMV portfolio. Here, the equally weighted portfolio is not included as there is no variation in the portfolio weights. The coefficient of interest for $H_{2.0}$ can be found in row two and nine. The coefficient estimate, denoted as β_1, equals 1.444 for the tangency portfolio and -2.687 for the GMV portfolio. This indicates that if inflation increases by one unit, the asset weights of the GSCI within the tangency portfolio increases by 1.444 percentage points, while for the GMV portfolio, a one unit increase in inflation decreases the asset weights of the GSCI by 2.687 percentage points. However, although the coefficient estimates reveal the direction of the relationship, they are not statistically significant different from zero for both portfolios and thus they need to be viewed at with attention.

The second coefficient of interest reveals a reversed pattern for both portfolios regarding the interaction term between the inflation dummy and the inflation rate. For the tangency portfolio the coefficient estimate β_8 constitutes -3.942, and 2.584 for the GMV portfolio. Although this pattern seems to be interesting to observe, the coefficients for both portfolios are not statistically different from zero resulting in little evidence to reject $H_{2.0}$ in favor of $H_{2.1}$.

Additionally, Table 7 presents the control variables. It is important to highlight that within the tangency portfolio, GSCI weights show a negative reaction to rising liquidity, measured in trading volume, for both the GSCI and the S&P 500, with coefficient estimates of -0.117 and -0.196, respectively. In contrast, for the GMV portfolio, there is a positive relation between trading volume and GSCI weights, with coefficients of 0.118 and 0.334. However, these coefficient estimates, except for the S&P 500 trading volume within the GMV portfolio, are not statistically different from zero indicated by the p-value. The GSCI and S&P 500 returns appear to influence changes in GSCI weights within the tangency portfolio. Specifically, the coefficient of interest stands at 10.835 for the GSCI return and -11.313 for the S&P 500 return. This means that a one-unit increase in GSCI or S&P 500 return leads to a 10.835 and -11.313 percentage point change in GSCI weights, respectively. These findings hold statistical significance at the 1% confidence level. In contrast, for the GMV portfolio, only the S&P 500 return demonstrates a statistically significant impact on GSCI weight, with a coefficient estimate of -2.687. Nevertheless, it is important to note that GSCI asset weights are determined through mean-variance optimization, which considers factors like expected return and return variance. This can also be seen in the adjusted R-squared that increases substantially if returns of the indices are added as control variables.

Furthermore, to discover inflation hedging properties to a further extent, I assess the real returns for all the portfolios and for the GSCI index as outlined in Chapter 4. The average monthly real return for the GSCI over the whole sample period is negative, -0.39%, whereas the portfolios, namely tangency, GMV and equally weighed portfolio, earn a positive real return: 0.8%, 0.26%, 0.05%. Thereby, the real return of the equally weighted portfolio offers the best hedging opportunity against inflation over the whole sample period, as the real return is closest to 0%. During the high-inflationary time in 2021/2022, the GSCI reveals a positive real return of 0.42% monthly. Whilst the GSCI reversed the real return, the portfolios all increased their magnitude to 1.02%, 0.47% and 0.25% for the tangency, the GMV and the equally weighted portfolio.

In conclusion, the distinction between the tangency and the GMV portfolio leads to different results. Generally, the tangency portfolio invests more into commodities if inflation rate increases. In contrast, the GMV portfolio does the opposite: invest less in commodities if inflation rate is high. However, during the high-inflationary time of 2021/2022, the pattern reverses, indicating less allocation of the tangency portfolio towards commodities, whilst the GMV allocates more towards commodities. Hence, for the tangency portfolio $H_{2.0}$ needs to be confirmed and for the GMV portfolio $H_{2.0}$ needs to be rejected in favor of $H_{2.1}$. However, these results need to be looked at carefully as the results are not statistically different from zero and thus, there is limited meaningfulness. Moreover, it seems that the portfolios as well as the GSCI offer inflation hedging properties during the high-inflationary time in 2021 and 2022.

5.4 Implication of unexpected inflation on portfolios (3rd Hypothesis)

Whether inflation has an impact on portfolio weights has already been analyzed in the previous sections. However, inflation is usually split up in two components: the expected and unexpected inflation. Therefore, this section presents the empirical findings of the 3rd hypothesis outlined in Chapter 2 with the appropriate methodology of Chapter 4, that is whether unexpected inflation does not have a greater impact on portfolio weights than expected inflation. Hence, $H_{0.3}$: Unexpected inflation does not have a greater influence than expected inflation on portfolio weights.

Table 8 displays findings from the regression analysis conducted. Thereby, I explore the connection between GSCI weights, representing the dependent variable, and inflation variables, representing the explanatory variables, across the tangency and the GMV portfolio. Here again, the equally weighted portfolio is neglected as there is no variation in portfolio weights and hence, the statistical inference may be limited.

In the tangency portfolio, expected inflation shows a coefficient estimate of -1.217, albeit lacking statistical significance. On the other hand, unexpected inflation yields a statistically insignificant positive coefficient estimate of -0.665, suggesting a potential negative correlation with GSCI weights. Additionally, the interaction term with expected inflation demonstrates a positive and statistically insignificant coefficient (6.922), indicating variable effects with changes in expected inflation. Moreover, the interaction term reveals a reversed pattern, indicating that during 2021 and 2022, expected inflation had a positive effect on portfolio weights, whereas unexpected inflation had a negative impact on portfolio weights. More specifically, one unit of increased expected inflation increases portfolio weights of the GSCI by 6.922 percentage points during 2021 and 2022. On the other hand, one unit increase in unexpected inflation decreases the GSCI asset weights in the tangency portfolio by -5.771 percentage points.

Table 8: Coefficient estimates for the regression analysis of hypothesis 3

	Dependent variable: GSCI weights	
Variables	**Tangency**	**GMV**
Constant	0.449*	0.237
	(0.239)	(0.154)
$Inflation_{expected}$	-1.217	3.939
	(7.969)	(4.168)
$Inflation_{unexpected}$	-0.665	0.613
	(3.343)	(1.743)
$D_{Inflation} * Inflation_{expected}$	6.922	-7.883
	(15.148)	(11.102)
$D_{Inflation} * Inflation_{unexpected}$	-5.771	5.801
	(8.067)	(5.349)
r_{GSG}	11.096***	0.308
	(1.335)	(0.954)
r_{SPY}	-11.096***	-4.066***
	(1.961)	(1.171)
Observations	120	120
Adj. R-squared	0.419	0.099

*Notes: This table shows the output of the regression analysis conducted for hypothesis 3. Hence, the dependent variable is the asset weight of the GSCI in the constructed portfolio consisting of S&P 500 and the GSCI. The first column consists of the explanatory variables of the regression equation. The second column represent the coefficient estimate for the explanatory variables of the first column for the tangency portfolio. The second column represent the coefficient estimate for the explanatory variables of the first column for the GMV portfolio. The robust standard errors are in parentheses. *** p < 0.01; ** p < 0.05; *p <0.1. Source: Own creation.*

Within the GMV portfolio, expected inflation exhibits a positive coefficient of 3.939, though lacking statistical significance. Moreover, unexpected inflation similarly yields a positive coefficient (0.613). The interaction of dummy with the unexpected inflation also maintains positive with a coefficient estimate of 5.801 that is statistically insignificant on the 10% confidence level. Whereas the expected inflation during this time yields a negative impact on portfolio weights.

In conclusion, if we dismiss statistical significance, unexpected inflation yields a lower influence on portfolio weights for the tangency portfolio than expected inflation. On the other hand, for the GMV portfolio, the expected inflation rate has a greater impact on the portfolio weights. Therefore, in general, $H_{0.3}$ is rejected for both the tangency portfolio and GMV portfolio. However, as the coefficient estimates lack statistical significancy, I have to conclude that the estimates are not different from zero, resulting again in a cautious application of the findings. For the period of high inflation, expected inflation has a greater impact on portfolio weights for the GMV portfolio, though lacking statistical significance. Hence, I reject $H_{0.3}$ for the tangency and GMV portfolio during the high-inflationary time. Moreover, the adjusted R-squared points toward a greater fit of the data for the tangency portfolio.

5.5 Changes in inflation forecasts and its implications for the portfolio (4th Hypothesis)

Investors not only react to actual inflation rates, but they also pay attention to inflation forecasts. Therefore, this section examines how shifts in inflation forecasts affect the weighting of GSCI assets in both the tangency and GMV portfolios. In other words, this section tests $H_{0.4}$: Changes in inflation expectations do not significantly affect portfolio allocation towards commodity futures. The results of the regression analysis from Chapter 4 can be found in Table 9.

Table 9 illustrates how changes in inflation forecasts affect the portfolio weights. In the tangency portfolio, a change in the inflation forecasts by one unit increases the weighting of the GSCI by 2.047 percentage points. In the GMV portfolio, on the other hand, the coefficient estimate is slightly negative at -0.730. Remarkably, the impact of changes in inflation forecasts is more pronounced in periods of high inflation: the coefficient estimates are -11.276 for the tangency portfolio and 14.473 for the GMV portfolio. However, none of these coefficient estimates is statistically significant. Interestingly, the pattern appears to reverse for both portfolios in 2021 and 2022.

Table 9: Coefficient estimates for the regression analysis of hypothesis 4

	Dependent variable: GSCI weights	
Variables	**Tangency**	**GMV**
Constant	0.419***	0.308***
	(0.039)	(0.022)
$\Delta_{inflation}$	2.047	-0.730
	(8.693)	(5.124)
$\Delta_{inflation} * D_{Inflation}$	-11.276	14.473
	(21.036)	(9.929)
r_{GSG}	11.044***	0.356
	(1.359)	(0.914)
r_{SPY}	-10.848***	-4.483***
	(1.888)	(1.091))
Observations	120	120
Adj. R-squared	0.419	0.102

*Notes: This table shows the output of the regression analysis conducted for hypothesis 4. Hence, the dependent variable is the asset weight of the GSCI in the constructed portfolio consisting of S&P 500 and the GSCI. The first column consists of the explanatory variables of the regression equation. The second column represent the coefficient estimate for the explanatory variables of the first column for the tangency portfolio. The second column represents the coefficient estimate for the explanatory variables of the first column for the GMV portfolio. The robust standard errors are in parentheses. *** p < 0.01; ** p < 0.05; *p <0.1. Source: Own creation.*

In conclusion, there is indication that shifts in inflation expectations influence portfolio allocations to commodities, but these changes are not statistically significant. Thus, I confirm the hypothesis $H_{0.4}$, stating that relative changes in inflation expectations do not significantly impact portfolio weights of commodities within the constructed portfolio.

The empirical result section only covers the most necessary results found during the analysis. Hence, I refer to the Appendix for more detailed and elaborate insights on the findings. As mentioned earlier, this section delves into the main findings, presenting them in an appropriate manner. However, the discussion and interpretation of the findings can be found in the subsequent chapter.

6 Discussion

6.1 Mean-variance optimization

Whether an investor invests into commodities during severe times of market disruptions depends on the macroeconomic environment. During my analysis, I investigate four different types of investors: An investor who invests into the tangency, the GMV, the equally weighted and the Black-Litterman portfolio. This variety of portfolios incorporates a variation of interests. The tangency portfolio invests optimally based on a mean-variance optimization approach, whereas the GMV portfolio minimizes risk. The equally weighted portfolio distributes weights equally amongst assets, while the Black-Litterman approach bases their investments on the mean-variance approach, however, this portfolio further embraces the views and confidence of the investor.

To begin, I explore the different allocations to commodities across the portfolios. I found that the tangency portfolio only allocates funds towards commodities in 2016 (1.6%), 2021 (26.9%) and 2022 (100%). The reasons for these exceptions are so-called commodity rallies, where prices of commodities increase rapidly. In 2016, after five consecutive years of negative returns, commodities earnt positive returns and further outperformed equity and fixed-income assets. According to Morningstar (2017), reduced concerns about the future outlook of the Chinese economy, diminishing expectations of a rise in US interest rates, and encouraging signals from the supply and demand fronts were some of the reasons for the commodity rally. Moreover, the surge in commodity prices in 2021 was due to an increased manufacturing-based demand after the economic downturn in the wake of COVID-19, which led to the recovery of the global demand. Furthermore, also the supply side had its recovery on a global scale. Increasing energy and labor shortages, higher transportation costs and the weather led to increased prices in commodities. (United States International Trade Comission, 2021). The year 2022 was greatly affected by a significant event: the invasion of Ukraine by Russia. This event had a massive impact on commodity prices, particularly on oil, gas, steel, and grains. As if this was not enough, the subsequent imposition of sanctions by almost all countries further fueled the rise in commodity prices. Meanwhile, amidst the ongoing invasion, the United States held its midterm elections, which influenced the nation's energy policies due to increased Republican control in the White House. These, along with several other smaller events, contributed to the escalation in commodity prices. Consequently, the mean-variance approach of Markowitz (1952), which optimizes the risk-return tradeoff, led to an increased allocation towards commodities during these years for the tangency portfolio. Interestingly, between March 2021 and December 2022, the tangency portfolio invests in 12 out of 19 months into commodities, signaling the importance of commodities for the tradeoff in the mean-variance dimension.

On the other hand, the GMV portfolio consistently invests into commodities over the sample period with an annual average of 28.4%. Therefore, for the GMV portfolio I disagree with Conover et al. (2010) who claims that 15% allocated to commodities delivers the greatest reduction of portfolio risk. However, I am consistent with the findings of Gorton and Rouwenhorst (2006) and Bhardwaj et al. (2015) who find that commodities offer diversification benefits. On the other hand, I agree with Conover et al. (2010), stating that diversification benefits being successfully reached at a minimum allocation of 5% to commodities. Nevertheless, even though the tangency as well as the GMV portfolio invests into commodities, the years of when they invest differs. Not surprisingly, the GMV reacts more heavily to increased volatility indicated by a peak volatility of the VIX in 2020, whereas the tangency portfolio seems to be more sensitive towards inflation and interest rates. This can be seen especially in the year 2020, where the GMV portfolio invests 52.8% in commodities whereas the tangency allocates 0% towards commodities. Moreover, the mean-variance approach benefits from the correlation of the assets. Therefore, it seems that the correlations vary over time. Hence, the combination of risk, expected returns and correlation leads to different portfolio allocations.

The same pattern is visible for the Black-Litterman model which on average invests 7% annually in commodities over the sample period. By doing so, incorporating views and confidence intervals of investors reveals the preference of the investor during the sample period. An investor was more torn to commodities in 2016 and 2021, while in 2022 and 2023 the investor was more torn to the equity.

Overall, the correlation matrix in Table 3 reveals the correlations amongst the different asset types and the different components of inflation. Therefore, I am consistent with Fama (1981), Schwert (1981) and Eckmeier and Hofmann (2022) who reveal negative correlation between equity and inflation. On the other hand, equity and commodities are positively correlated in this thesis which aligns with Gorton and Rouwenhorst (2006) and Bhardwaj et al. (2015), while being inconsistent with Bhardway and Dunsby (2013). Hence, incorporating commodities in an all-equity portfolio does offer diversification benefits and seems to improve the performance of the portfolio according to the literature and my analysis. However, one needs to bear in mind that the constraints of no short selling and no leverage might influence the results and that loosening up these constraints might have an impact on the outcome.

6.2 Sharpe ratios improvement (1st Hypothesis)

Speaking of portfolio performance improvements, Table 5 reveals the true differences over the whole sample period on a monthly basis. Therefore, it is observable that the tangency portfolio improves the risk-return tradeoff by 0.362 per month, while the GMV and the equally weighted portfolio does not improve the tradeoff compared to the all-equity portfolio (-0.022 and -0.222). Therefore, for the tangency portfolio the null hypotheses $H_{0.0}: \Delta = 0$ and $H_{0.1}: \Delta = 0$ can be rejected. Simultaneously, for the GMV and the equally weighted portfolio the null hypotheses $H_{0.0}: \Delta = 0$ and $H_{0.1}: \Delta = 0$ hold true. Hence, I draw parallels with several scholars that are consistent to an improvement of the risk-return tradeoff, such as can for the tangency portfolio[7]. However, the GMV and equally weighted portfolios are consistent with Yan Garcia (2017) and Lean et al. (2023) stating that the inclusion of commodities in a portfolio does not always increase the Sharpe ratio of a portfolio. Hence, the interest of an investor matters when it comes to the portfolio weights. In particular, risk-aversion and target excess returns are crucial to the investor.

Regarding monthly differences, the analysis showed no significant improvement in the Sharpe ratios across all portfolios. There are several reasons regarding the insignificancy. One reason might be the cost of rebalancing monthly, which inherently decreases net returns. Additionally, tax considerations of realized gains after rebalancing might also play a crucial role. Therefore, allocating funds to commodities monthly may not be attractive as it decreases net returns, leaving the investor to remain 100% invested in equity. A third reason might also be the imposition of increasing interest rates on a monthly scale that led to lower or even negative excess returns. Lastly, De Roon et al. (2012) state that there is no free lunch, referring to decreasing portfolio risk comes with decreasing excess returns.

In conclusion, including commodities to an all-equity portfolio might not always increase the performance and hence the Sharpe ratio. However, over the whole sample period, the inclusion of commodities does significantly improve the performance on a monthly average for the tangency portfolio, whereas for monthly differences there is no significant improvement. Although, bearing in mind that rebalancing, tax considerations, interest rates and the backward-looking approach in the analysis may influence the outcome significantly. The GMV and the equally weighted portfolio do not improve the Sharpe ratios and hence the portfolio performance. Therefore, I agree with De Roon et al. (2012) that there is no free lunch in the US market, indicating that one cannot have higher returns while reducing risk.

6.3 Commodities as a hedge for inflation in 2021/2022 (2nd Hypothesis)

The analysis around hedging properties for the tangency and GMV portfolios disclose intriguing relationships. For instance, a 1% increase in inflation prompts the tangency portfolio to raise commodity weights by 1.444 percentage points, whereas the GMV portfolio reduces commodities by 2.687 percentage points. Interestingly, this relationship reverses during the high-inflation years of 2021 and 2022. In these years, the tangency portfolio reduces commodity weights by 3.942 percentage points for each unit increase in inflation, while the GMV portfolio increases commodities by 2.584 percentage points. Consequently, concerning the tangency portfolio, I generally align with literature[8] suggesting that commodities serve as a partially effective hedge against inflation, supported by the positive

[7] See for example Jensen et al. (2002), Wang et al. (2022), Gorton and Rouwenhorst (2006) and Bhardwaj et al. (2015).
[8] See for example Bodie (1983), Gorton and Rouwenhorst (2006) and Spierdijk and Umar (2014)

coefficient observed. Backing up literature, I further align with Podkaminer et al. (2021) indicating that sparingly allocating to commodities increases hedging against inflation. However, the consideration of rebalancing and tax effects may render it a costly hedge rather than an efficient one, consistent with the findings of Futerman and Sarjanovic (2022). Conversely, regarding the period of 2021 and 2022, my findings contradict those of Findsen (2023) and Futerman and Sarjanovic (2022), who argue that commodities serve as a short-term hedge but not in the long run. Conversely, focusing on the GMV portfolio, my findings disagree with scholars who assert that commodities are not a long-term hedge but are effective in the short term. This idea is further investigated and backed up by applying the Fisher equation, where the monthly real return of the GSCI over the whole sample period is -0.39%, while during 2021 and 2022, the index earns a positive real return of 0.42%, which is consistent with Spierdijk and Umar (2014). However, the combination of the S&P 500 and the GSCI earns a positive real return across all portfolios, indicating that equity appears to be a more suitable hedge against inflation according to the Fisher equation.

Understanding these variations is crucial for effective portfolio management. Hence, the differences in these relationships can be explained through three main factors. Firstly, the tangency portfolio predominantly invests in commodities for only three out of ten years, resulting in minimal rebalancing costs and tax implications. Conversely, the GMV portfolio requires monthly rebalancing, making hedging comparatively expensive. Therefore, the GMV invests less in commodities with regards to inflation while the tangency invests more. However, this is contradicted by the fact that commodity weights decrease for the tangency portfolio if liquidity of both indices increase. Whereas the GMV portfolio increases its weight as a response to greater liquidity, signaling the reaction to buy commodity futures if the rebalancing costs are lower. Secondly, the GMV portfolio allocates 52% to commodities in 2020, leading to already heightened initial weights for the event window of 2021, which subsequently decreased in 2022. However, increasing speculations about a potential recession in 2021 heightened volatility and uncertainty, prompting risk-minimizing investors (GMV) to increase their exposure to commodities. This can also be seen in the greater magnitude of the sentiment control variable, where the tangency portfolio reacts more sensitive to market dispositions. Thirdly, amidst the COVID-19 pandemic in 2020, the equity markets rebounded to peak levels in response to the downturn, leading to a subsequent correction in 2021 and 2022. Consequently, despite rising inflation during these years, the S&P 500 corrected the previous positive market overreaction of 2020. This dynamic could account for the inverse relationship between inflation and commodity weights observed in those years for the tangency portfolio.

In conclusion, my analysis reveals interesting insights of the tangency and GMV portfolios when it comes to the relationship between inflation and commodity weights. While the tangency portfolio appears to be consistent with some literature suggesting commodities as a partially effective hedge against inflation, considering rebalancing and tax effects may render it costly. Nevertheless, during the high-inflation periods of 2021 and 2022, the findings disagree with existing scholars. On the other hand, the GMV portfolio, aligns with scholars who view commodities as effective in the short term but not as an effective long-term hedge. Incorporating the potential influences of portfolio weights during high-inflationary times is crucial for an effective portfolio management. Especially, considering factors such as market dynamics, rebalancing costs and taxes. However, it is important to not just look at inflation as a single construct, rather, the components of expected and unexpected inflation should be investigated. However, bearing in mind the lack of statistical significance makes it hard to statistically infer to the general population.

6.4 Portfolio implications of unexpected inflation (3rd Hypothesis)

The unexpected component of the inflation influences portfolio weights differently as the expected components. For the tangency portfolio, I generally observe a negative relation between to unexpected inflation and commodity weights (-0.665), whereas for the GMV portfolio this relation is positively (0.613). However, looking at the high-inflationary window, the magnitudes of these relations increase substantially. For the tangency portfolio, every unit increase in unexpected inflation decreases portfolio weights allocated to commodities by 5.771 percentage points, while the GMV increase the weights by 5.801 percentage points. Interestingly and conversely, over the whole sample period the total realized inflation influences the portfolio weights positively for the tangency (1.444) and negatively for the GMV (-2.687), as mentioned earlier. The expected component moves in the same direction over the whole sample period as the unexpected component for both portfolios, however, throughout 2021 and 2022, the direction becomes positive for the tangency (6.922) and negative for the GMV portfolio (7.883).

Over the whole sample period, my results are consistent with Cieslak and Pfluger (2022) stating that investors are more concerned over the expected movements rather than the unexpected fluctuations, indicating higher coefficients for the expected inflation for both portfolios. Moreover, I align with the statement of Marotta (2022) revealing that commodities are more sensitive to high inflation, which can be seen in the increased magnitude for both the expected and unexpected components during 2021 and 2022. Surprisingly, even though the correlations of equity to both components of inflation are negative, the equity part in the portfolio seems to increase over the whole sample period if both inflation types increase. The reversed pattern can be seen for the GMV, aligning with Schwert (1981) and Fama (1981).

Specifically, there are two reasons for these patterns during 2021 and 2022: Uncertainty and expectations. An investor that invests into the tangency portfolio invests optimally based on the risk-return tradeoff. As a result, this tradeoff is optimized over the whole sample period as well as during high-inflationary times with less investments towards commodities. This can either be explained through higher risk and lower expected returns for commodities or vice versa, lower risk and higher expected returns for equity. The GMV investor only cares about minimizing risk. Therefore, unexpected inflation seems to increase uncertainty for equity to a greater extent. Thus, the GMV investor allocates more towards commodities if the expectations are not met and implicitly uncertainty rises. The reversed pattern can be seen for the expected inflation across portfolios. In addition, volatility seems to positively increase the weights of commodities for both the tangency and the GMV portfolio, which inherently increases uncertainty. Furthermore, the portfolio weights are also dependent on the magnitude of the inflation components. Unexpected inflation has been generally low over the whole sample period, compared to the increased magnitude throughout 2021 and 2022.

To conclude this section, the expected and unexpected inflation has the same influence over the sample period, whilst having a different influence during high-inflationary times. Moreover, the components influence the portfolios differently. However, due to lack of statistical significance, once again, I cannot draw any conclusions for the bigger picture. Hence, inference from the sample on to the population cannot be made here.

6.5 Relative changes in inflation expectations (4th Hypothesis)

What happens to the portfolio weights if there is a relative change in inflation expectations? Table 9 reveals interesting insights into this relation. The tangency portfolio increases its allocation towards commodities (2.047) for 1% increases inflation expectations, whereas the GMV portfolio decreases its allocations (-0.730). During the high-inflationary times, the patterns reverse. Hence, the tangency portfolio invests 11.276 percentage points less into commodities while the GMV increases the commodity part in the portfolio by 14.473 percentage points. Hence, my results are consistent with Feldstein (1980) stating that a change in inflation expectations alter the real value of an asset and thus its investments. Nevertheless, the increased and reversed magnitude during the high inflation in 2021 and 2022 can be explained according to Marotta (2022). If there is a relative change in expectations, increased uncertainty exists of another change in expectations. Therefore, the GMV investor invests more in commodities whereas the tangency invests less into commodities. On the other hand, changes in inflation expectations from t to t+1 might be associated with an increasing price of commodities. Hence, the tangency investor invests more since its optimally for the whole sample period.

Overall, changes in inflation expectations from t to t+1 reveal a different impact on both portfolios. While the tangency portfolio invests more in commodities during the sample period, the GMV portfolio invests less. This can be explained due to higher expected prices in commodities. On the other hand, during the high inflationary time, the pattern reverses and the GMV portfolio invests more into commodities. This can mainly be attributed due to a greater uncertainty of another change in expectations and hence greater volatility on the markets. Nevertheless, again these results must be applied with cautious as there is no statistical evidence to reject the null hypothesis $H_{0.4}$.

6.6 Limitations and further research

Even though the empirical findings shed insightful light on the relationship between commodities, equity and inflation, there are limitations to the methodology and the data. The data could be extended to more individual stocks rather than only two indices, implying more precise portfolio weights. Furthermore, the inflation continued to be rather above the central banks targeted inflation rate of 2% in 2023, thus, an extension of the event window could potentially influence the outcome. Moreover, implementing transaction costs for the sake of rebalancing or tax considerations could further sharpen the outcome. Additionally, the regression analysis could benefit from more control variables and other factors that might influence portfolio weights. However, as the portfolio weights are computed by the mean-variance approach, the analysis must be associated with an increasing estimation uncertainty as there might be higher moments for the calculation of portfolio weights. Conversely, the mean-variance approach is based on historical return data. Hence, if markets are efficient the results cannot be used as an indicator for future perspectives. Lastly, as most of the coefficient estimates do not show statistical significance, the results must be viewed at with cautious. Therefore, I cannot generalize the results to the whole population.

Further research should focus on different data, as inflation rates as well as commodity and equity returns differ from country to country. For example, the European inflation is different from the US inflation. Furthermore, research should also focus on a more forward-looking approach, where returns for the assets are estimated. For example, returns could be estimated according to the Fama and French five-factor asset pricing model (Fama & French, 2014). Nevertheless, I do believe that this study can potentially be insightful, even though there are limitations.

7 Conclusion

In this study I have investigated the optimal portfolio weights of a constructed portfolio consisting of the S&P 500 and the GSCI Commodity Index over the sample period of ten years, with a focus of the heightened inflation during 2021 and 2022. Previous research has shown that commodities or more specifically, commodity futures offer diversification benefits and hedging properties against inflation. Yet, no research scholar has conducted the high-inflationary time in the wake of the COVID-19 pandemic (2021) and throughout the eruption of the war between Russia and Ukraine (2022). Therefore, the research question that was studied in this thesis was: "*How can equity portfolio weights be strategically optimized by adding commodity futures to hedge effectively against inflation during high-inflationary times?".*

To answer this research question, I investigated the mentioned indices above. With these indices I perform the mean-variance optimization approach that is found by Markowitz (1952). By doing so, I found the tangency, the global-minimum variance and equally weighted portfolios. Furthermore, to complement the optimization undertaking, I chose four individual stocks that represent the S&P 500 to perform the Black-Litterman optimization (Black & Litterman, 1992). Subsequently, I explored the relation of the weights of commodities in the portfolios and the inflation. In particular, I focused on the general inflation, and split it up into the unexpected and expected component. Minimizing the risk of the portfolio, the GMV continuously invests into commodities each year, while the tangency portfolio only invests into commodities in 2016, 2021 and 2022. Like the GMV, the Black-Litterman portfolio allocates consistently funds towards commodities. Interestingly, only the tangency portfolio improves the Sharpe ratio of the all-equity portfolio if commodities are added. Furthermore, the weights of commodities in the portfolio increase if inflation increases for the tangency portfolio, whereas for the GMV portfolio, the weights decrease. This represents the whole sample period. During 2021 and 2022, the pattern reverses. In addition, the tangency portfolio reacts negatively when investing in commodities for an increase in unexpected inflation, while the GMV portfolio reacts positively. The magnitude becomes even greater when the unexpected inflation increases during 2021 and 2022. Moreover, a change in inflation expectations from t to t+1 led to an increase in portfolio weights for commodities in the tangency portfolio. On the other hand, the GMV portfolio decreased its allocation to commodities.

Hence, although the literature shows that commodities offer diversification benefits and hedging properties, I conclude that this is somewhat only occasionally the case. If an investor invests optimally, he/she can improve the risk-return tradeoff of an all-equity portfolio by adding commodities. On the other hand, if an investor aims to minimize the portfolio risk, he/she also encounters a decrease in expected returns. Combined with previous studies, I find that investing in commodities does not offer free lunch. An improved portfolio performance only comes with more risk. Hence, only in the optimal case the portfolio performance seems to improve. But its portfolio weights alter throughout the high-inflationary time. However, lacking statistical significance, I fail to derive a conclusion that can be applied in the general population.

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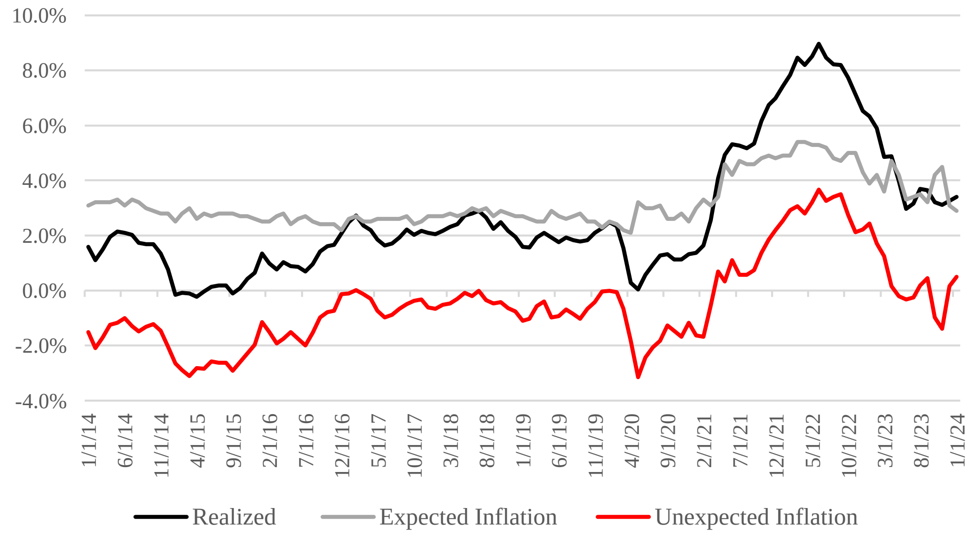
II Appendix

This section offers an elaborate understanding of the data used, the methodology applied, and the empirical results found. The section consists of inflation rate, 3-month treasury bills, monthly trading liquidity, sentiment and volatility, the results of the robustness checks, the efficient frontier over the whole sample period, tables for monthly portfolio allocation for each year reaching from 2014 to 2024, and the computations of the real returns. Moreover, the relevant notes can be found below the figures and tables.

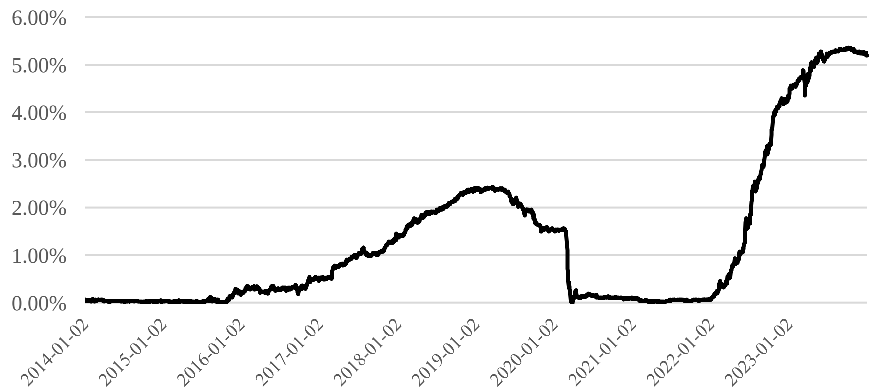
Appendix A: Data

Figure 7: Inflation and its components



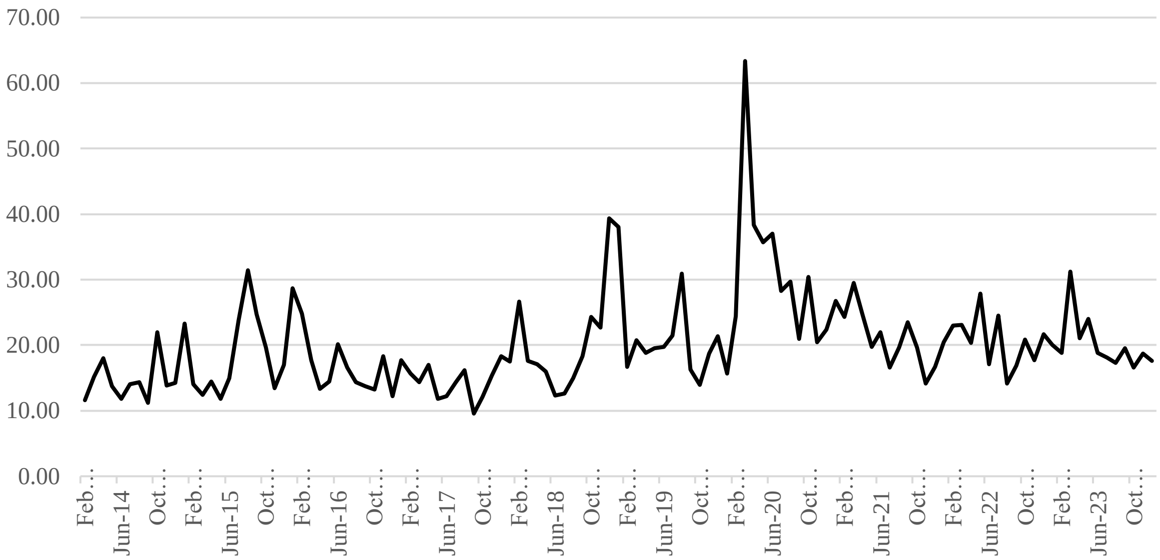
Notes: This figure depicts the realized inflation over the sample period. Furthermore, the expected inflation is represented by inflation forecasts. The unexpected inflation is the difference of the realized inflation minus the expected inflation. The inflation is measured by the Consumer Price Index. Source: Own creation.

Figure 8: 3-month US Treasury Bill



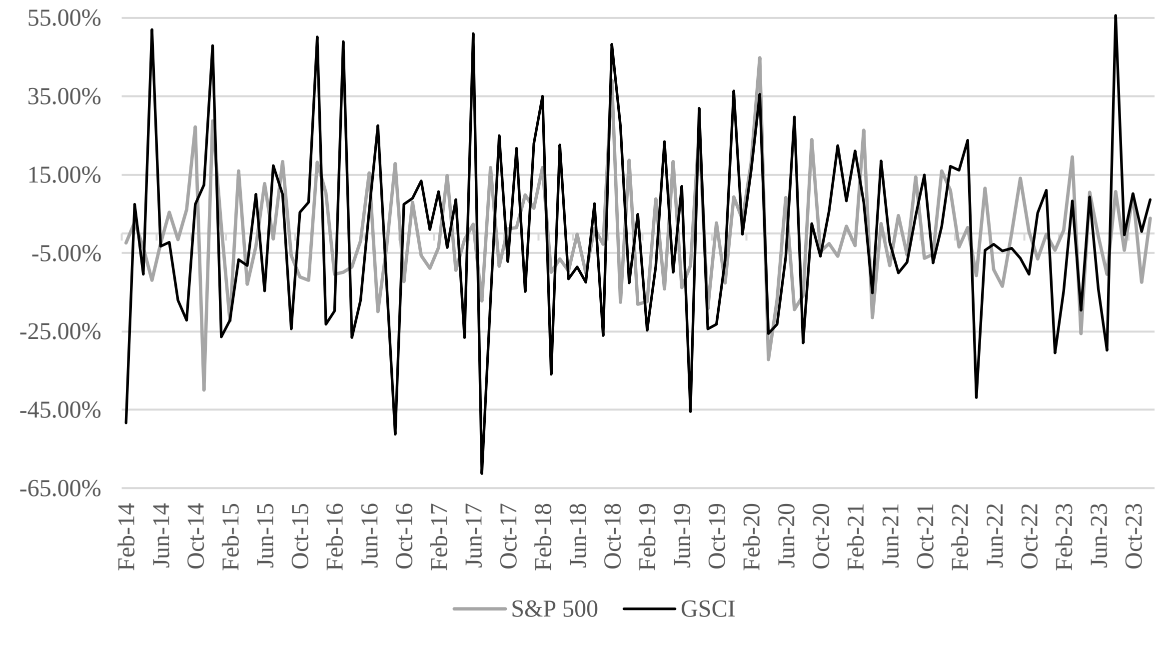
Notes: This figure represent the risk-free rate that is measured by the 3-month Treasury Bill on the secondary market in the US. Source: Own creation.

Figure 9: VIX



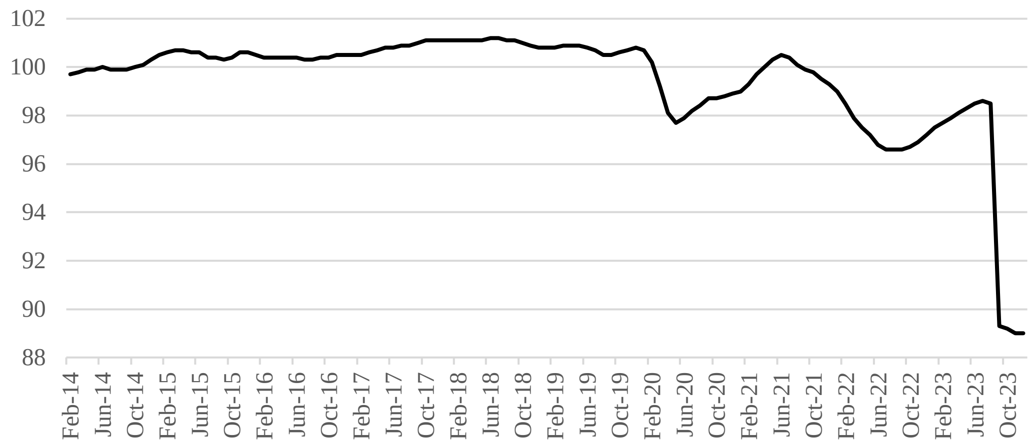
Notes: This graph depicts the monthly volatility measured by the VIX. Source: Own creation.

Figure 10: Relative changes in trading volume of the S&P 500 and GSCI



Notes: This graph depicts the relative changes of monthly trading volume for the S&P 500 and GSCI. Hence, the trading volume indicates the liquidity of the indices. Source: Own creation.

Figure 11: Market sentiment



Notes: This figure shows the market sentiment over the sample period measured by the US Consumer Confidence Index (CCI). Source: Own creation.

Appendix B: Methodology

This section serves as a more comprehensive part for the methodology. In particular, I delve into the robustness of the regression. Thereby, as already mentioned earlier I test for heteroskedasticity, autocorrelation amongst errors and normality of the distribution. Thus, the checks can be found below. With regards to testing homoskedasticity of all three regressions, I conclude that the regression to test the 2nd hypothesis (Eq. 21) rejects the concept of homoskedasticity on a 10% confidence level with a p-value of 0.089. Furthermore, to test the 3rd hypothesis (Eq. 24) I also reject homoskedasticity on the 10% confidence level with a p-value of 0.011. Finally, the 4th hypothesis and its regression (Eq. 26) also rejects homoskedasticity in favor of heteroskedasticity on the 10% confidence level with a p-value of 0.007. Hence, I apply robust standard errors that account for heteroskedasticity. I do so by applying the Breusch-Pagan test. In contrast, the White test tells that all three regressions are subject to homoskedasticity. Hence, due to uncertainty in the rejection of homoskedasticity, I apply robust standard errors.

In addition, I employ the Shapiro-Wilk test to assess normality in the data. I find that all regressions fail to confirm normality, hence, I reject the assumption of normality with p-values of 2.685e-06 (Eq. 21), 2.942e-06 (Eq. 24) and 3.683e-06 (Eq. 26). Moreover, I assess the assumption of no error correlation by employing the Durbin-Watson test. I find that there is no autocorrelation amongst residuals. More particularly, I do not reject the null hypotheses of autocorrelation for all regressions with a p-value of 0.395 (Eq. 21), 0.349 (Eq. 24) and 0.418 (Eq. 26).

Appendix C: Results

This section further delves into the monthly portfolio weights. I perform portfolio optimization for each month over the sample period. Therefore, each table represent the tangency, the GMV and the equally weighted portfolio as well as their relevant Sharpe ratios.

Table 10: Monthly portfolio allocation in 2014

2014	Tangency		GMV		Sharpe Ratios		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	0%	100%	28%	72%	0.031	-0.051	-0.108
February	23%	77%	25%	75%	0.463	0.463	0.428
March	99%	1%	54%	46%	0.058	0.043	0.038
April	24%	76%	35%	65%	0.077	0.075	0.069
May	100%	0%	34%	66%	0.212	0.059	0.117
June	64%	36%	66%	34%	0.395	0.395	0.370
July	100%	0%	41%	59%	-0.097	-0.463	-0.411
August	100%	0%	67%	33%	0.376	0.242	0.123
September	100%	0%	72%	28%	-0.114	-0.246	-0.322
October	100%	0%	40%	60%	0.088	-0.140	-0.096
November	100%	0%	100%	0%	0.526	0.526	-0.267
December	100%	0%	72%	28%	-0.014	-0.235	-0.373

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.

Table 11: Monthly portfolio allocation in 2015

2015	Tangency		GMV		Sharpe Ratios		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	100%	0%	78%	22%	-0.144	-0.208	-0.244
February	100%	0%	100%	0%	0.487	0.487	0.275
March	100%	0%	84%	16%	-0.082	-0.129	-0.188
April	36%	64%	87%	13%	0.365	0.205	0.361
May	100%	0%	77%	23%	0.093	0.040	-0.034
June	0%	100%	86%	14%	-0.011	-0.120	-0.068
July	100%	0%	85%	15%	0.136	-0.022	-0.356
August	0%	100%	66%	34%	0.005	-0.126	-0.092
September	100%	0%	76%	24%	-0.087	-0.118	-0.147
October	100%	0%	81%	19%	0.469	0.399	0.215
November	100%	0%	82%	18%	0.016	-0.102	-0.293
December	100%	0%	65%	35%	-0.079	-0.1969	-0.238

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.

Table 12: Monthly portfolio allocation in 2016

2016	Tangency		GMV		Sharpe Ratios		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	0%	100%	86%	14%	-0.129	-0.149	-0.150
February	100%	0%	94%	6%	-0.017	-0.022	-0.051
March	100%	0%	94%	6%	0.399	0.392	0.256
April	0%	100%	92%	8%	0.322	0.065	0.269
May	95%	5%	85%	15%	0.097	0.097	0.072

June	0%	100%	76%	24%	0.011	0.005	0.008
July	100%	0%	100%	0%	0.350	0.350	-0.213
August	0%	100%	100%	0%	0.054	-0.028	0.041
September	0%	100%	80%	20%	0.136	0.027	0.086
October	0%	100%	94%	6%	-0.104	-0.237	-0.171
November	100%	0%	94%	6%	0.227	0.224	0.126
December	34%	66%	71%	29%	0.296	0.261	0.294

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.

Table 13: Monthly portfolio allocation in 2017

2017	**Tangency**		**GMV**		**Sharpe Ratios**		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	100%	0%	90%	10%	0.070	0.059	0.005
February	100%	0%	89%	11%	0.616	0.559	0.206
March	100%	0%	88%	12%	-0.059	-0.103	-0.205
April	100%	0%	78%	22%	0.025	-0.070	-0.169
May	100%	0%	86%	14%	0.040	0.007	-0.062
June	100%	0%	83%	17%	-0.043	-0.091	-0.127
July	71%	29%	86%	14%	0.218	0.194	0.200
August	100%	0%	81%	19%	-0.064	-0.089	-0.102
September	68%	32%	80%	20%	0.203	0.193	0.189
October	78%	22%	100%	0%	0.187	0.166	0.173
November	100%	0%	80%	20%	0.201	0.181	0.089
December	0%	100%	87%	13%	0.141	0.035	0.124

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.

Table 14: Monthly portfolio allocation in 2018

2018	**Tangency**		**GMV**		**Sharpe Ratios**		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	100%	0%	70%	30%	0.306	0.277	0.238
February	100%	0%	33%	67%	-0.155	-0.254	-0.241
March	0%	100%	26%	74%	0.029	-0.049	-0.113
April	0%	100%	40%	60%	0.128	0.056	0.035
May	100%	0%	81%	19%	0.025	0.013	-0.005
June	0%	100%	84%	16%	-0.032	-0.125	-0.080
July	100%	0%	99%	1%	0.142	0.138	-0.131
August	100%	0%	100%	0%	0.080	0.080	-0.017
September	0%	100%	84%	16%	0.126	-0.147	0.019
October	100%	0%	35%	65%	-0.303	-0.382	-0.379
November	100%	0%	64%	36%	-0.025	-0.259	-0.338
December	100%	0%	39%	61%	-0.336	-0.376	-0.379

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.

Table 15: Monthly portfolio allocation in 2019

2019	Tangency		GMV		Sharpe Ratios		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	47%	53%	44%	56%	0.272	0.272	0.271
February	81%	19%	89%	11%	0.100	0.099	0.090
March	100%	0%	39%	61%	-0.047	-0.081	-0.077
April	100%	0%	94%	6%	0.185	0.177	0.084
May	100%	0%	88%	12%	-0.450	-0.469	-0.463
June	100%	0%	84%	16%	0.342	0.326	0.216
July	100%	0%	88%	12%	-0.069	-0.091	-0.123
August	100%	0%	38%	62%	-0.119	-0.224	-0.206
September	100%	0%	94%	6%	0.004	0.001	-0.008
October	100%	0%	43%	57%	0.016	-0.033	-0.026
November	100%	0%	100%	0%	0.284	0.284	0.034
December	0%	100%	71%	29%	0.369	0.241	0.306

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.

Table 16: Monthly portfolio allocation in 2020

2020	Tangency		GMV		Sharpe Ratios		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	100%	0%	67%	33%	-0.164	-0.447	-0.562
February	100%	0%	32%	68%	-0.320	-0.372	-0.368
March	100%	0%	1%	99%	-0.109	-0.399	-0.250
April	100%	0%	75%	25%	0.223	0.134	0.023
May	0%	100%	85%	15%	0.379	0.216	0.319
June	0%	100%	28%	72%	0.140	0.114	0.091
July	100%	0%	72%	28%	0.309	0.284	0.253
August	100%	0%	84%	16%	0.615	0.595	0.477
September	100%	0%	45%	55%	-0.121	-0.163	-0.161
October	100%	0%	58%	42%	-0.093	-0.122	-0.124
November	49%	51%	55%	45%	0.628	0.626	0.628
December	55%	45%	69%	31%	0.419	0.406	0.417

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.

Table 17: Monthly portfolio allocation in 2021

2021	Tangency		GMV		Sharpe Ratios		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	0%	100%	61%	39%	0.239	0.138	0.170
February	0%	100%	63%	37%	0.495	0.340	0.393
March	100%	0%	87%	13%	0.187	0.156	0.044
April	54%	46%	60%	40%	0.640	0.632	0.638
May	36%	64%	62%	38%	0.110	0.094	0.106

June	58%	42%	78%	22%	0.248	0.233	0.246
July	100%	0%	100%	0%	0.157	0.157	0.089
August	100%	0%	100%	0%	0.250	0.250	0.011
September	0%	100%	80%	20%	0.256	-0.166	0.027
October	78%	22%	73%	27%	0.501	0.499	0.444
November	100%	0%	100%	0%	-0.050	-0.050	-0.231
December	0%	100%	96%	4%	0.252	0.194	0.233

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.

Table 18: Monthly portfolio allocation in 2022

2022	**Tangency**		**GMV**		**Sharpe Ratios**		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	0%	100%	30%	70%	0.634	0.371	0.140
February	0%	100%	45%	55%	0.349	0.210	0.166
March	74%	26%	81%	19%	0.160	0.154	0.127
April	0%	100%	57%	43%	0.104	-0.160	-0.120
May	0%	100%	55%	45%	0.104	0.043	0.050
June	100%	0%	39%	61%	-0.258	-0.378	-0.373
July	100%	0%	91%	9%	0.274	0.243	0.059
August	100%	0%	62%	38%	-0.256	-0.305	-0.290
September	100%	0%	67%	33%	-0.407	-0.423	-0.420
October	67%	33%	36%	64%	0.112	0.102	0.109
November	100%	0%	46%	54%	0.029	-0.084	-0.074
December	0%	100%	81%	19%	-0.205	-0.409	-0.337

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.

Table 19: Monthly portfolio allocation in 2023

2023	**Tangency**		**GMV**		**Sharpe Ratios**		
Date	SPY	GSG	SPY	GSG	Tangency	GMV	1/N
January	100%	0%	70%	30%	0.110	0.063	0.024
February	0%	100%	63%	37%	-0.358	-0.509	-0.507
March	100%	0%	62%	38%	-0.066	-0.153	-0.173
April	100%	0%	83%	17%	-0.214	-0.255	-0.283
May	100%	0%	94%	6%	-0.293	-0.318	-0.398
June	100%	0%	86%	14%	0.063	0.045	-0.007
July	0%	100%	79%	21%	0.310	-0.060	0.142
August	0%	100%	58%	42%	-0.281	-0.455	-0.441
September	0%	100%	50%	50%	-0.119	-0.514	-0.513
October	100%	0%	68%	32%	-0.415	-0.628	-0.573
November	100%	0%	85%	15%	0.218	0.089	-0.193
December	100%	0%	100%	0%	-0.064	-0.064	-0.281

Notes: This table presents the monthly portfolio weights of the tangency and the GMV portfolio. In addition, the monthly Sharpe ratios for the tangency, the GMV and the equally weighted (1/N) portfolio can be found. Source: Own creation.